# On Radio Mean D-Distance Number of Graph Obtained from Graph Operation

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***Abstract***

*A Radio Mean D-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G)$, where $d^D(u, v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio mean D-distance number of f, $rmn^D(f)$ is the maximum label assigned to any vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the minimum value of $rmn^D(f)$ taken over all radio mean D-distance labeling f of G. In this paper we find the radio mean D-distance number of graph obtained from graph operation.*

**Keywords -** *D-distance, radio D-distance coloring, radio D-distance number, radio mean D-distance, radio mean D-distance number.*

**AMS Subject Classification.** 05C78.

## I. INTRODUCTION

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling (multi-level distance labeling) can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al. [2] introduced the concept of radio labeling of graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu and Zhu [10]. However Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O.Togni [8]. M.M.Rivera et al. [21] gave the radio number of $C_n \times C_n$, the cartesian product of $C_n$. In [4] C.Fernandez et al. found the radio number for complete graph, star graph, complete bipartite graph, wheel graph and gear graph. M.T.Rahim and I.Tomescu [17] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The concept of D-distance was introduced by D. Reddy Babu et al. [18, 19, 20].If u, v are vertices of a connected graph *G,* the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices *w* of *s* and $\ell(s)$ is the length of the path. The *D*-distance, $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*.

In [12], we introduced the concept of Radio D-distance. The radio D-distance coloring is a function f : $V(G) \to \mathbb{N} \cup \{0\}$ such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$. It is denoted by $rn^D(G)$. A radio D-distance coloring $f$ of G is a minimum radio D-distance coloring if $rn^D(f) = rn^D(G)$, where $rn^D(G)$ is called radio D-distance number.

Radio mean labeling was introduced by R. Ponraj et al [14, 15, 16]. A radio mean labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G). \quad (1.1)$$

for every u, v ∈V(G). The span of a labeling $f$ is the maximum integer that f maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. The condition (1.1) is called radio mean condition.

In [13], we introduce the concept of radio mean D-distance number. A radio mean D-distance labeling is a one to one mapping $f$ from V(G) to $\mathbb{N}$ satisfying the condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G). \quad (1.2)$$

for every u, v ∈V(G). The span of a labeling $f$ is the maximum integer that $f$ maps to a vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the lowest span taken over all radio mean D-distance labelings of the graph G. The condition (1.2) is called radio mean D-distance condition. In this paper we determine the radio mean D-distance number of graph obtained from graph operation. The function f : $V(G) \to \mathbb{N}$ always represents injective map unless otherwise stated.

## II. MAIN RESULT

**Theorem 2.1.** The radio mean D-distance number of a $K_2 + mK_1$, $rmn^D(K_2 + mK_1) = m + 2$, $m \geq 1$

Proof.

It is obvious that $diam^D(K_2 + mK_1) = 2m + 3$. Let $V(K_2 + mK_1) = \{x_j / j = 1, 2\} \cup \{v_i / i = 1, 2, \ldots, m\}$ and E = $\{x_jx_{j+1}, x_jv_i, x_{j+1}v_i / i = 1, 2, 3, \ldots, m \text{ and } j = 1\}$. Since $(K_2 + mK_1)$ has m + 2 vertices it requires m + 2 labels. The D-distance is $d^D(x_1, x_2) = 2m + 3$, $f(x_1) = 1$ then the label 0 is forbidden.

$rmn^D(K_2 + mK_1) \geq 0 + m + 2$

$\geq m + 2$

Now we shall give the following label to set the equality. Let $f(v_i) = i + 2$, $1 \leq i \leq m$, $f(x_j) = j$, j = 1, 2. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(K_2 + mK_1) + 1 = 2m + 4$, for every pair of vertices (u, v) where $u \neq v$.

For $(x_1, x_2)$, $d^D(x_1, x_2) + \left\lceil \frac{f(x_1)+f(x_2)}{2} \right\rceil \geq 2m + 3 + \left\lceil \frac{1+2}{2} \right\rceil \geq 2m + 4$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq m + 4 + \left\lceil \frac{i+2+j+2}{2} \right\rceil \geq 2m + 4$.

For $(v_i, x_j)$, $d^D(v_i, x_j) + \left\lceil \frac{f(v_i)+f(x_j)}{2} \right\rceil \geq m + 7 + \left\lceil \frac{i+2+j}{2} \right\rceil \geq 2m + 4$.

