

Curvature Tensors in A Kaehler Manifold with Semi-Symmetric Metric Connection

P. K. Dwivedi*, Ankit Maurya**

*Department of Mathematics, K.S. Saket (P.G.) College, Ayodhya, Uttar Pradesh, India

**Department of Mathematics, Sri. L.B.S. Degree College, Gonda, Uttar Pradesh, India

ABSTRACT

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In the present communication studies have been carried out with special reference to Semi-Symmetric curvature tensor, Weyl's projective curvature tensor, Concircular curvature tensor, Conharmonic curvature tensor and H-Projective curvature tensor.

Keywords : Kaehler manifold, Semi-Symmetric curvature tensor, Weyl's projective curvature tensor, Concircular curvature tensor, Conharmonic curvature tensor and H-Projective curvature tensor.

INTRODUCTION:

We consider the $2n$ dimensional Kaehler manifold M_n with structure $\{F, g\}$, here F stands for a real valued function and g stands for the metric tensor, the two satisfy the following:

$$(1.1) \text{ (a) } \bar{X} + X = 0, \quad \text{(b) } g(\bar{X}, \bar{Y}) = g(X, Y).$$

$$(1.2) g(X, Y) = g(Y, X)$$

$$(1.3) (\nabla_X F)(Y) = 0$$

where (1.4) $\bar{X} \stackrel{\text{def}}{=} FX$.

Here, X and Y are arbitrary vector fields and ∇ is the Riemannian connection on the manifold M_{2n} .

In the Kaehler manifold under consideration let us consider that K is the curvature tensor with respect to the connection ∇ defined by [3]

$$(1.5) K(X, Y, Z) \stackrel{\text{def}}{=} \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z.$$

As per this definition the following properties may easily be derived [3]

$$(1.6) \text{ (a) } K(X, Y, \bar{Z}) = \overline{K(X, Y, Z)},$$

$$\text{(b) } K(X, Y, Z) + \overline{K(X, Y, Z)} = 0,$$

$$\text{(c) } K(X, Y, \bar{Z}, W) + K(X, Y, Z, \bar{W}) = 0,$$

$$\text{(d) } K(X, Y, \bar{Z}, \bar{W}) + K(\bar{X}, \bar{Y}, Z, W) = 0,$$

$$\text{(e) } K(\bar{X}, \bar{Y}, \bar{Z}, \bar{W}) = K(X, Y, Z, W) = K(X, Y, \bar{Z}, \bar{W}),$$

$$\text{(f) } K(X, \bar{Y}, Z, W) = K(X, Y, Z, \bar{W}),$$

$$\text{(g) } K(X, \bar{Y}, \bar{Z}, W) = K(\bar{X}, Y, Z, \bar{W}),$$

where (1.7) $K(X, Y, Z, W) = g(K(X, Y, Z), W)$.

The Ricci tensor of $2n$ dimensional Kaehler manifold M_{2n} denoted by Ric . possesses the following properties

$$(1.8) \text{ (a) } Ric.(\bar{X}, \bar{Y}) = Ric.(X, Y),$$

$$(b) Ric. (X, \bar{Y}) + Ric. (\bar{X}, Y) = 0,$$

$$(c) R\bar{X} = \bar{R}\bar{X},$$

$$\text{and } (d) -\bar{R}\bar{X} = RX.$$

Here, X, Y, Z and W appearing in (1.7) and (1.8) are arbitrary vector fields.

The Weyl's projective curvature tensor W is given by

$$(1.9) W(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-1} \{ Ric. (X, Z)Y - Ric. (Y, Z)X \}.$$

This tensor satisfies the following properties [4]

$$(1.10) (a) W(\bar{X}, \bar{Y}, \bar{Z}, \bar{u}) = W(X, Y, Z, u),$$

$$(b) W(\bar{X}, \bar{Y}, Z, u) = W(X, Y, \bar{Z}, \bar{u}),$$

$$(c) W(X, Y, Z, u) \stackrel{\text{def}}{=} g(W(X, Y, Z), u),$$

and X, Y, Z and u are arbitrary vector fields.

