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A Queuing-inventory Model in Multiproduct Supply Chains

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***Abstract*—we consider a supply chain that includes a manufacture which produces more than one product that are demanded by several retailers. After production, each all type of products is hold in separated warehouses. Each warehouse has different holding cost and each product has a different backorder cost. We formulate a linear cost function to aggregate all the costs of the holding, back ordering and ordering. The manufacture incurs a setup time whenever it switches from producing one product type to another and it has a finite production rate and stochastic production times. The manufacturer is modeled as a FCFS, GI/G/1 queue. In order to mitigate the effect of setups, products are produced in batches. Customers' arriving orders are sent to warehouses from retailers of each product and by sending production authorization (PA) to manufacturing plants. Moreover, we extend the proposed model in order to analyze the logistics process to three-echelon inventory model. At last, several numerical examples of manufacturing supply chain network are given in order to analysis performance evaluation.**

***Index Terms*—Order batching, inventory control, queuing system, supply chain.**

I. INTRODUCTION

Supply chain management includes materials/supply management from the supplier of raw materials to ultimate product and also, network of organizations that are involved, through upstream and downstream stages, in the different processes and activities that produce value in the form of products and services in the hands of the consumer.

Therefore a supply chain consists of all parties involved in satisfying a customer request. And also, the supply chain includes not only the manufacturer and suppliers, but also transporters, retailers, and even customers themselves.

The supply chain activities constitute a mega process and various decisions are involved in their successful design and operation. Decisions regarding stocking and control of inventory of stocks are a common problem to all enterprises. Asset managers of large enterprises have the responsibility of determining the approximate inventory level in the form of components and finished goods to hold at each level of supply chain in order to guarantee specified end customer service levels. Given the size and complexity of the supply chain, a common problem for this asset manager is to know how to quantify the trade-off between service level and investment in inventory required to supporting these service levels. The problem is made even more difficult because the supply chains are highly dynamic with uncertainty in demand, variability in processing times at each stage of the supply chain, multiple dimensions for customer satisfaction, finite resources, etc.

The goal of a supply chain should be to maximize overall supply chain profitability. Supply chain profitability is the difference between the revenue generated from the customer and the total cost incurred across all stages of the supply chain.

One of the challenges in supply chain management is to control the capital in inventories. The objective of inventory control is therefore to balance conflicting goals like keeping stock levels down to have cash available for other purposes and having high stock levels for the continuity of the production and for providing a high service level to customers. A good inventory management system has always been important in the workings of an effective manufacturing supply chain.

Queuing systems are the natural models when dealing with problems where the main characteristics are congestion and jams. In this paper, we use $GI/G/1$ queue as tool for performance measures of the manufacturing supply chain and also, we use queue to analyze logistics processes.

We review the articles of inventory management in logistics chains including single-product multi-stage systems and multi-product systems (Table1).

In the next section, we present our model in detail, and then in Section III, we analyze the proposed model with an example and also, we present the results of our computational analysis. Finally, we give some concluding remarks in section IV.

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Table2. Coding reviewed articles of inventory management in logistics chains

Reference article(s)	Article's code (problem definition/constraints/outputs/objective functions)
[1] and [2]	Queueing, Single Production, Decomposition Method
[3]	Queueing, Assemble, Single Production, Stochastic, $M/M/1$, Longest Path Analysis
[4]	Inventory Queueing, Single Production, Decomposition Method
[5]	Multi Production, $M/G/1$, Lead Time
[6]	Queueing, Multi Production, Decomposition Method
[7]	Inventory Queueing, Multi Production, Continuous, $M^X/G/\infty$
[8]	Queueing, Single Production, Batch, $GI^X/G/1$
[9]	Production Inventory, Multi Production, Stochastic, Batch
[10]	Production Inventory, Stochastic, Batch, Decomposition Method, Monte Carlo simulation

II. Problem Definition

We consider a three echelon supply chain network including n retailers, L warehouses and a manufacturing plant as shown in Fig1. This network offers L types of product to the customers arrived to retailers' node. Customers' demands enter to the retailers and the whole demand accumulation for each product is forwarded to warehouses of that product. Then production authorization (PA) is sent to the manufacturing plant of that product with regards to the fact total number of orders should be Q_j for each products' warehouse $j = 1,2,...,L$ (orders of each product from warehouse to manufacturing plant are sent in batch size). We consider a multi-item production–inventory system where a manufacturing plant produces L type of products and separated inventory buffers are kept for each product. If it is available, an order is satisfied from buffer stock. If not, the demand is backordered. We assume that manufacturing plant serves producer of each product with $GI/G/1$ queue. After production process, products batch is delivered to the warehouse of that product and fulfill the retailers' demand.

