Patient waiting time management through fuzzy based failure mode and effect analysis

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Abstract. The amount of time patients spends on services to be delivered in clinics, still is a major problem of some health 8 centers. To solve this problem, various methods proposed by researchers. Failure Mode and Effects Analysis (FMEA) is one of 9 the most used approaches to identify influential failure modes in prolongation of waiting time. In the FMEA method, numeric 10 scores assign to failure modes, using the Risk Priority Number (RPN), but RPN criticized for its shortcoming and leads to 11 unreal results. In this paper, to cover the conventional FMEA shortcoming, firstly, eleven risk factors result in prolongation 12 of waiting time introduced by experts. Secondly, integration of the triangular fuzzy number (TFN) with the Best Worth 13 Method (fuzzy-BWM) was utilized to determine the weights of effective criteria. In the following, failure modes ranked 14 through fuzzy Multi-Objective Optimization by Ratio Analysis (fuzzy-MOORA). Finally, the ranks of eleven failure modes 15 compared in three different methods (Conventional FMEA, conventional MOORA, and fuzzy-MOORA). The potential usage 16 of this method is covering the shortcoming of previous methods and contribute certainty in identifying significant failure 17 18 modes of the patient waiting time reduction in Out-Patient Departments (OPD). According to the analysis, three main failures for managing waiting time are: the patients never follow up for a later date by the center which can result in chaos in OPD, 19 because of phone or in-person referrals. Secondly, unable to manage canceling/postponing an appointment in emergency 20 cases, Thirdly, office visit not done in the estimated time, which results in a disordering in the center. 21

Keywords: Patient waiting time, out-patient departments, failure mode and effects analysis, fuzzy best-worst method, fuzzy
 multi-objective optimization by ratio analysis

24 **1. Introduction**

One of the important issues that out-patient departments (OPD) or any other section in health centers facing with is patient waiting time management [1]. The OPD is part of the hospital that clinical services are provided to patients who do not need to stay in the hospital over the night. In the last decade, reducing waiting time and service management were decisive factors in choosing a service sector [2]. Accurate time management, gaining patients' trust and produce profit, cost savings and market share benefits [3, 4]. On the contrary, long patient waiting time not only results in dissatisfaction of patients, but also the effects on the quality of services [5, 6]. This is the reason why health service sectors in many countries focus on service time management and finding a definite solution to overcome this problem.

There have been numerous studies to investigate the best solution of reducing the waiting time in OPD [7, 8]. Among all, one of most used methods, is recognition the failure modes and their effects on patient

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waiting time. For instance, Zhu et al. [9], used a sim-45 ulation study to analyze the factors influencing the 46 prolongation of the waiting time in OPDs. Mahesh 47 et al. [10] assessed factors result on the patient wait-48 ing time in the cardiology department by using of 49 DMAIC (Define, Measure, Analyze, Improve, and 50 Control) methodology. Alkuwaiti et al. [5] applied 51 cross section study to analyze effective variables on 52 the patient's satisfaction. 53

Historically, the FMEA method is one of the most 54 well-known methods for failure modes evaluation 55 [11]. The FMEA is a team-based systematic tool 56 and pre-occurrence prevention principle which can 57 be used to identify risks, cause of the occurrence, and 58 impacts of potential risks. FMEA is often applied to 59 either validate or to improve a process [12]. FMEA 60 determines the risk priorities of failure modes of an 61 organization through the risk priority number (RPN). 62

RPN is calculating through the multiplication of 63 occurrence (O: indicates failure frequency), sever-64 ity (S: indicates the seriousness of the effect of the 65 failure) and detection (D: indicates the possibility of 66 failure detection before its effects) of potential fail-67 ures [13]. The higher the RPN, the more urgently 68 corrective action is required, because of the higher 69 probability of future failure risks [14]. 70

The FMEA is used to support decision maker to 71 solve various challenges in the healthcare industry 72 such as preventing medication errors in hospitals 73 [15], analyzing the effects of chemotherapy for both 74 patient and nurses [16], assessing failures at a health-75 care diagnosis service [17], improving medication 76 management process to reduce risks and errors [18], 77 discovering risks in the intensive care unit and reduce 78 or eliminate them [19]. However, besides many 79 advantages of FMEA, its main weakness is being 80 team-motivated, that leads to uncertainty in consid-81 ering the determination of RPN [20]. Therefore, for 82 achieving more robust results against the opinions 83 of different individuals, it is vital to prioritize the 84 risks with regard to uncertainties inherent in these 85 criteria. In addition, the shortage of full ranking (the 86 distinction between various risk priorities) and the 87 assumption of the equal importance of determinant 88 factors are other shortcomings of traditional RPN 89 [21]. Consequently, researchers have tried to cover 90 some of the drawbacks of the RPN by utilizing alter-91 native approaches, including MCDM [22]. 92

Throughout the last decades, various MCDM
methods presented and used in a different field.
Some of the well-known methods are Technique
for the Order of Preference by Similarity to Ideal

Solution (TOPSIS) [23, 24], Analytic Hierarchy Process (AHP) [25, 26], Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) and Multi-MOORA [27], and Analytic Network Process (ANP) [28], and Best Worth Method (BWM) [29–31]

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Under the situation where data cannot be expressed quantitatively, fuzzy set theory can be used. The fuzzy set theory has enabled doing various studies in health care management. For more information, one can refer to [27] for the use of fuzzy -MOORA and -Multi-MOORA techniques, and [32] for the use of fuzzy AHP in health service management and patient safety.

With regards to the gaps like not considering certainty in managing patient waiting time at OPDs and the weakness of existing approaches, the contribution of this study is aimed to provide a new full score ranking method to improve and cover the deficiencies of traditional methods. The proposed approach is extended version of the FMEA, and fuzzy-BWM and fuzzy-MOORA are utilized in suggested method. Therefore, in the first place, risk factors that play an important role in the prolongation of waiting time are defined by experts. Secondly, BWM in fuzzy environment is used for weighing the triple factors (SOD), considering uncertainty in the group decision-making process and solving the problem in assigning different weights to the three factors. Fewer paired wise comparison and including certainty in decision making are some of the advantages of the proposed method in comparison with conventional methods. In third place, for ranking failure modes, fuzzy-MOORA is utilized. In the proposed approach, failure modes are decision making alternatives and factors that weighted by fuzzy-BWM, are failure assessment criteria. In this paper, by considering certainty in both weighting criteria and ranking failure modes, full prioritization is possible. The advantage of full prioritization is the facilitation of identifying significant failure modes and implementing appropriate action to solve problems.