The concircular curvature tensor C is defined as

$$(1.11) C(X, Y, Z) = K(X, Y, Z) - \frac{r}{n(n-1)} \{ g(Y, Z)X - g(X, Z)Y \}$$

where, r is the scalar curvature tensor in M_n .

The concircular curvature tensor as has been defined in (1.11) satisfies the following:

$$(1.12) (a) C(\bar{X}, \bar{Y}, \bar{Z}, \bar{u}) = 'C(X, Y, Z, u),$$

$$(b) C(\bar{X}, \bar{Y}, Z, u) = 'C(X, Y, \bar{Z}, \bar{u}),$$

$$(c) C(X, Y, Z, u) \stackrel{\text{def}}{=} g(C(X, Y, Z), u),$$

and X, Y, Z, u are arbitrary vector fields.

The Conharmonic curvature tensor L is defined as

$$(1.13) L(X, Y, Z) = K(X, Y, Z) - \frac{1}{n-2} \{ Ric(Y, Z)X - Ric(X, Z)Y \} + \{ g(Y, Z)RX - g(X, Z)RY \}$$

and this curvature tensor satisfies the following properties

$$(1.14) (a) L(\bar{X}, \bar{Y}, Z, \bar{u}) = L(X, Y, Z, u)$$

$$(b) L(\bar{X}, \bar{Y}, Z, u) = L(X, Y, \bar{Z}, \bar{u})$$

$$\text{where } (c) L(X, Y, Z, u) = g(L(X, Y, Z), u).$$

The H-projective curvature tensor has the following properties

$$(1.16) (a) P(X, Y, Z, u) = P(Y, X, Z, u),$$

$$(b) P(X, Y, Z, u) + P(Y, Z, X, u) + P(Z, X, Y, u) = 0$$

$$\text{where } (c) P(X, Y, Z, u) = g(P(X, Y, Z), u).$$

In this continuation, we shall also have the following [5]

$$(1.17) \tilde{K}(X, Y)Z = K(X, Y)Z - \alpha(Y, Z)X - \alpha(X, Z)Y - g(Y, Z)AX + g(X, Z)AY$$

$$\text{where } (1.18) \alpha(Y, Z) = g(A, Y, Z) = (\nabla_Y \omega)(Z) - \omega(Y)\omega(Z) + \frac{1}{2} \omega(\rho)g(Y, Z)$$

and A is a 1-form.

2. SEMI-SYMMETRIC CURVATURE TENSOR \tilde{K} AND ITS PROPERTIES IN CASE $\tilde{\nabla}_X \omega = 0$

A semi symmetric metric connection is a linear connection $\tilde{\nabla}$ on M_{2n} provided the torsion tensor T of the connection $\tilde{\nabla}$ and the metric tensor g of the manifold satisfies the following conditions:

$$(2.1) (a) (\tilde{\nabla}_Z g)(X, Y) = 0,$$

$$(b) T(X, Y) = \omega(Y)X - \omega(X)Y$$

where ω is a 1-form and X, Y are arbitrary vector fields.

The 1-form ω vector fields are usually termed as 1-form vector fields associated with the metric tensor g by the relation

$$(2.2) (a) \omega(X)X = g(X, \rho)$$

$$\text{and } (b) (\tilde{\nabla}_Z \omega)(Y) = (\nabla_X \omega)(Y) - \omega(X)\omega(Y) + \omega(\rho)g(X, Y)$$