Therefore, $f(v_m) = m + 2$ is the largest label.

Hence, $rmn^D(K_2 + mK_1) = m + 2$, $m \geq 1$. ■

- ❖ The *crown* $(C_n \odot K_1)$ is obtained by joining a pendant edge to each vertex of $C_n$.

**Theorem 2.2.** The radio mean D-distance number of a crown graph,

$$rmn^D(C_n \odot K_1) = \begin{cases} 6\left(\frac{n-1}{2}\right) + 3 \text{ if } n \text{ is odd } n \geq 3. \\ 6\left(\frac{n}{2}\right) + 2 \text{ if } n \text{ is even } n \geq 4. \end{cases}$$

Proof.

It is obvious that $diam^D(C_n \odot K_1) = 4\left(\frac{n-1}{2}\right) + 7$ (n is odd) and $diam^D(C_n \odot K_1) = 4\left(\frac{n}{2}\right) + 7$ ($n$ is even). Let $V(C_n \odot K_1) = \{v_i, u_j / i, j = 1, 2, 3, \ldots, n\}$ and $E = \{v_iv_j, v_iu_i / i, j = 1, 2, 3, \ldots, n\}$.

n is odd, Since ($C_n \odot K_1$) has $4\left(\frac{n-1}{2}\right)+2$ vertices it requires $4\left(\frac{n-1}{2}\right)+2$ labels. The D-distance is $d^D(u_i, v_j) = 5$ (i = j), $f(u_i) = 2\left(\frac{n-1}{2}\right)+1+i$ then the label $2\left(\frac{n-1}{2}\right)+1$ is forbidden.

$rmn^D(C_n \odot K_1) \geq 2\left(\frac{n-1}{2}\right)+1+4\left(\frac{n-1}{2}\right)+2$

$\geq 6\left(\frac{n-1}{2}\right)+3$

Now we shall give the following label to set the equality. Let $f(u_i) = 2\left(\frac{n-1}{2}\right)+i+1$, $f(v_i) = 6\left(\frac{n-1}{2}\right)+4-i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(C_n \odot K_1) + 1 = 4\left(\frac{n-1}{2}\right)+8$, for every pair of vertices (u, v) where u ≠ v.

For $u_i$ and $u_j$ where $|i-j| \geq \left\lceil\frac{n}{2}\right\rceil, j > \left\lceil\frac{n}{2}\right\rceil$,

$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+7 + \left\lceil\frac{2\left(\frac{n-1}{2}\right)+i+1+2\left(\frac{n-1}{2}\right)+j+1}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+8.$

For $u_i$ and $u_j$ where $|i-j| \leq \left\lceil\frac{n}{2}\right\rceil, j \leq \left\lceil\frac{n}{2}\right\rceil$,

$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+3 + \left\lceil\frac{2\left(\frac{n-1}{2}\right)+i+1+2\left(\frac{n-1}{2}\right)+j+1}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+8.$

For $v_i$ and $v_j$ where $|i-j| \geq \left\lceil\frac{n}{2}\right\rceil, j > \left\lceil\frac{n}{2}\right\rceil$,

$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+3 + \left\lceil\frac{6\left(\frac{n-1}{2}\right)+4-i+6\left(\frac{n-1}{2}\right)+4-j}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+8.$

For $v_i$ and $v_j$ where $|i-j| \leq \left\lceil\frac{n}{2}\right\rceil, j \leq \left\lceil\frac{n}{2}\right\rceil$,

$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)-1 + \left\lceil\frac{6\left(\frac{n-1}{2}\right)+4-i+6\left(\frac{n-1}{2}\right)+4-j}{2}\right\rceil \geq 4\left(\frac{n-1}{2}\right)+8.$

Therefore, $f(v_n) = 6\left(\frac{n-1}{2}\right)+3$ is the largest label.

n is even, Since ($C_n \odot K_1$) has $4\left(\frac{n}{2}\right)$ vertices it requires $4\left(\frac{n}{2}\right)$ labels. The D-distance is $d^D(u_i, v_j) = 5$ (i = j), $f(u_i) = 2\left(\frac{n}{2}\right)+2+i$ then the label $2\left(\frac{n}{2}\right)+2$ is forbidden.