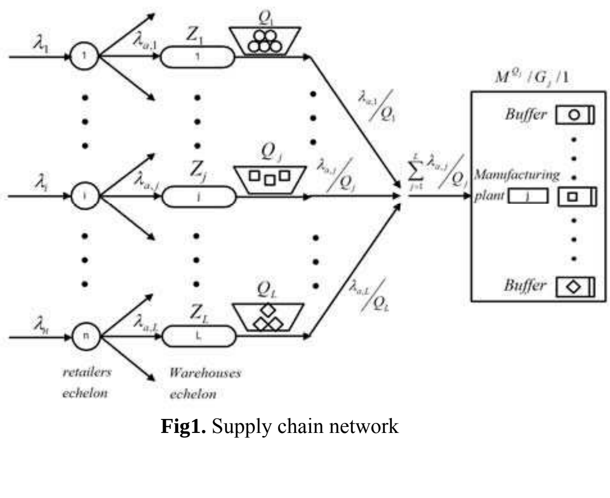

Fig1. Supply chain network

A. Assumptions

Assumptions of the developed model are as follow:
Customers' demand includes all types of products. We assume the orders of retailer i is as an independent renewal process with a constant rate $\lambda_i \geq 0$ and Squared Coefficient of Variation (SCV) C_i^2.The probability vectors, $q_i = (q_{i1}, q_{i2},..., q_{iL})$ define customers' demand from each kind of product at retailer i, ($\sum_{j=1}^{L} q_{ij} = 1, 0 \leq q_{ij} \leq 1; i = 1,2,...,n$).Therefore, the orders of warehouse j are as the processes with a constant rate $\lambda_{a,j} = \sum_{i=1}^{n} \lambda_i q_{ij}$ and SCV, $C_{a,j}^2 = \frac{1}{\lambda_{a,j}} \sum_{i=1}^{n} \lambda_i q_{ij} C_i^2$. In other words, $\lambda_{a,j}$ and $C_{a,j}^2$ are mean arrival rates and SCV of the aggregated input streams of product j, respectively.
In our problem it is assumed that each warehouse holds one product type in batch size Q_j which maximum number of batches is K_j. Therefore, maximum inventory level of warehouse of product j is $Z_j = K_j \times Q_j$.
We apply production authorization (PA) system to produce each product type. The PA system is a generalized pull-based production control system. We assume that products in inventories are stored in batches for each product j, and there is a PA card attached to each batch. In this paper, we consider the case when the number of PA cards is the same as the number of batches. The PA system operates in the following way whenever Q_j units are depleted from a batch in the inventory; the corresponding PA card is transmitted to the manufacturing plant and also is served as new production orders that trigger the manufacturing plant to begin its production process. In general, the manufacturing plant uses a FCFS discipline to produce these orders. Once the manufacturing plant produces Q_j units, the finished units and the PA card are sent to the warehouse. In the event when a customer places an order and there is no production inventory available, we assume that this customer wait until the product becomes available.
Production policy is make to stock strategy which is based on forecast and operates under $(K_j - 1, K_j)$ inventory control rule for warehouse of product j.
The base of above assumptions, batch arrival streams in manufacturer is following a Poisson process with rate $\lambda_a(B) = \sum_{j=1}^{L} \frac{\lambda_{a,j}}{Q_j}$. Following the asymptotic approach suggested by [11], SCV of batch arrival streams in manufacturer is $C_a^2(B) = \frac{\sum_{j=1}^{L} \frac{\lambda_{a,j}}{Q_j} C_{a,j}^2}{\lambda_a(B)}$.