The rest of this study organized as follows: In Section 2, fuzzy set and triangular fuzzy number (TFN) explained and all steps of transferring BWM and MOORA to fuzzy-BWM and fuzzy-MOORA, respectively, presented. In Section 3, Proposed approach explained in detail. In Section 4, the results presented, analysis and discussion are described, and the final results of the proposed method compared with conventional FMEA and MOORA method. Finally, the conclusion presented and corrective actions to reduce waiting time in OPD or any other section of the healthcare centerexplained.

151 **2. Methodology**

In this section, as prerequisite methods, a brief
 explanation of fuzzy sets theory, fuzzy BWM and
 fuzzy MOORA approach, presented. The list of ter minologies used in this article are as follows:

Α	Set
\tilde{A}, \tilde{B}	Fuzzy sets
C_n	Criteria for <i>n</i> -th component
a	Lower bound of fuzzy set
b	Middle bound of fuzzy set
с	Upper bound of fuzzy set
$\mu_{\tilde{A}}(x)$	Membership function
X	Reference set
TM_k	k-th decision maker group
C_B	Best criteria
C_W	Worst criteria
\tilde{P}_B	Best-to-Others vector
$ ilde{P}_W$	Others-to-Worst vector
$ ilde{w}$	Fuzzy weight of criteria
W	Worst criteria
В	Best criteria
\tilde{y}_i	Performance value
\tilde{v}_{ij}	Weighted normalized decision fuzzy matrix

156 2.1. Fuzzy set theory

The fuzzy set theory can solve the ambiguous and 157 imprecise conceptual problems as a practical tool in 158 uncertain conditions and environment [33]. The fuzzy 159 theory is a framework that has the ability to model 160 reality as it is. It tries to bring the model and reality 161 closer together and reduce the gap between model-162 ing and human thinking. This framework provides a 163 suitable opportunity for the definition of fuzzy terms 164 such as low, medium, and high, which corresponds 165 well with human thinking and feelings [34]. 166

A fuzzy set represents elements' membership degrees in the defined interval, [0,1], which is specified as a membership function. To define the basic fuzzy set, consider a set *A* defined in reference *X* as

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) | x \in X \right\}$$
(1)

where $\mu_{\tilde{A}}(x): X \to [0, 1]$ is the membership function of the set \tilde{A} . The membership value represents the degree of dependency of $x \in X$ in A and \tilde{A} is called a fuzzy set.

2.1.1. The TFN

A TFN represents by three real numbers, the upper bound (c) as the maximum value, the lower bound (a) as the minimum value, and the medium value (b) of TFN like $\tilde{A} = (a, b, c)$. The membership function of a TFN is:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b\\ \frac{c-x}{c-b} & b \le x \le c\\ 0 & \text{otherwise} \end{cases}$$

Consider $\tilde{A} = (a_1, b_1, c_1)$, $\tilde{B} = (a_2, b_2, c_2)$ as two positive TFN, the basic operations for TFNs are as followed:

$$A \oplus B = (a_1 + a_2, b_1 + b_2, c_1 + c_2),$$

 $\tilde{A} \oplus \tilde{B} = (a_1 - c_2, b_1 - b_2, c_1 - a_2),$

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$$\tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2),$$
$$\tilde{A} \otimes \tilde{B} = (a_1/c_2, b_1/b_2, c_1/a_2),$$
$$\lambda \tilde{A} = (\lambda a_1, \lambda b_1, \lambda c_1), \lambda \ge 0,$$

2.2. Fuzzy BWM

One of the powerful methods of the MCDM technique for determining the weights of the criteria is BWM [29]. When the comparison system is fully consistent with every criterion, or there are two or more criteria in the MCDM, the BWM method can be used to lead the decisions into to a single solution [35].

Fuzzy BWM determines fuzzy weights from the fuzzy reference comparisons, and it is based on the best and the worst criteria [36]. The traditional BWM method uses crisp values for comparisons [29]. However, in uncertain and non-deterministic conditions, it cannot determine weights of criteria accurately. This is one of the reasons that BWM extended to fuzzy BWM [30, 31]. The fuzzy BWM has the outstanding features of the BWM method and yields the weight of the criteria based on TFN. Therefore, it leads to keep the originality of the information.

Weighting criteria by using fuzzy BWM included four steps.

Step 1: Building a set of decision criteria, including *n* components, $\{C_1, C_2, ..., C_n\}$.

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Table 1
Linguistics variable and CIs for assessing the weight of risk
factors

Linguistics terms	Fuzzy membership value	CIs
Equally important (EI)	(1,1,1)	3.00
Weakly important (WI)	(2/3, 1, 3/2)	3.8
Fairly important (FI)	(3/2,2,5/2)	5.29
Important (I)	(5/2,3,7/2)	6.69
Very important (VI)	(7/2,4,9/2)	8.04

¹⁹⁵ In this step, in order to assess the alternatives, the decision criteria system is built.

¹⁹⁷ **Step 2:** Determining the best (C_B) and the worst (C_W) ¹⁹⁸ criteria based on the judgment of the *k* groups of ¹⁹⁹ decision-makers, { $TM_1, TM_2, ..., TM_k$ }.

Step 3: Determining \tilde{p}_{ij} as a fuzzy reference comparison.

In this step, the qualitative preferences of the best and worst criterion over every other criterion can be made by utilizing the linguistic terms in Table 1. After transforming linguistic variable to TFN, the obtained fuzzy Best-to-Others (BO) vector is:

$$\tilde{P}_B = (\tilde{p}_{B1}, \tilde{p}_{B2}, ..., \tilde{p}_{Bn})$$
 (2)

where \tilde{p}_{Bj} , j = 1, 2, ..., n is the fuzzy preference of the best criterion, and $\tilde{p}_{BB} = (1, 1, 1)$.

> In a similar way, we can determine the qualitative preference of the risk factors over the worst risk factor. Therefore, the fuzzy Others-to-Worst (OW) vector is:

$$\tilde{P}_W = (\tilde{p}_{1W}, \tilde{p}_{2W}, ..., \tilde{p}_{nW})$$
 (3)

where \tilde{p}_{jW} , j = 1, 2, ..., n, is the fuzzy preference of the worst criterion, and $\tilde{p}_{WW} = (1, 1, 1)$.