$rmn^D(C_n \odot K_1) \geq 2\left(\frac{n}{2}\right)+2+4\left(\frac{n}{2}\right)$

$\geq 6\left(\frac{n}{2}\right)+2$

Now we shall give the following label to set the equality. Let $f(u_i) = 2\left(\frac{n}{2}\right)+i+2$, $f(v_i) = 6\left(\frac{n}{2}\right)+3-i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(C_n \odot K_1) + 1 = 4\left(\frac{n}{2}\right)+8$, for every pair of vertices (u, v) where u ≠ v.

For $u_i$ and $u_j$ where $|i-j| \geq \left\lceil\frac{n}{2}\right\rceil, j > \left\lceil\frac{n}{2}\right\rceil$,

$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+7 + \left\lceil\frac{2\left(\frac{n}{2}\right)+i+1+2\left(\frac{n}{2}\right)+j+1}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+8.$

For $u_i$ and $u_j$ where $|i-j| \leq \left\lceil\frac{n}{2}\right\rceil, j \leq \left\lceil\frac{n}{2}\right\rceil$,

$d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+3 + \left\lceil\frac{2\left(\frac{n}{2}\right)+i+1+2\left(\frac{n}{2}\right)+j+1}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+8.$

For $v_i$ and $v_j$ where $|i-j| \geq \left\lceil\frac{n}{2}\right\rceil, j > \left\lceil\frac{n}{2}\right\rceil$,

$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+3 + \left\lceil\frac{6\left(\frac{n}{2}\right)+4-i+6\left(\frac{n}{2}\right)+4-j}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+8.$

For $v_i$ and $v_j$ where $|i-j| \leq \left\lceil\frac{n}{2}\right\rceil, j \leq \left\lceil\frac{n}{2}\right\rceil$,

$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)-1 + \left\lceil\frac{6\left(\frac{n}{2}\right)+4-i+6\left(\frac{n}{2}\right)+4-j}{2}\right\rceil \geq 4\left(\frac{n}{2}\right)+8.$

Therefore, $f(v_n) = 6\left(\frac{n}{2}\right)+2$ is the largest label.

$$rmn^D(C_n \odot K_1) = \begin{cases} 6\left(\frac{n}{2}\right) + 2 \text{ if } n \text{ is even } n \geq 4. \\ 6\left(\frac{n-1}{2}\right) + 3 \text{ if } n \text{ is odd } n \geq 3. \end{cases}$$ ■

**Theorem 2.3.** The radio mean D-distance number of a Ladder graph, $rmn^D(L_n) = 5n - 5$ if $n \geq 3$.

**Proof.**

It is obvious that $diam^D(L_n) = 4n$. Let $V(L_n) = \{v_1, v_2, v_3, \ldots, v_{2n}\}$ and $E = \{v_iv_j / i, j = 1, 2, 3, \ldots, n\}$. Since $L_n$ has 2n vertices it requires 2n labels. The D-distance is 9, $f(v_3) = 3n - 4$ then the label $3n - 5$ is forbidden.

$rmn^D(L_n) \geq 2n + 3n - 5$

$\geq 5n - 5$

Now we shall give the following label to set the equality. Let $f(v_1) = 4n - 5$, $f(v_{2i-1}) = 4n - 7 + i$, $2 \leq i \leq n - 1$, $f(v_{2n+2-2i}) = 4n - 5 + i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(L_n) + 1 = 4n + 1$, for every pair of vertices (u, v) where $u \neq v$.

If $v_i$ and $v_j$ are adjacent,

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 5 + \left\lceil \frac{4n-5+4n-5+i}{2} \right\rceil \geq 4n + 1.$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 6 + \left\lceil \frac{4n-5+4n-7+i}{2} \right\rceil \geq 4n + 1.$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{4n-5+i+4n-5+j}{2} \right\rceil \geq 4n + 1.$

If $v_i$ and $v_j$ are not adjacent,

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 9 + \left\lceil \frac{4n-5+i+4n-5+j}{2} \right\rceil \geq 4n + 1.$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 4n + \left\lceil \frac{4n-7+i+4n-5+j}{2} \right\rceil \geq 4n + 1.$

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 4n - 3 + \left\lceil \frac{4n-5+i+4n-7+i}{2} \right\rceil \geq 4n + 1.$

Therefore, $f(v_2) = 5n - 5$ is the largest label.