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We define the probability that an arrival in manufacturer is for product j as: $p_j = \frac{\lambda_{a,j}}{Q_j \lambda_a(B)}$

The manufacturer incurs a setup time whenever it switches from producing one product type to another. We assume U and W be random variables that denote the setup time and process time experienced by a batch, respectively.
The probability that a batch of product j experiences a setup (a random variable with mean τ_j and variance η_j) is $1-p_j$, we can obtain the mean and SCV of setup time experienced by an arbitrary batch as:

$$E[U] = \sum_{j=1}^{L} p_j(1-p_j)\tau_j \quad (1)$$

$$C_U^2 = \frac{Var[U]}{E^2[U]} = \frac{\sum_{j=1}^{L} p_j(1-p_j)\eta_j}{\left(\sum_{j=1}^{L} p_j(1-p_j)\ \tau_j\right)^2} \quad (2)$$

We assumed that unit production times at manufacturer for product j are i.i.d. generally distributed random variables, which is denoted by B_j, with $1/\mu_j = E(B_j)$ and SCV, C_j^2 .Thus, mean production time for batch product j is Q_j/μ_j and coefficient of variation C_j^2/Q_j .
Similarly, we can obtain mean and SCV of processing time for an arbitrary batch as:

$$E[W] = \sum_{j=1}^{L} p_j \frac{Q_j}{\mu_j} \quad (3)$$

$$C_W^2 = \sum_{j=1}^{L} Q_j C_j^2 \quad (4)$$

We can obtain the mean and SCV of the effective batch service time S, ($S = U + W$) of an arbitrary batch, from which we can then compute the corresponding mean and coefficient of variation as:

$$E[S] = E[U] + E[W] \quad (5)$$

$$C_S^2 = C_U^2 + C_W^2 \quad (6)$$

The utilization of the manufacturing plant is given by

$$\rho = \sum_{j=1}^{L} \frac{\lambda_{a,j}}{Q_j} E[S] = \lambda_a(B) E[S] \quad (7)$$

The system incurs a holding cost h_j per unit of inventory of product j per unit time, a backordering cost b_j per unit of product j per unit time and C_{s_j} order setup cost for product j (\$ per set up).
The goal of modeling such a supply network is to minimize supply chain total cost in order to find optimal values of K_j, Q_j.

Costs contain inventory holding cost (h_j), *back ordering cost* (b_j), *and order set up cost* (C_{s_j}).

B. Notations

The notations used in this paper are as follow:

Q_j	Number of units in one bucket of product j; $j = 1,2,\ldots,L$
K_j	Total number of buckets at warehouse of product
Z_j	Maximum inventory at warehouse of product j; $K_j Q_j$
$\lambda_{a,j}$	Retailers' demand arrival rate of product j ($\sum_{i=1}^{n} \lambda_i q_{ij}$)
$C_{a,j}^2$	SCV of retailers' demand arrival rate of product j ($\frac{1}{\lambda_{a,j}} \sum_{i=1}^{n} \lambda_i q_{ij} C$)
A_j	Number of orders of product j arrived at manufacturer
μ_j	Service rate of product j in manufacturing plant units/unit time
C_j^2	SCV of service rate of product j in manufacturing plant units/unit time
p_j	The probability that an arrival is for product j
τ_j	Mean of setup time of product j
η_j	Variance of setup of product j
U	Setup time
W	Process time of batch
I_j	inventory level of product j
N_j	Number of orders of product j at manufacturing plant in a $GI/G/1$ queue
B_j	Backorders level of product j at warehouse
R_j	Number of orders arrived at warehouse of product j, but after the last batch was released for processing
h_j	Inventory holding cost for product j (\$ per unit per unit time)
b_j	Back order cost for product j (\$ per unit per unit time)
C_{s_j}	Order set up cost for product j (\$ per set up)
ρ	Intensity of the manufacturing plant
l_{ji}	Lead time for logistics for retailer i to receive items from warehouse of product j,
Γ_{ji}	Expected number of orders from product j at retailer i, in the queue $M^{C_j}/M/\infty$ in steady state
ξ	Service rate of logistics process
ρ'_j	Intensity of the logistics hub $\lambda_{a,j}/\xi < 1$
W_j	Expected waiting time at warehouse of product j just due to backordering
L_{ji}	Mean lead time (including backordering delay) for an order of items from retailer i to be filled from warehouse of

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product j, $L_{j1} = L_{j2} = \ldots = L_{jn} = L_j$

θ_{ji} Expected demand for product j during replenishment lead time for each item at retailer i ($\lambda_{a,j} L_{ji}$)

C. Problem formulation

In this paper, we would like to minimize the expected total cost at the warehouses.