Step 4: Determining optimal fuzzy weights $(\tilde{w}_1^*, \tilde{w}_2^*, ..., \tilde{w}_n^*).$

In this step, in order to obtain the constrained optimization problem for determining optimal fuzzy weights, (4) is used. The purpose of obtaining \tilde{w}_n^* is to consider decision makers' preference in each criterion. The optimal weight for the criteria is the one where for each pair of $\frac{\tilde{w}_B}{\tilde{w}_J} = \tilde{p}_{Bj}$, and $\frac{\tilde{w}_j}{\tilde{w}_W} = \tilde{p}_{jW}$. With regards to sum condition for weights, the following problem results:

min max
$$\left\{ \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{p}_{Bj} \right|, \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{p}_{jW} \right| \right\}$$

s.t.
$$\begin{cases} \sum_{j=1}^{n} \tilde{w}_{j} = 1\\ 0 \le a_{j}^{w} \le b_{j}^{w} \le c_{j}^{w}, j = 1, 2, ..., n \end{cases}$$
(4)

where $\tilde{p}_{jW} = (a_{jW}, b_{jW}, c_{jW})$ and $\tilde{p}_{Bj} = (a_{Bj}, b_{Bj}, c_{Bj})$.

The minimax model in (4) can be transferred to the nonlinear constrained optimization problem [37] as follows:

$$\min \tilde{\xi}$$
s.t.
$$\begin{cases} \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{p}_{Bj} \right| \leq \tilde{\xi} \\ \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{p}_{jW} \right| \leq \tilde{\xi} \\ \sum_{j=1}^n \tilde{w}_j = 1 \\ 0 \leq a_j^w \leq b_j^w \leq c_j^w, \, j = 1, 2, ..., n \end{cases}$$
(5)

where $\tilde{\xi}$ is a TFN.

Because $a^{\xi} \leq b^{\xi} \leq c^{\xi}$, we suppose that $\tilde{\xi}^* = (k^*, k^*, k^*), k^* \leq a^{\xi}$ then (5) can be transferred to (6):

min $\tilde{\xi}^*$

$$.t. \begin{cases} \left| \frac{(a_B^w, b_B^w, c_B^w)}{(a_j^w, b_j^w, c_j^w)} - (a_{Bj}, b_{Bj}, c_{Bj}) \right| \le (k^*, k^*, k^*) \\ \left| \frac{(a_g^w, b_j^w, c_j^w)}{(a_W^w, b_W^w, c_W^w)} - (a_{jW}, b_{jW}, c_{jW}) \right| \le (k^*, k^*, k^*) \\ \sum_{j=1}^n \tilde{w}_j = 1 \\ 0 \le a_j^w \le b_j^w \le c_j^w, j = 1, 2, ..., n \end{cases}$$
(6)

By solving the model in (6), the optimal fuzzy weights of all DMs $\tilde{w}_j^* = (\tilde{w}_1^*, \tilde{w}_2^*, ..., \tilde{w}_n^*)$ and optimal value of ξ^* are obtainable.

In order to calculate the consistency ratio (*CR*), ξ^* is used. The *CR* can be obtained according to *CR* = $\frac{\xi^*}{CI}$. This ratio is acceptable when *CR* < 0.1 [38]. The maximum possible value of consistency index (*CI*) in linguistic variables for fuzzy BWM, is given in Table 1. 213 214

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Table 2

Linguistic variables for ranking failure modes									
Linguistic variables	Very low (VL)	Low (L)	Slightly Low (SL)	Medium (M)	Slightly High (SH)	High (H)	Very High (VH)		
TFNs	(0,0,1)	(0,1,3)	(1,3,5)	(3,5,7)	(5,7,9)	(7,9,10)	(9,10,10)		

229 2.3. Fuzzy MOORA

The fuzzy MOORA is developed in three different approaches, the ratio method, reference point approach, and full multiplicative form [39]. In this study, the fuzzy ratio approach in [40] is considered for further investigation. In this method, linguistic variables in Table 2 are used for rating failure modes; for implementing this method, steps are as follow:

Step 1: A set of TFNs are used to create a decision matrix with *m* alternatives and *n* criteria:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & & & \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}$$
(7)

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where $\tilde{x}_{ij} = (a_{ij}^x, b_{ij}^x, c_{ij}^x)$ is TFN and i = 1, 2, ..., m, j = 1, 2, ..., n

Step 2: Normalized elements of decision matrix:

$$t_{ij}^{x*} = \frac{t_{ij}^{x}}{\sqrt{\sum_{i=1}^{m} \left[(a_{ij}^{x})^{2} + (b_{ij}^{x})^{2} + (c_{ij}^{x})^{2} \right]}}, t = a, b, c$$
(8)

Step 3: The multiplication of \tilde{w}_j by the normalized decision matrix results in the formation of the weighted normalized decision matrix as:

$$\tilde{v}_{ij} = (a_{ij}^{v}, b_{ij}^{v}, c_{ij}^{v}) = (a_{j}^{w} a_{ij}^{x}, b_{j}^{w} b_{ij}^{x}, c_{j}^{w} c_{ij}^{x})$$
(9)

Step 4: The \tilde{v}_{ij} is used to calculate the performance of the normalized value by subtracting the useless criteria from the total of useful criteria:

$$\tilde{y}_i = \sum_{j=1}^g \tilde{v}_{ij} - \sum_{j=g+1}^n \tilde{v}_{ij}$$
(10)

where \tilde{y}_i is a TFN, $\sum_{j=1}^{g} \tilde{v}_{ij}$ and $\sum_{j=g+1}^{n} \tilde{v}_{ij}$ are beneficial and non-beneficial criteria, respectively; g and n - g are numbers of beneficial and non-beneficial criteria, respectively. 243

Step 5: The best non-fuzzy performance (BNP) [41] is used to convert the fuzzy performance values that are normalized to a non-fuzzy value:

$$BNP_{i}(\tilde{y}_{i}) = y_{i} = \frac{(c_{i}^{y} - a_{i}^{y}) + (b_{i}^{y} - a_{i}^{y})}{3} + a_{i}^{y}$$
(11)

The ranking of the failure modes can be performed using BNP when the values are sorted from the largest to the smallest. The largest value is considered to be the most important one.