Let us write

$$(2.3) (\tilde{\nabla}_Y \omega)(Z) = 0.$$

Using (2.2) in (2.3), we get

$$(2.4) (\nabla_Y \omega)(Z) = \omega(Y)\omega(Z) - \omega(\rho)g(Y, Z).$$

Using (1.1b), we have

$$(2.5) \omega(Z) = g(Z, \rho).$$

Differentiating (2.5) covariantly with respect to y , we get

$$(2.6) (\nabla_Y \omega)(Z) + \omega(\nabla_Y Z) = g(\nabla_Y Z, \rho) + g(Z, \nabla_Y \rho).$$

Using (2.5) in (2.6), we get

$$(2.7) (\nabla_Y \omega)(Z) = g(Z, \nabla_Y \rho).$$

with the help of (2.4) and (2.7), we get

$$(2.8) \alpha(Y, Z) = -\frac{1}{2}\omega(\rho)g(Y, Z)$$

and

$$(2.9) AY = -\frac{1}{2}\omega(\rho)Y.$$

Using (2.8) and (2.9) in (1.17), we get

$$(2.10) \tilde{K}(X, Y, Z) = K(X, Y, Z) + \omega(\rho)\{g(Y, Z)X - g(X, Z)Y\}$$

where, \tilde{K} is the curvature tensor with respect to the semi-symmetric connection $\tilde{\nabla}$.

In the light of all these observations, we can therefore state:

Theorem (2.1):

The semi-symmetric curvature tensor \tilde{K} with respect to the semi-symmetric metric connection $\tilde{\nabla}$ always satisfies (2.10).

We now define

$$(2.11) \tilde{K}(X, Y, Z, u) \stackrel{\text{def}}{=} g(\tilde{K}(X, Y, Z), u).$$

Using (2.10) in (2.11), we get

$$(2.12) \tilde{K}(X, Y, Z, u) = K(X, Y, Z, u) + \omega(\rho)\{g(Y, Z)g(X, u) - g(X, Z)g(Y, u)\}.$$

Barring X, Y, Z, u in (2.12) and thereafter using (1.2) and (1.6d), we get

$$(2.13) \tilde{K}(\bar{X}, \bar{Y}, \bar{Z}, \bar{u}) = \tilde{K}(X, Y, Z, u).$$

We further put bars on X and Y and (2.13) and get

$$(2.14) \tilde{K}(X, Y, \bar{Z}, \bar{u}) = \tilde{K}(\bar{X}, \bar{Y}, Z, u).$$

From (2.6), we get the following

$$(2.15) \tilde{K}(\bar{X}, Y, Z, u) = \tilde{K}(\bar{X}, Y, Z, u) + \omega(\rho)\{g(Y, Z)g(\bar{X}, u) - g(\bar{X}, Z)g(Y, u)\},$$

$$(2.16) \tilde{K}(X, \bar{Y}, Z, u) = \tilde{K}(X, \bar{Y}, Z, u) + \omega(\rho)\{g(\bar{Y}, Z)g(X, u) - g(X, Z)g(\bar{Y}, u)\},$$

$$(2.17) \tilde{K}(X, Y, \bar{Z}, u) = \tilde{K}(X, Y, \bar{Z}, u) + \omega(\rho)\{g(Y, \bar{Z})g(X, u) - g(X, \bar{Z})g(Y, u)\},$$

and (2.18) $\tilde{K}(X, Y, Z, \bar{u}) = \tilde{K}(X, Y, Z, \bar{u}) + \omega(\rho)\{g(Y, Z)g(X, \bar{u}) - g(X, Z)g(Y, \bar{u})\},$

Adding (2.15), (2.16), (2.17) and (2.18) and thereafter using (1.6c), we get

$$(2.19) \tilde{K}(\bar{X}, Y, Z, u) = \tilde{K}(X, \bar{Y}, Z, u) + \tilde{K}(X, Y, \bar{Z}, u) + \tilde{K}(X, Y, Z, \bar{u}) + \tilde{K}(\bar{X}, Y, Z, u)$$

where, we have taken into account

$$(2.20) (a) F(X, Y) + F(Y, X) = g(\bar{X}, Y) + g(X, \bar{Y}) = 0$$

$$(b) F(X, Y) = g(\bar{X}, Y).$$

In the light of all these observations, we can therefore state:

Theorem (2.2):

In a Kaehler manifold the curvature tensor \tilde{K} associated with semi-symmetric metric connection $\tilde{\nabla}$ satisfies (2.13), (2.14) and (2.19) provided $(\tilde{\nabla}_Y \omega)(Z) = 0$.