Mathematically, we can express:

$$Min \sum_{j=1}^{L} TC(K_j, Q_j) = \sum_{j=1}^{L} (h_j E[I_j] + b_j E[B_j] + C_{s_j} (\frac{\lambda_{a,j}}{Q_j})) \tag{8}$$

$$s.t.$$

$$K_j, Q_j \in Z^+$$

For computing inventory and backorders, we use stochastic equations which capture the properties of the system as in [12]. Observing that,

$$R_j = A_j - \left\lfloor \frac{A_j}{Q_j} \right\rfloor Q_j, \quad j = 1,2,\ldots,L \tag{9}$$

$$B_j = \max\lfloor N_j Q_j + R_j - K_j Q_j, 0 \rfloor, \quad j = 1,2,\ldots,L \tag{10}$$

$$I_j = \max\lfloor K_j Q_j - N_j Q_j - R_j, 0 \rfloor, \quad j = 1,2,\ldots,L \tag{11}$$

The corresponding steady state probability distribution for R_j, N_j, B_j, I_j are as follow:

R_j is uniformly distributed from 0 to $Q_j - 1$. Thus,

$$P\{R_j = m\} = \frac{1}{Q_j}, \; m = 0,1,\ldots,Q_j - 1 \tag{12}$$

We use a development described in [12] to approximate the probability distribution of batches in the queue $GI/G/1$ using a geometric distribution of the following form:

$$P\{N = m\} \approx \begin{cases} 1-\rho & m = 0 \\ \rho(1-\sigma)\sigma^{m-1} & m = 1,2,\ldots \end{cases} \tag{13}$$

Where $\sigma = (\hat{N} - \rho)/\hat{N}$, $\hat{N} = \lambda_a(B) w_0 + \rho$ and $w_0 \approx \left\{ \frac{\rho^2 (1 + C_S^2)}{1 + \rho^2 C_S^2} \right\} \left\{ \frac{C_a^2 + \rho^2 C_S^2}{2\lambda_a (1-\rho)} \right\}$.

From [13], we also obtain the approximation of the distribution of the number of orders in the queue $GI/G/1$ of product j:

$$P_j\{N_j = m_j\} \approx \begin{cases} 1 - \left(\frac{\rho}{\sigma}\right) r_j & m_j = 0 \\ \left(\frac{\rho}{\sigma}\right)(1-r_j) r_j^{m_j} & m_j = 1,2,\ldots \end{cases} \tag{14}$$

Where $r_j = \frac{p_j \sigma}{1 - \sigma(1-p_j)}$, $p_j = \frac{\lambda_{a,j}}{Q_j \lambda_a(B)}$ and steady state probability distributions I_j, B_j are as follow:

$$P\{I_j = m\} = \frac{1}{Q_j} P_{N_j} (\left\lfloor \frac{Z_j - m}{Q_j} \right\rfloor); m = 1,2,\ldots,K_j Q_j \tag{15}$$

$$P\{B_j = m\} = \frac{1}{Q_j} P_{N_j} (\left\lfloor \frac{Z_j + m}{Q_j} \right\rfloor); m = 1,2,\ldots \tag{16}$$

We can obtain $E[I_j]$ and $E[B_j]$ as:

$$E[I_j] = \sum_{i=1}^{Z_j} \frac{Z_j - i}{Q_j} P_{N_j}(i) \tag{17}$$

$$E[B_i] = \sum_{i=0}^{\infty} \frac{i}{Q_j} P_{N_j} (\left\lfloor \frac{Z_j + i}{Q_j} \right\rfloor) \tag{18}$$

$$\begin{aligned} E[B_j] &= \sum_{m=1}^{\infty} \frac{m}{Q_j} P_{N_j} (K_j + \left\lfloor \frac{m}{Q_j} \right\rfloor) \\ &= \frac{1}{2}\left(\frac{\rho}{\sigma}\right)(1-r_j) r_j^{K_j} [\frac{Q_j - 1}{1 - r_j} + \frac{2Q_j}{1 - r_j} \times \frac{r_j}{1 - r_j}] \\ &= \frac{1}{2}\left(\frac{\rho}{\sigma}\right) r_j^{K_j} [(Q_j - 1) + \frac{2Q_j r_j}{1 - r_j}] \end{aligned} \tag{19}$$