3. Proposed approach

Using the fuzzy BWM for weighting criteria and fuzzy MOORA for ranking failure modes, the proposed method in this study is divided in three main stages.

In the first stage, five experienced clerks in the OPD section of a hospital, defined 11 key failure modes resulted in patient waiting time management, using brainstorming (see Table 3). The values of the three factors of RPN are also given in Table 4.

In the second step, the fuzzy BWM method is used to determine the importance of RPN factors and weigh them, such that at first, the best and worst criteria are determined and then paired comparisons are made based on the linguistic data. Consequently, by using the fuzzy BWM model in (6), the optimal weight of the criteria is determined.

In the third step, the ranking of failure modes for managing patient waiting time at OPD was performed by utilizing the fuzzy MOORA method using linguistic variables. The output of this model is to prioritize key criteria in managing patient waiting time at OPD. Finally, the results of the proposed method are compared with Conventional RPN, Conventional MOORA, and fuzzy MOORA. 245 246 247

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Failure Modes	Definitions
F ₁	Long check-in time of patients in OPD
F ₂	Postpone/cancel appointments due to emergency cases
F ₃	Delay of the visit because of first new
	patient appointment.
F ₄	Disturb others because of boring waiting time for patients results in disorder
F ₅	Patient exit the office without checkout process
F ₆	Completed visit but not within the expected time
F ₇	Patient exit office, but lost to adjoin in primary care offices since no direction provided on location of checkout
F ₈	Office visit completed but patient have not given any recommendations or booklet
F9	Returned patients make disorder because they are familiar with the checkout office and the checkout process
F ₁₀	Patients exit the office, considering
	receiving a phone call later
F ₁₁	No entertainment or special space for kids results in chaos

Table 3 Selected failure modes

Ta	ble	4		
Traditional ratings	for	RPN	factors	[42]

Rating	S	0	D
10	Hazardous without warning	Very high: Almost failure is	Absolute uncertainty
9	Hazardous with warning	inevitable	
8	Very high	High: repeated	High: repeated
7	High	failures	failures
6	Moderate	Moderate:	Moderate:
5	Low	occasional failures	occasional
4	Very low		failures
3	Minor	Low: relatively	Low: relatively
2	Very minor	few failures	few failures
1	None	Remote: unlikely failure	Remote: failure is unlikely

Figure 1, illustrates a summary of the proposed method.

4. Results and discussion

In this section, the results of implementing the proposed approach in order to reduce the patient's waiting time presented and discussed. In the first step, according to the first phase of this approach, conventional RPN method, failure modes are identified by



the FMEA team and the values of the three effective criteria SOD for each failure mode are determined (see Table 3).

Then, according to FMEA teams' opinions linguistic variables are assigned into each risk factors (Table 5) and consequently corresponding TFN in Table 2 are assigned to each linguistic variable.

Thereafter, the weights of the TFNs are determined using the fuzzy BWM method. For this purpose, the experts identified the best and worst factor in prolongation of waiting time due are identified based on experts' experience and their importance relative to other factors (paired comparisons) in the form of linguistic variables in Table 1 (see Table 6). For instance, for making first best vector, TM_1 identified O as a best criterion, then the importance of O compared with the other factors. The comparison results are written in fuzzy number and by using (6), all limitations are found and the BO and OW vectors are calculated as follows:

$$\tilde{P}_W = [(5/2, 3, 7/2), (3/2, 2, 7/2), (1, 1, 1)]$$

The mathematical programming model in (6) is 301 updated as

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Failure		S	everity (S	5)			Occurrence (O)				Detection (D)				
Modes								TM No.							
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F ₁	1(L)	2(SL)	1(L)	1(L)	2(SL)	2(SL)	2(SL)	1(L)	2(SL)	1(L)	2(SL)	2(M)	1(L)	2(SL)	1(SL)
F ₂	7(H)	9(H)	9(VH)	9(H)	5(SH)	7(H)	5(SH)	9(H)	8(H)	7(SH)	7(H)	5(H)	9(SH)	8(H)	7(SH)
F ₃	2(SL)	1(L)	1(L)	2(SL)	2(SL)	1(L)	2(SL)	2(SL)	1(L)	2(SL)	1(L)	2(VL)	2(L)	1(VL)	2(VL)
F ₄	3(M)	5(SH)	7(SH)	3(M)	5(SH)	3(M)	2(SL)	3(M)	1(SL)	3(M)	3(M)	2(SH)	3(H)	1(SH)	3(SH)
F ₅	5(SH)	3(M)	7(SH)	5(SH)	8(H)	2(SL)	3(M)	2(SL)	2(SL)	3(M)	2(SH)	3(H)	2(H)	2(SH)	3(SH)
F ₆	7(SH)	3(M)	5(SH)	7(SH)	3(M)	3(M)	3(M)	3(M)	6(SH)	2(SL)	3(SH)	3(SH)	3(M)	6(SH)	2(M)
F ₇	1(L)	2(SL)	2(SL)	3(M)	1(L)	2(SL)	3(M)	6(SH)	3(M)	3(M)	2(M)	3(M)	6(M)	3(SH)	3(M)
F ₈	3(M)	6(SH)	3(M)	3(M)	2(SL)	3(M)	3(M)	3(M)	2(SL)	3(M)	3(SL)	3(SL)	3(SL)	2(SL)	3(L)
F9	7(SH)	3(M)	2(SL)	3(M)	3(M)	3(M)	2(SL)	2(SL)	3(M)	2(SL)	3(L)	2(VL)	2(VL)	3(L)	2(VL)
F ₁₀	5(SH)	9(H)	7(H)	7(SH)	8(H)	7(H)	9(VH)	7(H)	9(H)	8(H)	7(VH)	9(VH)	7(VH)	9(H)	8(H)
F11	3(M)	7(SH)	2(SL)	3(M)	3(M)	7(SH)	3(M)	3(M)	2(SL)	3(M)	7(M)	3(SL)	3(SL)	2(M)	3(M)