3. PROPERTIES OF WEYL'S PROJECTIVE CURVATURE TENSOR \tilde{W} , CONCIRCULAR CURVATURE TENSOR \tilde{L} WITH RESPECT TO SEMI-SYMMETRIC METRIC CONNECTION $\tilde{\nabla}$

The Weyl's projective curvature tensor \tilde{W} is defined as

$$(3.1) \tilde{W}(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-1}\{\tilde{R}ic. (X, Z)Y - \tilde{R}ic. (Y, Z)X\}$$

where $\widetilde{Ric.}$ is the Ricci tensor with respect to semi-symmetric connection $\widetilde{\nabla}$.

Contracting (2.10), we get

$$(3.2) \quad \widetilde{Ric.}(Y, Z) = Ric.(Y, Z) + (n - 1)\omega(\rho)g(Y, Z).$$

Using (2.10) and (3.2) in (3.1), we get

$$(3.3) \quad \widetilde{W}(X, Y, Z) = K(X, Y, Z) + \omega(\rho)\{g(Y, Z)X - g(X, Z)Y\} + \frac{1}{n-1}\{Ric.(X, Z)Y + (n - 1)\omega(\rho)g(X, Z)Y - Ric.(Y, Z)X + (n - 1)\omega(\rho)g(Y, Z)X\}.$$

with the help of (3.3), we can easily derive

$$(3.4) \quad \widetilde{W}(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-1}\{Ric.(X, Z)Y - Ric.(Y, Z)X\}.$$

We now assume the following

$$(3.5) \quad \widetilde{W}(X, Y, Z, u) = g(\overline{W}(X, Y, Z), u).$$

Using (3.5) in (3.4), we get

$$(3.6) \quad \widetilde{W}(X, Y, Z, u) = K(X, Y, Z, u) + \frac{1}{n-1}\{Ric.(X, Z)g(Y, u) - Ric.(Y, Z)g(X, u)\}$$

Using (1.9) and (1.10c), we get

$$(3.7) \quad \widetilde{W}(X, Y, Z, u) = W(X, Y, Z, u).$$

On barring X, Y, Z, u in (3.7) and thereafter (1.10a), we get

$$(3.8) \quad \widetilde{W}(\overline{X}, \overline{Y}, \overline{Z}, \overline{u}) = \widetilde{W}(X, Y, Z, u).$$

Again we put bars on X, Y in (3.7) and then use (1.10b) and get

$$(3.9) \quad \widetilde{W}(\overline{X}, \overline{Y}, Z, u) = \widetilde{W}(X, Y, \overline{Z}, \overline{u}).$$

We again put bars on Y and Z in (3.7) and get

$$(3.10) \quad \widetilde{W}(X, \overline{Y}, \overline{Z}, u) = \widetilde{W}(\overline{X}, Y, Z, \overline{u}).$$

All these observations enable us to state:

Theorem (3.1):

The Weyl's projective curvature tensor $\widetilde{W}(X, Y, Z)$ with respect to the semi-symmetric metric connection $\widetilde{\nabla}$ respectively satisfies (3.8), (3.9) and (3.10).