Now, we can calculate optimal inventory level of every product. (By (8))

D. Performance measure of warehouses

The stock-out probability at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is zero and is obtained as follows:

$$\begin{aligned} P\{I_j = 0\} &= P\{Z_j \le N_j(t) Q_j + R_j(t)\} = P\left\{\frac{Z_j - R_j(t)}{Q_j} \le N_j(t)\right\} \\ &= P\{K_j - \frac{R_j(t)}{Q_j} \le N_j(t)\} \\ &= \frac{1}{Q_j} P\{K_j \le N_j(t)\} + \frac{Q_j - 1}{Q_j} P\{K_j - \frac{R_j(t)}{Q_j} \le N_j(t)\} \\ &= \frac{1}{Q_j}\left(\frac{\rho}{\sigma}\right) r_j^{K_j} + \frac{Q_j - 1}{Q_j}\left(\frac{\rho}{\sigma}\right) r_j^{K_j - 1} \end{aligned} \tag{20}$$

And also, the fill rate at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is greater than zero:

$$\begin{aligned} P\{I_j > 0\} &= P\{Z_j > N_j Q_j + R_j\} = 1 - P\{I_j = 0\} \\ &= 1 - \frac{1}{Q_j}\left(\frac{\rho}{\sigma}\right) r_j^{K_j} - \frac{Q_j - 1}{Q_j}\left(\frac{\rho}{\sigma}\right) r_j^{K_j - 1} \end{aligned} \tag{21}$$

Also the lead time of product j at its manufacturing plant is given by

$$W_{s_j} = \frac{(Q_j - 1)}{2} (1/\lambda_{a,j}) + w_0 + (Q_j/\mu_j) \tag{22}$$

Where $\frac{(Q_j - 1)}{2} (1/\lambda_{a,j})$ is batch forming time of product j and Q_j/μ_j is mean production time for product j batch.

E. The squared coefficient of variation of the inter-departure times which is produced from the warehouses

In this section, we show how to derive the characteristics of batching departure streams from the manufacturing plant with known $\lambda_d(B)$, $C_d^2(B)$ which are obtained as:

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$$\lambda_d(B) = \lambda_a(B) = \sum_{j=1}^{L} \frac{\lambda_{a,j}}{Q_j} \qquad (22)$$

We use the approximation of SCV of the inter-departure times for the batches from the manufacturing plant with batch setups in the $GI/G/1$ queue which is given in [12] and shown in (23):

$$C_d^2(B) = (1-\rho^2)\left\{\frac{C_a^2(B)+\rho^2 C_S^2}{(1+\rho^2 C_S^2)}\right\} + \rho^2 C_S^2 \qquad (23)$$

Also, we can obtain the characteristics of batching departure stream of product j from the manufacturing plant as following equation:

$$\lambda_{d,j} = \lambda_{a,j} = \sum_{i=1}^{L} q_{ij}\lambda_i \qquad (24)$$

$$C_{d,j}^2(B) = p_j C_a^2(B) + 1 - p_j \qquad (25)$$

And also, we use the following approximation of the SCV of the inter-departures of individuals from the warehouse of product j:

$$C_{d,j}^2(I) = Q_j C_a^2(B) + Q_j - 1 \qquad (26)$$

Where Q_j denoting the size of fixed batches of product j. When a product departs from the warehouse, there is a probability q_{ij} that the product will be routed to retailer i therefore the mean inter-arrival time and SCV for arrivals to retailer i are given by

$$\lambda_{a,ji} = \lambda_i q_{ij} \qquad (27)$$

$$C_{a,ji}^2 = q_{ij} C_{d,j}^2(I) + 1 - q_{ij} \qquad (28)$$

F. *Logistics Process*

In continue, we extend the model by adding logistics processes. We assume that there is some logistics time to supply products from warehouses to retailers. We model the logistics process of product j by using $M/M^{c_j}/\infty$ queue in continuous time, where c_j is vehicle capacity which is deterministic and logistics time is exponential. We assume that the logistics process depends on the demands of customers for its arrival process.