 Table 5

 FMEA teams' opinions for risk factors scoring in managing patients' waiting time

Table 6	
Best and worst of triple factors based on FMEA teams'	opinions

No. of Team		BO v risk	ector of factors		OW vector of risk factors			
	Best	S	0	D	Worst	S	0	D
ГM ₁	0	EI	FI	Ι	D	Ι	FI	EI
ΓM_2	S	Ι	EI	WI	D	1	FI	EI
ГM ₃	0	EI	Ι	FI	D	FI	WI	EI
ΓM_4	S	FI	EI	Ι	D	VI	Ι	EI
ГM ₅	0	EI	FI	WI	S	VI	EI	WI

 $Min = \xi$

$$s.t.\begin{cases} -\xi c_{2} \leq a_{1} - 1.5c_{2} \leq \xi c_{2} \\ -\xi c_{3} \leq a_{1} - 2.5c_{3} \leq \xi c_{3} \\ -\xi c_{3} \leq a_{2} - 1.5c_{3} \leq \xi c_{3} \\ -\xi b_{2} \leq b_{1} - 2b_{2} \leq \xi b_{2} \\ -\xi b_{3} \leq b_{1} - 3b_{3} \leq \xi b_{3} \\ -\xi b_{3} \leq b_{2} - 2b_{3} \leq \xi b_{3} \\ -\xi a_{2} \leq c_{1} - 2.5a_{2} \leq \xi a_{2} \\ -\xi a_{3} \leq c_{2} - 2.5a_{3} \leq \xi a_{3} \\ -\xi a_{3} \leq c_{2} - 2.5a_{3} \leq \xi a_{3} \\ \frac{a_{1} + 4b_{1} + c_{1}}{6} + \frac{a_{2} + 4b_{2} + c_{2}}{6} + \frac{a_{3} + 4b_{3} + c_{3}}{6} = 1 \\ 0 \leq a_{i} \leq b_{i} \leq c_{i}, i = 1, 2, 3 \\ \xi \geq 0 \end{cases}$$
(12)

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Model (12) solved and the results are presented in Table 7. Given that the largest linguistic variable based on experts' opinion for the best factor is selected as Important (I), the CR calculated and all result showed the value smaller than 0.1, which verify that results are acceptable.

In the third phase of the proposed approach, based on the results of the first and second phases, risk scenario ranking is performed using the fuzzy MOORA method. Initially, the weighted normalized matrix is obtained by considering the weights of the three SOD factors (see Table 8).

As outlined in the proposed approach, in this section, the fuzzy ratio system approaches from the fuzzy MOORA method is implemented. Table 9 shows the results of the BNP, taking into account the uncertainty in the SOD factors.

The uncertainty of the SOD factors and weighing of these factors are considered, and failure modes have been re-ranked using RPN, conventional MOORA and fuzzy MOORA methods and the result is summarized in Table 10.

According to Table 10 and based on the traditional RPN, the risk of F_{10} with RPN = 720 has been addressed in the first priority. In addition, the risks F_1 and F_9 with RPN = 12 are jointly in the eighth priority and the risks F_7 and F_8 with RPN = 60 are jointly in the seventh priority. With a general review of the prioritization of risks based on traditional FMEA, it can be concluded that prioritization of risks has been

	Weights of factors listed as a TFN										
No. of	No. of S			S O				D	ξ*	CR	
Team	a	b	с	a	b	с	a	b	с		(CI = 6.69)
TM ₁	0.517	0.517	0.597	0.261	0.288	0.400	0.161	0.161	0.202	0.209	0.031
TM_2	0.267	0.282	0.338	0.418	0.489	0.597	0.216	0.216	0.216	0.438	0.065
TM_3	0.181	0.181	0.181	0.262	0.280	0.280	0.402	0.551	0.642	0.504	0.075
TM_4	0.123	0.148	0.148	0.222	0.444	0.556	0.370	0.444	0.444	0.228	0.028
TM_5	0.247	0.342	0.342	0.190	0.262	0.262	0.314	0.445	0.445	0.203	0.025
w _j *	0.375	0.448	0.493	0.271	0.353	0.419	0.186	0.210	0.218		

Table 7 Weights of factors listed as a TFN

	Table 8	
Normalized	fuzzy assessment	matri

			Normalized fuzzy assessment matrix						
Failure Modes	S			0			D		
F ₁	0.02	0.07	0.14	0.01	0.05	0.11	0.04	0.09	0.15
F ₂	0.20	0.27	0.31	0.21	0.26	0.29	0.18	0.24	0.28
F ₃	0.02	0.07	0.14	0.02	0.07	0.12	0.00	0.01	0.05
F ₄	0.07	0.14	0.20	0.12	0.18	0.24	0.15	0.21	0.26
F ₅	0.02	0.07	0.14	0.15	0.21	0.26	0.17	0.23	0.28
F ₆	0.10	0.16	0.23	0.12	0.18	0.24	0.12	0.18	0.24
F ₇	0.10	0.16	0.23	0.03	0.08	0.14	0.10	0.16	0.22
F ₈	0.08	0.15	0.22	0.09	0.15	0.21	0.02	0.08	0.14
F9	0.06	0.12	0.19	0.09	0.15	0.21	0.00	0.01	0.05
F ₁₀	0.24	0.30	0.33	0.18	0.24	0.28	0.24	0.28	0.30
F ₁₁	0.10	0.16	0.23	0.09	0.15	0.21	0.07	0.12	0.18

Table 9 The value of BNP for each failure mode

Failure modes	\tilde{y}_i	<i>Yi</i>
F ₁	(0.02, 0.07, 0.15)	0.078
F ₂	(0.17,0.26, 0.34)	0.256
F ₃	(0.01, 0.06, 0.13)	0.067
F ₄	(0.09, 0.17, 0.26)	0.172
F ₅	(0.08, 0.15, 0.24)	0.157
F ₆	(0.09, 0.18, 0.27)	0.179
F ₇	(0.06, 0.13, 0.22)	0.138
F ₈	(0.06, 0.14, 0.22)	0.140
F9	(0.05, 0.11, 0.19)	0.116
F ₁₀	(0.19, 0.28, 0.34)	0.270
F11	(0.07, 0.15, 0.24)	0.155

Table 10 Comparison of prioritized results

Failure Modes	Conventional FMEA		Conventional MOORA		Fuzzy MOORA	
	RPN	Rank	Rank y_i Rank			Rank
F1	12	8	0.152	9	0.078	10
F2	576	2	0.543	2	0.256	2
F3	4	9	0.109	10	0.067	11
F4	245	3	0.412	3	0.172	4
F5	224	4	0.412	3	0.157	5
F6	210	5	0.391	4	0.179	3
F7	60	7	0.282	6	0.138	8
F8	60	7	0.261	7	0.140	7
F9	12	8	0.195	8	0.116	9
F10	720	1	0.586	1	0.270	1
F11	125	6	0.326	5	0.155	6

done in a way that risks are grouped into eight categories. It indicates that the prioritization based on this traditional index is not fully ranked and confuses the decision-maker in risk management and corrective/preventive action planning.