We now consider the concircular curvature tensor $\widetilde{C}(X, Y, Z)$ defined by

$$(3.11) \quad \widetilde{C}(X, Y, Z) = \widetilde{K}(X, Y, Z) + \frac{r}{n-1}\{g(Y, Z)X - g(X, Z)Y\}.$$

Making use of (2.10) in (3.11), we get

$$(3.12) \quad \widetilde{C}(X, Y, Z) = \widetilde{K}(X, Y, Z) + \omega(\rho)\{g(Y, Z)X - g(X, Z)Y\} - \frac{r}{n(n-1)}\{g(Y, Z)X - g(X, Z)Y\}.$$

Using (1.11) in (3.12), we get

$$(3.13) \quad \widetilde{C}(X, Y, Z) = C(X, Y, Z) + \omega(\rho)\{g(Y, Z)X - g(X, Z)Y\}.$$

We now assume the following

$$(3.14) \quad \widetilde{C}(X, Y, Z) = g(\overline{C}(X, Y, Z), u).$$

Using (3.14) in (3.13), we get

$$(3.15) \quad \widetilde{C}(X, Y, Z, u) = \widetilde{C}(X, Y, Z, u) + \omega(\rho)\{g(Y, Z)g(X, u) - g(X, Z)g(Y, u)\}.$$

We now put bars in (3.15) and thereafter use (1.2) and (1.12a), we respectively get

$$(3.16) \quad \widetilde{C}(\overline{X}, \overline{Y}, \overline{Z}, \overline{u}) = \widetilde{C}(X, Y, Z, u)$$

and

$$(3.17) \quad \widetilde{C}(X, Y, \overline{Z}, \overline{u}) = \widetilde{C}(\overline{X}, \overline{Y}, Z, u).$$

We can therefore state:

Theorem (3.2):

The concircular curvature tensor $\widetilde{C}(X, Y, Z)$ with respect to semi-symmetric metric connection $\widetilde{\nabla}$ respectively satisfies (3.15), (3.16) and (3.17).

The Conharmonic curvature tensor $\widetilde{L}(X, Y, Z)$ admitting semi-symmetric metric connection $\widetilde{\nabla}$ is defined as

$$(3.18) \quad \widetilde{L}(X, Y, Z) = \widetilde{K}(X, Y, Z) - \frac{1}{n-2}\{\widetilde{Ric.}(Y, Z)X - \widetilde{Ric.}(X, Z)Y + g(Y, Z)\widetilde{R}X - g(X, Z)\widetilde{R}Y\}.$$

From (2.10), we get

$$(3.19) \quad \tilde{R}X = RX + (n-1)\omega(\rho)X,$$

where $\tilde{R}X$ is obtained by contracting Ric. which is Ricci tensor with respect to semi-symmetric metric connection $\tilde{\nabla}$.

Using (2.10), (3.2) and (3.19) in (3.18), we get

$$(3.20) \quad \tilde{L}(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-2} \{Ric. (Y, Z)X - Ric. (X, Z)Y\} + \{g(Y, Z)RX - g(X, Z)RY\} - \frac{n-1}{n-2} \{\omega(\rho)g(Y, Z)X - \omega(\rho)g(X, Z)Y\} - \frac{n-1}{n-2} \{\omega(\rho)g(X, Z)Y - \omega(\rho)g(Y, Z)X\}.$$

After allowing a suitable rearrangement in (3.20), we get the following

$$(3.21) \quad \tilde{L}(X, Y, Z) = L(X, Y, Z) - 2\omega(\rho) \left(\frac{n-1}{n-2} \right) \{g(Y, Z)X - g(X, Z)Y\}.$$

Let us write $\tilde{L}(X, Y, Z, u) = g(\tilde{L}(X, Y, Z), u)$ then from (3.21), we get

$$(3.22) \quad \tilde{L}(X, Y, Z, u) = L(X, Y, Z, u) - 2\omega(\rho) \left(\frac{n-1}{n-2} \right) \{g(Y, Z)g(X, u) - g(X, Z)g(Y, u)\}.$$