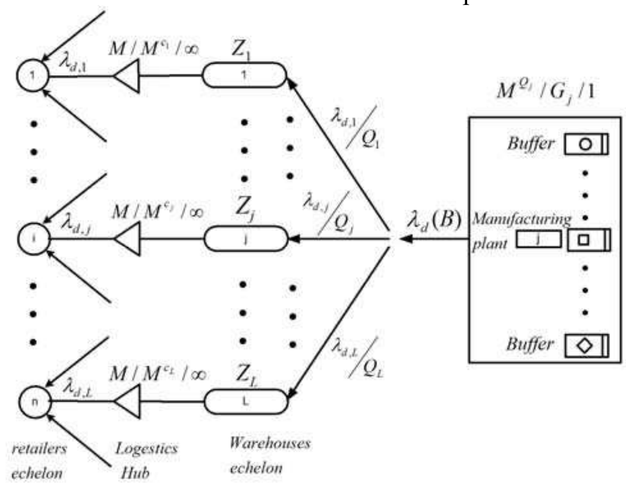

Fig.2 supply chain network with logistics hub and retailers

For the performance analysis of $M/M^{c_j}/\infty$ queue, we use the results of [15] and [16].
We obtain mean lead time of product j from its warehouse at retailer i, $L_{ji} = l_{ji} + W_j$ by using Little's law we have:

$$W_j = \frac{E\lfloor B_j \rfloor}{\lambda_{a,j}} \qquad (29)$$

$$L_{ji} = l_{ji} + W_j \qquad (30)$$

$$\Gamma_{ij} = \frac{q_{ij}\lambda_{a,j}}{\xi} C_j = \frac{1}{\xi}(q_{ij}\lambda_{a,j}C_j) \qquad (31)$$

$$l_{ji} = \frac{\Gamma_{ij}}{q_{ij}\lambda_{a,j}} = \frac{C_j}{\xi} \qquad (32)$$

We can compute expected demand of product j at retailer i during replenishment lead time as

$$\theta_{ji} = \lambda_{a,j,i} L_{ji} \qquad (33)$$

III. NUMERICAL EXAMPLE

In this section, we analyze the model by an example. We consider a supply chain network which produces three product types. The supply chain includes a manufacturing plant, three warehouses and two retailers where the demands for products are characterized by $\lambda_1 = 0.6$, $\lambda_2 = 0.8$ and $C_{a,1}^2 = C_{a,2}^2 = 1$.
The probability vectors, $q_1 = (0.2, 0.3, 0.5))$, $q_2 = (0.4, 0.3, 0.3)$ define customers' demand for three products at two retailers. Information of the manufacturing plant and costs of three warehouses are showed in Table 2 and information of logistics processes is showed in Table 3:

Table2. Information of the manufacturing plant

Product type	μ_j	C_j^2	τ_j	η_j	h_j	b_j	C_{S_j}
1	0.5	0.7	0.1	1	10	100	6
2	0.6	0.8	0.3	0.9	12	120	10
3	0.8	0.6	0.15	0.95	14	140	12

Table3. Information of logistics processes

Product type	l_j	c_j	ξ_j
1	1	3	5
2	2	5	6
3	3	4	7

We solved the problem with MATLAB.7 software coding. We obtain optimum value K_j by variety values of batch sizing Q_j for three products.

In the condition that $\frac{b_j}{h_j} = 1$, we increase Q_j, and optimum maximum inventory level and total cost of three products are increased. The results imply that if backorder costs are greater than holding costs, system tends to hold more inventories (Table4).