Based on conventional MOORA method, F_{10} with $y_i = 0.586$, F_2 with $y_i = 0.543$ and F_4 , F_5 with $y_i = 0.412$ are in first, second and third rank, respectively. The prioritization of failure modes based on conventional MOORA, are grouped in nine categories. Therefore, the aim of conventional MOORA utilization is partially improving the shortcoming of traditional FMEA, where the number of categories increased from eight to nine.

Using the fuzzy MOORA method, it is observed that all identified risks are in distinct priorities. In other words, the proposed method of this study, considering the uncertainty of the risk scenario, has tried to resolve some of the main deficiencies of the traditional RPN and the conventional MOORA method. In this method, the rank of F_{10} , F_2 and F_4 has not changed in comparison of two other methods, but failure modes are fully ranked in 11 categories.

In summary, the non-interference weight of SOD factors, as well as the certainty in the process, is the result of conventional FMEA deficiencies. In the conventional MOORA method, contributing experts' ideas in the decision-making matrix results on increasing the number of categorizations from

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Fig. 2. Prioritized results from the proposed method.

eight to nine groups. However, decisive decisionmaking matrix and uncertainty exist in the experts'
decisions leads to imperfect ranking. To cover the
deficiencies of above-mentioned traditional methods,
in the fuzzy MOORA, the decision of the experts contributed in the TFN form and the weight of the SOD
factors is obtained through the fuzzy BWM method.

369 4.1. Sensitivity analysis

In this article, for the uncertainty reduction in 370 the obtained outputs, sensitivity analysis is used. In 371 sensitivity analysis, weights of risk factors changed 372 according to the fuzzy group matrix, such that, the 373 obtained original weights from fuzzy BWM are used 374 in case 0. However, in other cases (case 1, case 2, 375 case 3 and case 4) different weights for risk factors 376 are defined. Table 11 and Fig. 3 present the result of 377 sensitivity analysis and indicate that the most impor-378 tant failure mode is F_{10} which is in the first place in all 379 cases. Furthermore, due to the same values of SOD, 380 in F_1 , F_2 , F_3 , F_4 , F_9 , F_{11} despite the weight changing, 381



there was no change in failure mode ranking. However, ranking of F_5 , F_6 , F_7 , F_8 was changed in various cases due to the different value of SOD.

According to the experts' opinions, in F_7 , D criteria have more value than S and O, therefore, if weights of D criteria increases, F_7 will place in higher ranks. Contrariwise, in F_8 , increment of D criteria's weight causes lower rank of this failure mode. In addition, there are big values of uncertainties in ranking F_5 , F_6 factors which cause by the proximity of RPN's amount. The result of sensitivity analysis shows that weights of risk factors have a significant effect on the final ranking order. Consequently, appropriate weights based on hospital conditions and experts' opinions can result in accurate ranking and correct actions.

. Conclusion

Patient waiting time is an essential factor in choosing health centers because of increasing demand for providing effective health service, and intensive competitiveness among health centers. In this

Sensitivity analysis for unreferr cases								
Failure Modes	Case 0	Case 1	Case 2	Case 3	Case 4			
	$W_{S} = 0.35$	$W_{\rm S} = 0.2$	$W_{S} = 0.2$	$W_{S} = 0.3$	$W_{S} = 0.5$			
	$W_0 = 0.443$	$W_0 = 0.65$	$W_0 = 0.4$	$W_0 = 0.4$	$W_0 = 0.25$			
	$W_{\rm D} = 0.207$	$W_{\rm D} = 0.15$	$W_{\rm D} = 0.4$	$W_{\rm D} = 0.3$	$W_{\rm D} = 0.25$			
F ₁	10	10	10	10	10			
F ₂	2	2	2	2	2			
F ₃	11	11	11	11	11			
F4	4	4	4	4	4			
F ₅	5	5	3	5	6			
F ₆	3	3	5	3	3			
F ₇	8	8	7	7	7			
F ₈	7	7	8	8	8			
F9	9	9	9	9	9			
F ₁₀	1	1	1	1	1			
F ₁₁	6	6	6	6	5			

 Table 11

 Sensitivity analysis for different cases

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paper, to identify the failure modes for patient wait-403 ing time management, fuzzy-BWM presented for 404 weighting criteria. Thereafter, failure modes ranked 405 through fuzzy-MOORA method. The main purpose 406 of these methods is to overcome the shortcoming 407 of conventional FMEA (RPN index). In contrast, 408 the fuzzy-MOORA contributes certainty and produce 409 full prioritization of failure modes. The comparison 410 of prioritization results through the proposed method 411 and convention RPN, MOORA, prove the effective-412 ness of the proposed method. 413

Generally, in order to manage patient waiting time, the following three key scenarios are prioritized:

1) The main failure occurs when the patient com-416 pletes office visit, and never receives a phone call 417 from health center for the next session. Human errors 418 are the main reason for this failure. Therefore, the 419 patient referral in next days (in-person or phone) 420 results in chaos in the OPD section and prolonga-421 tion of other patients waiting time. The recommended 422 action is that all other sections, direct patients to 423 checkout station. 424

2) Postponing/canceling appointments due to 425 emergency cases. The cause of failure is that patient 426 feels rest less and may cancel the appointment and opt 427 another hospital. The main effect is on the hospital 428 reputation and number of patients would be reduced 429 because of some disorders. The action proposed for 430 this failure is engaging interactive activities for spe-431 cialist clinics. 432

3) Failure happens when office visit completes
but not in the estimated time. The possible effect is
the patient's opinion changing about hospital. Preplanning appointments through online platforms can
be helpful to prevent this failure occurrence.