Barring X, Y, Z, u in (3.22) and thereafter using (1.14a) and (2.2), we get

$$(3.23) \quad \tilde{L}(\bar{X}, \bar{Y}, \bar{Z}, \bar{u}) = L(X, Y, Z, u).$$

Again barring X, Y in (3.23) and then using (1.14b), we get

$$(3.24) \quad \tilde{L}(\bar{X}, \bar{Y}, Z, u) = \tilde{L}(X, Y, \bar{Z}, \bar{u}).$$

with all such observations, we can thereafter state:

Theorem (3.3):

The Conharmonic curvature tensor $\tilde{L}(X, Y, Z)$ equipped with semi-symmetric metrics connection $\tilde{\nabla}$ always satisfies (3.23) and (3.24) respectively.

4. H- PROJECTIVE CURVATURE TENSOR:

The H-projective curvature tensor $\tilde{P}(X, Y, Z)$ is defined by

$$(4.1) \quad 2\tilde{P}(X, Y, Z) = 2\tilde{K}(X, Y, Z) - \frac{1}{n+1} \{\tilde{R}ic. (Y, Z)X - \tilde{R}ic. (X, Z)Y + \tilde{R}ic. (Y, \bar{Z})\bar{X} + \tilde{R}ic. (X, \bar{Z})\bar{Y} + 2\tilde{R}ic. (Y, \bar{Y})\bar{Z}\}.$$

We now write

$$(4.2) \quad \tilde{P}(X, Y, Z, u) = g(\tilde{P}(X, Y, Z), u)$$

Using (4.2) in (4.1), we get

$$(4.3) \quad 2\tilde{P}(X, Y, Z, u) = 2\tilde{K}(X, Y, Z, u) - \frac{1}{n+1} \{\tilde{R}ic. (Y, Z)g(X, u) - \tilde{R}ic. (X, Z)g(Y, u) - \tilde{R}ic. (Y, \bar{Z})g(\bar{X}, u) + \tilde{R}ic. (X, \bar{Z})g(\bar{Y}, u) - 2\tilde{R}ic. (Y, \bar{Y})g(\bar{Z}, u)\}.$$

Using (2.12) and (3.2) in (4.3), we get

$$(4.4) \quad 2\tilde{P}(X, Y, Z, u) = 2K(X, Y, Z, u) + \omega(\rho) \{g(Y, Z)g(X, u) - g(X, Z)g(Y, u)\} - \frac{1}{n+4} [g(X, u)Ric. (Y, Z) - g(Y, u)Ric. (X, Z) + (n-1)\omega(\rho) \{g(Y, Z)g(X, u) - g(Y, u)g(X, Z)\} - g(\bar{X}, u)Ric. (Y, \bar{Z}) - (n-1)\omega(\rho)g(Y, \bar{Z})g(\bar{X}, u) + g(\bar{Y}, u)Ric. (X, \bar{Z}) + (n-1)\omega(\rho)g(X, \bar{Z})g(\bar{Y}, u) + 2g(\bar{Z}, u)Ric. (X, \bar{Y}) + 2(n-1)\omega(\rho)g(\bar{Z}, u)g(X, \bar{Y})].$$

Using (1.15) in (4.4), we get

$$(4.5) \quad 2\tilde{P}(X, Y, Z, u) = 2K(X, Y, Z, u) - \frac{1}{n+1} \{g(X, u)Ric. (Y, Z) - g(Y, u)Ric. (X, Z) - g(\bar{X}, u)Ric. (Y, \bar{Z}) + g(\bar{Y}, u)Ric. (X, \bar{Z}) + 2g(\bar{Z}, u)Ric. (X, \bar{Y})\}.$$

Using (4.5) in (4.4), we get

$$(4.6) \quad 2\tilde{P}(X, Y, Z, u) = 2P(X, Y, Z, u) + \frac{2\omega(\rho)}{n+1} \{g(Y, Z)g(X, u) - g(X, Z)g(Y, u)\} + \omega(\rho) \left(\frac{n-1}{n+1} \right) \{g(\bar{Y}, u)g(X, \bar{Z}) - g(Y, \bar{Z})g(\bar{X}, u) + 2g(\bar{Z}, u)g(X, \bar{Y})\}.$$