Hence, $rmn^D(L_n) = 5n - 5$ if $n \geq 3$. ■

- Bistar $B_{n,n}$ is the graph obtained by joining the center(apex) vertices of two copies of $K_{1,n}$ by an edge.

**Theorem 2.4.** The radio mean D-distance number of a bistar, $rmn^D(B(n, n)) = 3(n + 1)$ if $n \geq 2$.

**Proof.**

It is obvious that $diam^D(B(n, n)) = 2n + 7$. Let $V(B(n, n)) = \{x_j / j = 1, 2\} \cup \{v_i, u_i / i = 1, 2, \ldots, n\}$ and $E = \{x_jx_{j+1}, x_jv_i, x_{j+1}u_i / i = 1, 2, 3, \ldots, n$ and $j = 1\}$. Since B(n, n) has 2n + 2 vertices it requires 2n + 2 labels. The D-distance is $d^D(v_i, v_j) = n + 5$, $f(v_i) = n + 2(d^D(u_i, u_j) = n + 5$, $f(u_i) = n + 2)$ then the label n + 1 is forbidden.

$rmn^D B(n, n) \geq n + 1 + 2n + 2$

$\geq 3n + 3$

Now we shall give the following label to set the equality. Let $f(u_i) = n + i + 1$, $f(v_i) = 2n + i + 1$, $1 \leq i \leq n$, $f(x_j) = 3n + 4 - j$, $j = 1, 2$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(B(n, n)) + 1 = 2n + 8$, for every pair of vertices (u, v) where $u \neq v$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq n + 5 + \left\lceil \frac{n+i+1+n+j+1}{2} \right\rceil \geq 2n + 8.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 5 + \left\lceil \frac{2n+1+i+2n+1+j}{2} \right\rceil \geq 2n + 8.$

For $(v_i, x_1)$, $d^D(v_i, x_1) + \left\lceil \frac{f(v_i)+f(x_1)}{2} \right\rceil \geq n + 3 + \left\lceil \frac{2n+1+i+3n+3}{2} \right\rceil \geq 2n + 8.$

For $(u_i, x_1)$, $d^D(u_i, x_1) + \left\lceil \frac{f(u_i)+f(x_1)}{2} \right\rceil \geq n + 3 + \left\lceil \frac{n+i+1+3n+3}{2} \right\rceil \geq 2n + 8.$

For $(v_i, x_2)$, $d^D(v_i, x_2) + \left\lceil \frac{f(v_i)+f(x_2)}{2} \right\rceil \geq 2n + 5 + \left\lceil \frac{2n+1+i+3n+2}{2} \right\rceil \geq 2n + 8.$

For $(u_i, x_2)$, $d^D(u_i, x_2) + \left\lceil \frac{f(u_i)+f(x_2)}{2} \right\rceil \geq 2n + 5 + \left\lceil \frac{n+i+1+3n+2}{2} \right\rceil \geq 2n + 8.$

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2n + 7 + \left\lceil \frac{n+i+1+2n+1+j}{2} \right\rceil \geq 2n + 8.$

Therefore, $f(x_1) = 3(n + 1)$ is the largest label.

Hence, $rmn^D B(n,n) = 3(n+1)$ if $n \geq 2$. ■

- A vertex switching $G_v$ of a graph G is the graph obtained by taking a vertex v of G, removing all the edges to v and adding edges joining v to every other vertex which are not adjacent to v in G.

**Theorem 2.5.** The radio mean D-distance number of a switching cycle,

$$rmn^D(SC_n) = \begin{cases} 9, & n = 5. \\ 13, & n = 6. \\ 14, & n = 7. \\ 3\left(\frac{n}{2}\right) + 3 & if\ nis\ even\ n \geq 8 \\ 3\left(\frac{n-1}{2}\right) + 4 & if\ nis\ odd\ n\ \geq 9 \end{cases}$$

Proof.

It is obvious that $diam^D(SC_n) = 2\left(\frac{n}{2}\right) + 9$( n is even) and $diam^D(SC_n) = 2\left(\frac{n-1}{2}\right) + 10$(n is odd). Let $V(SC_n)= \{v_i / i = 1, 2, \ldots, n\}$ and $E = \{ v_iv_j /i, j = 1, 2, 3, \ldots, n\}$.

n is even, Since $(SC_n)$ has n vertices it requires n labels. The D-distance is $d^D(v_{i+1}, v_n) = 2\left(\frac{n}{2}\right) + 1$, $f(v_n) = \left(\frac{n}{2}\right) + 4$ then the label $\left(\frac{n}{2}\right) + 3$ is forbidden.

$$rmn^D(SC_n) \geq n + \left(\frac{n}{2}\right) + 3$$
$$\geq 3\left(\frac{n}{2}\right) + 3$$

Now we shall give the following label to set the equality.