438 5.1. Limitations and future scope

Overlooking the cause and effect relation of failure 439 modes is the main limitation of this study. Future stud-440 ies can address this problem through the cognitive 441 map based on Z-number theory [43]. The proposed 442 approach in this study can also use for qualitative 443 assessment data in a complex decision-making envi-444 ronment based on Type II fuzzy sets, D-number [44], 445 R-number [45], and G-number [46]. 446

In addition, in order to manage patient waiting time efficiently in healthcare industry, we need to position the patients' order in the right place of the healthcare supply chain. The patient order penetration point [47–52] defines the stage in the healthcare value chain, where a personalized healthcare service such as treatment is linked to a specific patient order, such as organ transplants and the blood transfusion [53]. The challenge would be more critical for servicing and tracing the large scaled markets [54, 55].

References

- A.I. Almomani and A. AlSarheed, Enhancing outpatient clinics management software by reducing patients' waiting time, *Journal of infection and public health* 9(6) (2016), 734–743.
- [2] A. Wijewickrama and S. Takakuwa, Simulation analysis of appointment scheduling in an outpatient department of internal medicine, In *Proceedings of the Winter Simulation Conference*, 2005. (pp. 2264-2273). IEEE.
- [3] L. Alrubaiee and F. Alkaa'ida, The mediating effect of patient satisfaction in the patients' perceptions of healthcare quality-patient trust relationship, *International Journal* of Marketing Studies 3(1) (2011), 103–127.
- [4] B. Aman and F. Abbas, Patient's perceptions about the service quality of public hospitals located at District Kohat, J Pak Med Assoc 66(1) (2016), 72–75.
- [5] A. Alkuwaiti, T. Maruthamuthu and S. Akgun, Factors associated with the quality of outpatient service: The application of factor analysis-A case study, *International Journal of Healthcare Management* (2018), 1–6.
- [6] N. Mustafa, T.A. Salim and A. Watson, The Impact of Waiting Time on Hospital Service Perception and Satisfaction: The moderating role of Gender, *International Journal of Business & Management Science* 8(1) (2018), 131–150.
- [7] S. Mendoza, R.C. Padpad, A.J. Vael, C. Alcazar and R. Pula, A Web-Based "InstaSked" Appointment Scheduling System at Perpetual Help Medical Center Outpatient Department. In World Congress on Engineering and Technology; Innovation and its Sustainability, 2018, November, (pp. 3-14). Springer, Cham.
- [8] M. Ortiz-Barrios, G. Jiménez-Delgado, S. McClean and G. Polifroni-Avendaño, Using Computer Simulation for Reducing the Appointment Lead-Time in a Public Pediatric Outpatient Department, In *International Conference* on Human-Computer Interaction, 2019, July (pp. 75-86). Springer, Cham.
- [9] Z. Zhu, B.H. Heng and K.L. Teow, Analysis of factors causing long patient waiting time and clinic overtime in outpatient clinics, *Journal of Medical Systems* 36(2) (2012), 707–713.
- [10] B.P. Mahesh, B. Soragaon and A.R. Annigeri, Reduction of Patient Wait Time at a Multi-Specialty Hospital using DMAIC Methodology and Factor Analysis, *International Journal of Engineering & Technology* 7(4.25) (2018), 309–312.
- [11] H-C. Liu, L. Liu and N. Liu, Risk evaluation approaches in failure mode and effects analysis: A literature review, *Expert Systems with Applications* 40(2) (2013) 828–838.
- [12] M.J. Rezaee, A. Salimi and S. Yousefi, Identifying and managing failures in stone processing industry using cost-based FMEA, *The International Journal of Advanced Manufacturing Technology* 88(9-12) (2017), 3329–3342.
- [13] H. Safari, Z. Faraji and S. Majidian, Identifying and evaluating enterprise architecture risks using FMEA and fuzzy VIKOR, *Journal of Intelligent Manufacturing* 27(2) (2016), 475–486.

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[15] B. Duwe, B.D. Fuchs and J. Hansen-Flaschen, Failure mode and effects analysis application to critical care medicine, Critical Care Clinics 21(1) (2005), 21-30.

ficial Intelligence 34 (2014), 168-177.

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- [16] N.Y. Yang and M.H. Lee, Analysis of effects of chemotherapy using failure mode and effect analysis (FMEA) on patient safety and safe nursing, Journal of Korean Academy of Nursing Administration 21(3) (2015), 254-262.
- [17] S. Abbasgholizadeh Rahimi, A. Jamshidi, D. Ait-Kadi and A. Ruiz, Using fuzzy cost-based FMEA, GRA and profit-ability theory for minimizing failures at a healthcare diagnosis service, Quality and Reliability Engineering International 31(4) (2015), 601-615.
- [18] K. Jain, Use of failure mode effect analysis (FMEA) to improve medication management process, International Journal of Health Care Quality Assurance 30(2) (2017), 175 - 186
- [19] R. Askari, M. Shafii, S. Rafiei, M.S. Abolhassani and E. Salarikhah, Failure mode and effect analysis: Improving intensive care unit risk management processes, International Journal of Health Care Quality Assurance 30(3) (2017), 208-215.
- [20] D.M. Barends, M.T. Oldenhof, M.J. Vredenbregt and M.J. Nauta, Risk analysis of analytical validations by probabilistic modification of FMEA, Journal of Pharmaceutical and Biomedical Analysis 64 (2012), 82-86.
- [21] H.C. Liu, L. Liu and N. Liu, Risk evaluation approaches in failure mode and effects analysis: A literature review, Expert Systems with Applications 40(2) (2013), 828-838.
- [22] M. Velasquez and P.T. Hester, An analysis of multi-criteria decision making methods, International Journal of Operations Research 10(2) (2013), 56-66.
- [23] Ž. Rađenović and I. Veselinović, Integrated AHP-TOPSIS method for the assessment of health management information systems efficiency, Economic Themes 55(1) (2017), 121-142.
- [24] M. Shafii, S.M. Hosseini, M. Arab, E. Asgharizadeh and F. Farzianpour, Performance analysis of hospital managers using fuzzy AHP and fuzzy TOPSIS: Iranian experience, Global Journal of Health Science 8(2) (2016), 137-155.
- [25] G. Büyüközkan, G. Çifçi and S. Güleryüz, Strategic analysis 556 557 of healthcare service quality using fuzzy AHP methodology, Expert Systems with Applications 38(8) (2011), 558 9407-9424. 559
 - A. Calabrese, R. Costa, N. Levialdi and T. Menichini, [26] Integrating sustainability into strategic decision-making: A fuzzy AHP method for the selection of relevant sustainability issues, Technological Forecasting and Social Change 139 (2019), 155-168.
 - [27] F. Abdi, Hospital leanness assessment model: A Fuzzy MULTI-MOORA decision making approach, Journal of Industrial and Systems Engineering 11(3) (2018), 37–59.
- [28] H. Alilou, O. Rahmati, V.P. Singh, B. Choubin, B. Pradhan, 568 S. Keesstra, S.S. Ghiasi and S.H. Sadeghi, Evaluation of 569 watershed health using Fuzzy-ANP approach considering 570 geo-environmental and topo-hydrological criteria, Journal 571 of Environmental Management 232 (2019), 22-36. 572
- [29] J. Rezaei, Best-worst multi-criteria decision-making 573 method, Omega 53 (2015), 49-57. 574
- [30] J. Li, J.Q. Wang and J.H. Hu, Multi-criteria decision-575 making method based on dominance degree and BWM with 576 probabilistic hesitant fuzzy information, International Jour-577