Barring X and Y in (4.6), we get

$$(4.7) \quad 2\tilde{P}(\bar{X}, \bar{Y}, Z, u) = 2P(\bar{X}, \bar{Y}, Z, u) + \frac{2\omega(\rho)}{n+1} [g(\bar{Y}, Z)g(\bar{X}, u) - g(\bar{X}, Z)g(\bar{Y}, u)] + \omega(\rho) \left(\frac{n-1}{n+1}\right) [-g(Y, u)g(X, Z) + g(Y, Z)g(X, u) - 2g(\bar{Z}, u)g(\bar{X}, Y)].$$

Allowing a cyclic interchange of X, Y, Z in (4.7), we get

$$(4.8) \quad 2\tilde{P}(\bar{Y}, \bar{Z}, X, u) = 2P(\bar{Y}, \bar{Z}, X, u) + \frac{2\omega(\rho)}{n+1} [g(\bar{Z}, X)g(\bar{Y}, u) - g(\bar{Y}, X)g(\bar{Z}, u)] + \omega(\rho) \left(\frac{n-1}{n+1}\right) [g(X, Z)g(Y, u) - g(Z, u)g(Y, X) - 2g(\bar{X}, u)g(\bar{Y}, Z)].$$

and

$$(4.9) \quad 2\tilde{P}(\bar{Z}, \bar{X}, Y, u) = 2P(\bar{Z}, \bar{X}, Y, u) + \frac{2\omega(\rho)}{n+1} [g(\bar{X}, Y)g(\bar{Z}, u) - g(\bar{Z}, Y)g(\bar{X}, u)] + \omega(\rho) \left(\frac{n-1}{n+1}\right) [g(X, Y)g(Z, u) - g(X, u)g(Z, Y) - 2g(\bar{Y}, u)g(\bar{Z}, X)].$$

Adding (4.7), (4.8) and (4.9) and thereafter using (1.2) and (1.16), we respectively get the following

$$(4.10) \quad \tilde{P}(\bar{X}, \bar{Y}, Z, u) + \tilde{P}(\bar{Y}, \bar{Z}, X, u) + \tilde{P}(\bar{Z}, \bar{X}, Y, u) = g(X, \bar{Z})g(\bar{Y}, u) + g(Y, \bar{X})g(\bar{Z}, u) + g(\bar{Z}, Y)g(\bar{X}, u)$$

$$(4.11) \quad \tilde{P}(X, Y, \bar{Z}, \bar{u}) + \tilde{P}(Y, Z, \bar{X}, \bar{u}) + \tilde{P}(Z, X, \bar{Y}, \bar{u}) = g(\bar{X}, Z)g(Y, \bar{u}) + g(\bar{Y}, X)g(Z, \bar{u}) + g(\bar{Z}, Y)g(X, \bar{u}).$$

In the light of all these observations, we can therefore state:

Theorem (4.1):

The H-projective curvature tensor $\tilde{P}(X, Y, Z)$ admitting semi-symmetric metric connection $\tilde{\nabla}$ respectively satisfies (4.10) and (4.11).

Conclusion:

The first section of this communication is introductory in nature. In the second section we have established the relationships which are satisfied by the semi-symmetric curvature tensor \tilde{K} with respect to the semi-symmetric metric connection. In the third section we have established the relationships which are satisfied by Weyl's projective curvature tensor \tilde{W} , the concircular curvature tensor \tilde{C} and the conharmonic curvature tensor \tilde{L} all equipped with semi-symmetric metric connection. In the fourth and last section we have established the relationships which are satisfied by the H-projective curvature tensor \tilde{P} equipped with semi-symmetric metric connection.

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