Let

$f(v_2) = x_{\frac{n}{2}}$

$f(v_{2i+2}) = x_{i+1},\ 1 \leq i \leq \frac{n}{2} - 2$

$f(v_{2i+1}) = x_{2\left(\frac{n}{2}\right)-1-i}\ \ 0 \leq i \leq \frac{n}{2} - 2$

$f(v_{n-1}) = x_{2\left(\frac{n}{2}\right)}$

$f(v_n) = x_1$

$f(x_i) = \frac{n}{2} + 3 + i,\ 1 \leq i \leq 2\left(\frac{n}{2}\right)$.

We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(SC_n) + 1 = 2\left(\frac{n}{2}\right) + 10$, for every pair of vertices (u, v) where $u \neq v$.

If $v_i$ and $v_j$ are adjacent

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 5 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 7 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 5 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

If $v_i$ and $v_j$ are not adjacent

For $(v_i, v_j)$,$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 11 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 13 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$ .

For $(v_i, v_j)$,$d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 15 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

For$(v_i ,v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(u_i)+f(v_j)}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 9 + \left\lceil\frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2}\right\rceil \geq 2\left(\frac{n}{2}\right) + 10$.

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 11 + \left\lceil \frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 10.$

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 13 + \left\lceil \frac{\frac{n}{2}+3+i+\frac{n}{2}+3+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 10.$

Therefore, $f(v_{n-1}) = 3\left(\frac{n}{2}\right) + 3$ is the largest label.

n is odd, Since $(SC_n)$ has n vertices it requires n labels. The D-distance is $d^D(v_{i+1}, v_n) = 2\left(\frac{n-1}{2}\right) + 2$, $f(v_n) = \left(\frac{n-1}{2}\right) + 4$ then the label $\left(\frac{n-1}{2}\right) + 3$ is forbidden.

$rmn^D(SC_n) \geq n + \left(\frac{n-1}{2}\right) + 3$

$\geq 3\left(\frac{n-1}{2}\right) + 4$

Now we shall give the following label to set the equality.

Let

$f(v_{2i}) = x_{2\left(\frac{n-1}{2}\right)+1-i}$ $1 \leq i \leq \frac{n-1}{2} - 1$

$f(v_{2i+1}) = x_{i+1},$ $1 \leq i \leq \frac{n-1}{2} - 1$

$f(v_1) = x_{\left(\frac{n-1}{2}\right)-1}$

$f(v_{n-1}) = x_{2\left(\frac{n-1}{2}\right)}$

$f(v_n) = x_1$

$$f(x_i) = \begin{cases} \frac{n-1}{2} + 3 + i, i = 1 \\ \frac{n-1}{2} + 4 + i, 1 \leq i \leq 2\left(\frac{n-1}{2}\right) \end{cases}.$$

We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(SC_n) + 1 = 2\left(\frac{n-1}{2}\right) + 11$, for every pair of vertices (u, v) where $u \neq v$.

If $v_i$ and $v_j$ are adjacent

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 5 + \left\lceil \frac{\frac{n-1}{2}+3+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 5 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

If $v_i$ and $v_j$ are not adjacent

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 11 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 13 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 15 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 9 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 11 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + 13 + \left\lceil \frac{\frac{n-1}{2}+4+i+\frac{n-1}{2}+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + 11.$

Therefore, $f(v_{n-1}) = 3\left(\frac{n-1}{2}\right) + 4$ is the largest label.

Hence, $rmn^D(SC_n) = \begin{cases} 9, & n = 5. \\ 13, & n = 6. \\ 14, & n = 7. \\ 3\left(\frac{n}{2}\right) + 3 & if\ n is\ even\ n \geq 8 \\ 3\left(\frac{n-1}{2}\right) + 4 & if\ n is\ odd\ n \geq 9 \end{cases}$ ■

- ❖ The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G say G′ and G″, then join each vertex u′ in G′ to the neighbours of the corresponding vertex u″ in G″.