nal of Machine Learning and Cybernetics 10(7) (2019), 1671-1685.

- S.J. Ghoushchi, S. Yousefi and M. Khazaeili, An extended FMEA approach based on the Z-MOORA and fuzzy BWM for prioritization of failures, Applied Soft Computing 81 (2019), 105-505.
- [32] D. Adebanjo, T. Laosirihongthong and P. Samaranayake, Prioritizing lean supply chain management initiatives in healthcare service operations: A fuzzy AHP approach, Production Planning & Control 27 (12) (2016), 953-966.
- [33] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.
- [34] N. Werro, Fuzzy classification of online customers, 148 (2015). Heidelberg: Springer.
- Z.P. Tian, J.O. Wang and H.Y. Zhang, An integrated [35] approach for failure mode and effects analysis based on fuzzy best-worst, relative entropy, and VIKOR methods, Applied Soft Computing 72 (2018), 636-646.
- [36] S. Guo, and H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems 121 (2017), 23-31.
- [37] S.B. Tsai, J. Zhou, Y. Gao, J. Wang, G. Li, Y. Zheng, P. Ren and W. Xu, Combining FMEA with DEMATEL models to solve production process problems, PloS one 12 (8) (2017), e0183634.
- [38] T.L. Saaty, Analytic hierarchy process, Encyclopedia of Operations Research and Management Science (2013) 52-64. Springer, Boston, MA.
- [39] G. Akkaya, B. Turanoğlu and S. Öztaş, An integrated fuzzy AHP and fuzzy MOORA approach to the problem of industrial engineering sector choosing, Expert Systems with Applications 42(24) (2015), 9565-9573.
- [40] L.E. Wang, H.C. Liu and M.Y. Quan, Evaluating the risk of failure modes with a hybrid MCDM model under inter-valvalued intuitionistic fuzzy environments, Comput Ind Eng 102 (2016), 175-185.
- [41] J. Wu, J. Tian and T. Zhao, Failure mode prioritization by improved RPN calculation meth-od. In 2014 Reliability and Maintainability Symposium (2014) (pp. 1-6). IEEE.
- [42] A. Baležentis, T. Baležentis and W.K. Brauers, Personnel selection based on computing with words and fuzzy MUL-TIMOORA, Expert Systems with Applications 39(9) (2012), 7961-7967.
- [43] L.A. Zadeh, A note on Z-numbers, Information Sciences 181 (14) (2011), 2923-2932.
- [44] Y. Deng, D numbers: Theory and applications, Journal of Information & Computational Science 9(9) (2012), 2421-2428
- [45] H. Seiti, A. Hafezalkotob, L. Martínez and R-numbers, A new risk modeling associated with fuzzy numbers and its application to decision making, Information Sciences 483 (2019), 206–231.
- [46] S.J. Ghoushchi and M. Khazaeili, G-Numbers: Importance-Necessity Concept in Uncertain Environment, International Journal of Management and Fuzzy Systems 5(1) (2019), 27-32.
- [47] A. Hajfathaliha, Abbas, E. Teimoury, I. Ghaleh Khondabi and M. Fathi, Using queuing approach for locating the order penetration point in a two-echelon supply chain with customer loss, International Journal of Business and Management 6(1) (2011), 258-268.
- [48] E. Teimoury and M. Fathi, A queueing-game model for making decisions about order penetration point in supply chain in competitive environment, International Journal of Strategic Decision Sciences 4(4) (2013), 1–24.

641

- [49] E. Teimoury, M. Modarres, F. Ghasemzadeh and M. Fathi, A queueing approach to production-inventory planning for supply chain with uncertain demands: Case study of PAK-SHOO Chemicals Company, *Journal of Manufacturing Systems* 29(2–3) (2010), 55–62.
- [50] E. Teimoury, M. Modarres, A. Kazeruni Monfared and M.
 Fathi, Price, delivery time, and capacity decisions in an
 M/M/1 make-to-order/service system with segmented mar ket, *The International Journal of Advanced Manufacturing Technology* 57(1–4) (2011), 235–244.
- [51] E. Teimoury, M. Modarres, I. Ghaleh Khondabi and M.
 Fathi, A queuing approach for making decisions about order
 penetration point in multiechelon supply chains, *The Inter- national Journal of Advanced Manufacturing Technology* **63**(1-4) (2012), 359–371.
- [52] E. Teimoury and M. Fathi, An integrated operationsmarketing perspective for making decisions about order penetration point in multi-product supply chain: A queuing approach, *International Journal of Production Research* 51(18) (2013), 5576–5596.
- [53] M. Fathi and M. Khakifirooz, Kidney-related operations research: A review, *IISE Transactions on Healthcare Systems Engineering* (2019), 1–17.
- [54] M. Fathi, Mahdi, M. Khakifirooz and P.M. Pardalos, Optimization in Large Scale Problems: Industry 4.0 and Society 5.0 Applications, **152** (2019), Cham: Springer.
- [55] J. Velasquez, M. Khakifirooz and M. Fathi, Large Scale Optimization Applied to Supply Chain and Smart Manufacturing: Theory and Applications 149 (2019), Cham: Springer.