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Table4. Total cost variation by increasing Q_j if $b_j / h_j = 1$

Product type	Q_j	K_j^*	ρ_j	$E[I_j]$	$E[B_j]$	TC^*
1	2	1	0.6478	58.4301	100.7158	1.0880e+003
	4	2	0.1830	509.4270	70.3479	1.0567e+003
	6	3	0.0907	1.2278e+003	45.9475	1.1699e+003
	10	5	0.0394	3.3783e+003	26.9097	2.8384e+003
	16	8	0.0193	8.0370e+003	16.7974	1.0284e+004
	26	13	0.0098	1.8041e+004	11.0573	4.3463e+004
	40	1	0.0056	3.1748e+004	6.6081	3.1250e+004
2	3	1	0.3066	183.3429	98.1773	1.4273e+003
	6	3	0.0907	893.3124	51.4060	1.4192e+003
	9	4	0.0465	1.9782e+003	34.4818	2.6978e+003
	18	9	0.0163	6.6877e+003	18.4240	1.7453e+004
	30	1	0.0081	1.3367e+004	11.3783	4.9734e+004
3	5	2	0.1238	1.0603e+003	50.1774	1.3949e+003
	10	5	0.0394	4.3464e+003	23.6264	3.9171e+003
	15	7	0.0212	9.3725e+003	15.3870	1.1948e+004
	25	12	0.0103	2.3164e+004	9.7935	5.4166e+004
	30	1	0.0081	3.1185e+004	8.1782	5.8023e+004
	40	1	0.0056	4.7617e+004	6.6426	4.3750e+004
	50	1	0.0043	6.2078e+004	4.4199	3.5174e+004

In condition that $\frac{b_j}{h_j} = 10$, we increase Q_j, and optimum number of batches (K_j^*) and optimum maximum inventory level is variable but there is not any trend. Furthermore, total cost of three products is increasing in Q_j (Table5).

Table5. Total cost variation by increasing Q_j if $b_j / h_j = 10$

Product type	Q_j	K_j^*	ρ_j	$E[I_j]$	$E[B_j]$	TC^*	W_j
1	2	40	0.6478	58.4301	100.7158	1.0657e+004	296.6715
	4	4	0.1830	509.4270	70.3479	9.4051e+003	70.0239
	6	5	0.0907	1.2278e+003	45.9475	7.5990e+003	53.4018
	8	7	0.0563	2.1918e+003	33.8411	7.4128e+003	57.2372
	10	9	0.0394	3.3783e+003	26.9097	8.6126e+003	69.6334
	14	13	0.0235	6.3245e+005	19.0833	1.5250e+004	110.3028
	26	24	0.0098	1.8041e+004	11.0573	8.0534e+004	327.4489
	40	36	0.0056	3.1748e+004	6.6081	2.8930e+005	745.0176
2	3	3	0.3066	183.3429	98.1773	1.3665e+004	105.2937
	6	5	0.0907	893.3124	51.4060	9.2601e+003	50.8385
	9	8	0.0465	1.9782e+003	34.4818	9.5161e+003	56.3591
	15	14	0.0212	4.9555e+003	21.5401	2.1594e+004	104.6691
	24	22	0.0109	1.0238e+004	15.3412	7.6452e+004	233.0138
	30	27	0.0081	1.3367e+004	11.3783	1.4711e+005	351.4637
3	5	5	0.1238	1.0603e+003	50.1774	1.0615e+004	54.5614
	15	14	0.0212	9.3725e+003	15.3870	2.4822e+004	98.9385
	30	27	0.0081	3.1185e+004	8.1782	1.7157e+005	334.1556
	50	1	0.0043	6.2078e+004	4.4199	3.4855e+005	888.8527

IV. CONCLUSION

In this paper, we presented a model for the analysis of a three-layer supply chain which produces more than one product. We used $GI/GI/1$ queue operating under $(K_j - 1, K_j)$ inventory control policy to analyze the performance of warehouses. We obtained performance of measures such as stock-out probability, fill-rate and lead time of warehouses in proposed model. In the model, we used $M/M^{c_j}/\infty$ queue to analyze logistics process. In this paper, we survey the effect of order batching in multi-product multi-echelon supply chains. In future researches, we can consider a center warehouse that in the stock-out condition in each warehouse, customers' demands are satisfied (adding transmittal cost). Also, the pricing concept can be added to our model as an attractive aspect of future research.

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