**Theorem 2.6.** The radio mean D-distance number of a shadow graph of star, $rmn^D(D_2(K_{1,n})) = 4n + 2$ if $n \geq 2$.

**Proof.**

It is obvious that $diam^D(D_2(K_{1,n})) = 4(n + 1)$. Let $V(D_2(K_{1,n}))= \{x_j / j =1, 2\} \cup \{v_i,u_i / i = 1, 2, \ldots, n\}$ and $E = \{x_jx_{j+1}, x_jv_i, x_{j+1}u_i / i = 1, 2, 3, \ldots, n$ and $j= 1\}$. Since $D_2(K_{1,n})$ has 2n + 2 vertices it requires 2n + 2 labels. The D-distance is $d^D(x_1, x_2) = 4n + 4$, $f(x_2) = 2n + 1 (f(x_1) = 2n + 1)$ then the label 2n is forbidden.

$rmn^D(D_2(K_{1,n})) \geq 2n + 2n + 2$

$\geq 4n + 2$

Now we shall give the following label to set the equality. Let $f(u_i) = 3n + i + 2$, $f(v_i) = 2n + i + 2$, $1 \leq i \leq n$, $f(x_1) = 2n + 2$, $f(x_2) = 2n + 1$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(D_2(K_{1,n})) + 1 = 4n + 5$, for every pair of vertices (u, v) where $u \neq v$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil\frac{f(u_i)+f(u_j)}{2}\right\rceil \geq 2n + 6 + \left\lceil\frac{3n+i+2+3n+j+2}{2}\right\rceil \geq 4n + 5.$

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil\frac{f(v_i)+f(v_j)}{2}\right\rceil \geq 2n + 6 + \left\lceil\frac{2n+i+2+2n+j+2}{2}\right\rceil \geq 4n + 5.$

For $(v_i, x_1)$, $d^D(v_i, x_1) + \left\lceil\frac{f(v_i)+f(x_1)}{2}\right\rceil \geq 2n + 3 + \left\lceil\frac{2n+i+2+2n+2}{2}\right\rceil \geq 4n + 5.$

For $(u_i, x_1)$, $d^D(u_i, x_1) + \left\lceil\frac{f(u_i)+f(x_1)}{2}\right\rceil \geq 2n + 3 + \left\lceil\frac{3n+i+2+2n+2}{2}\right\rceil \geq 4n + 5.$

For $(v_i, x_2)$, $d^D(v_i, x_2) + \left\lceil\frac{f(v_i)+f(x_2)}{2}\right\rceil \geq 2n + 3 + \left\lceil\frac{2n+i+2+2n+1}{2}\right\rceil \geq 4n + 5.$

For $(u_i, x_2)$, $d^D(u_i, x_2) + \left\lceil\frac{f(u_i)+f(x_2)}{2}\right\rceil \geq 2n + 3 + \left\lceil\frac{3n+i+2+2n+1}{2}\right\rceil \geq 4n + 5.$

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil\frac{f(u_i)+f(v_j)}{2}\right\rceil \geq 2n + 6 + \left\lceil\frac{3n+i+2+2n+j+2}{2}\right\rceil \geq 4n + 5.$

For $(x_1, x_2)$, $d^D(x_1, x_2) + \left\lceil\frac{f(x_1)+f(x_2)}{2}\right\rceil \geq 4n + 4 + \left\lceil\frac{2n+2+2n+1}{2}\right\rceil \geq 4n + 5.$

Therefore, $f(u_n) = 4n + 2$ is the largest label.

Hence, $rmn^D(D_2(K_{1,n})) = 4n + 2$ if $n \geq 2$. ■

- ❖ The flower Fln is the graph obtained from a helm Hn by joining each pendant vertex to the apex of the helm.

**Theorem 2.7.** The radio mean D-distance number of a flower graph, $rmn^D(Fl_n) = 3(n + 1)$ if $n \geq 3$.

**Proof.**

It is obvious that $diam^D(Fl_n) = 2n + 10$. Let $V(Fl_n)= \{v\} \cup \{v_i,u_i / i = 1, 2, \ldots, n\}$ and $E = \{vv_i, vu_i, v_iu_i / i = 1, 2, 3, \ldots, n\}$. Since $Fl_n$ has 2n + 1 vertices it requires 2n + 1 labels. The D-distance is $d^D(v, v_j) = 2n + 5$, $f(v) = n + 3$ then the label n + 2 is forbidden.

$rmn^D(Fl_n) \geq n + 2 + 2n + 1$

$\geq 3n + 3$

Now we shall give the following label to set the equality. Let $f(u_i) = n + i + 3$, $f(v_i) = 3n - i + 4$, $1 \leq i \leq n$, $f(v) = n + 3$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq diam^D(Fl_n) + 1 = 2n + 11$, for every pair of vertices (u, v) where $u \neq v$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 2n + 8 + \left\lceil \frac{n+i+3+n+j+3}{2} \right\rceil \geq 2n + 11$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 9 + \left\lceil \frac{3n-i+4+3n-j+4}{2} \right\rceil \geq 2n + 11$.

For $(v_i, v)$, $d^D(v_i, v) + \left\lceil \frac{f(v_i)+f(v)}{2} \right\rceil \geq 2n + 5 + \left\lceil \frac{3n-i+4+n+3}{2} \right\rceil \geq 2n + 11$.

For $(u_i, v)$, $d^D(u_i, v) + \left\lceil \frac{f(u_i)+f(v)}{2} \right\rceil \geq 2n + 3 + \left\lceil \frac{n+i+3+n+3}{2} \right\rceil \geq 2n + 11$.

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 7 + \left\lceil \frac{n+i+3+3n-j+4}{2} \right\rceil \geq 2n + 11$.

For $(u_i, v_j)$, $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq n + 12 + \left\lceil \frac{n+i+3+3n-j+4}{2} \right\rceil \geq 2n + 11$.

Therefore, $f(v_n) = 3(n + 1)$ is the largest label.
Hence, $rmn^D(Fl_n) = 3(n + 1)$ if $n \geq 2$. ■

- A comb is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex.

**Theorem 2.8.** The radio mean D-distance number of a comb graph, $rmn^D(P_n \odot K_1) = 5n - 3$ if $n \geq 3$.

**Proof.**

It is obvious that $diam^D(P_n \odot K_1) = 4n + 1$. Let $V(P_n \odot K_1) = \{v_i, u_i / i = 1, 2, \ldots, n\}$ and $E = \{ v_i v_{i+1}, v_i u_i / i = 1, 2, 3, \ldots, n\}$. Since $(P_n \odot K_1)$ has 2n vertices it requires 2n labels. The D-distance is $d^D(u_1, u_2) = 10$, $f(u_1) = 3n - 2$ then the label $3n - 3$ is forbidden.

$$rmn^D(P_n \odot K_1) \geq 3n - 3 + 2n$$
$$\geq 5n - 3$$

Now we shall give the following label to set the equality. Let $f(u_i) = 3n - 3 + i$, $f(v_i) = 5n - 2 - i$, $1 \leq i \leq n$. We shall check the radio mean D-distance condition

$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(P_n \odot K_1) + 1 = 4n + 2$, for every pair of vertices $(u, v)$ where $u \neq v$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 4n + 1 + \left\lceil \frac{3n-3+i+3n-3+j}{2} \right\rceil \geq 4n + 2$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 4n - 6 + \left\lceil \frac{3n-3+i+3n-3+j}{2} \right\rceil \geq 4n + 2$.

For $(u_i, u_j)$, $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 4n - 7 + \left\lceil \frac{3n-3+i+3n-3+j}{2} \right\rceil \geq 4n + 2$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 6 + \left\lceil \frac{5n-2-i+5n-2-j}{2} \right\rceil \geq 4n + 2$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 4n - 3 + \left\lceil \frac{5n-2-i+5n-2-j}{2} \right\rceil \geq 4n + 2$.

For $(v_i, v_j)$, $d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 4n - 7 + \left\lceil \frac{5n-2-i+5n-2-j}{2} \right\rceil \geq 4n + 2$.

Therefore, $f(v_1) = 5n - 3$ is the largest label.
Hence, $rmn^D(P_n \odot K_1) = 5n - 3$ if $n \geq 3$. ■

## III. CONCLUSION

Though we have obtained the radio number of various different graphs with respect to the distance variants defined. the general results elude our attention; mainly because the radio numbers depend on the distance constraints, rather than structure of the graph. This certainly throws up more scope for further research. Moreover, equality can be tried for those cases ending up with sharp upper bounds.

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