SATELLITE IMAGE RESOLUTION ENHANCEMENT USING DISCRETE WAVELET TRANSFORM

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Abstract: In this paper, an image resolution enhancement algorithm based on discrete wavelet transform is proposed. The DWT uses the well known image resolution enhancement algorithm based on wavelet decomposition. The main focus of this paper lies in the optimization of DWT wavelet filters based on parameters related to edges in the image. Initially all the input images are classified into various bins based on the edges present in the image and each bin of images will be enhanced by applying unique wavelet filters. This library of wavelet filters are evolved using bio-inspired algorithm like genetic algorithm considering individual image bins. The individual image groups of satellite images are created based on the spatial frequency mean (SFM) and DWT wavelet filters are evolved for each groups for both near edge and far edge image namely local and global DWT filters. These enhancement algorithms are based on existing DWT-based methods are proposed to improve the resolution of the low-resolution satellite images. The proposed algorithms suitably enhance the resolution of the image shows a significant improvement in the PSNR compared with the existing techniques.

***IndexTerms* - Image Resolution Enhancement, Wavelet Transform, Genetic Algorithm, Evolved Wavelet Coefficients, and Satellite Imagery.**

I. INTRODUCTION

Improving the resolution of satellite images is important as the satellite imagery is of low resolution and needs to improve for any practical viewing and further applications. There are few image interpolation techniques which are good for conventional photographic images, but may not suite for satellite taken pictures. Also many authors have considered frequency domain techniques which involve wavelets for satellite pictures. In this research paper, we propose to use wavelet based resolution enhancement techniques but adding to the existing methods, the authors improves the algorithm in various levels to further enhance the resolution on the existing images. A discrete wavelet transform (DWT) based satellite image enhancement algorithms is proposed by Demirel et al [1] and it utilizes the DWT subbands and interpolation algorithms to improve the size of high-frequency subband images. The input low-resolution images have been interpolated directly to use it as low frequency LL subband image [1]. Celik et al used a complex wavelet-domain image resolution enhancement algorithm based on the estimation of wavelet coefficients. Using dual-tree complex wavelet transform high-resolution images are reconstructed from the wavelet coefficients [2]. A similar work using dual-tree complex wavelet transform (DT-CWT) is carried out for satellite images by Demirel et al [3]. Bendoumi et al proposed a hyperspectral (HS) image resolution enhancement algorithm based on spectral unmixing. The HS algorithm was used to fuse the high-spatial-resolution multispectral (MS) image and the low-spatial-resolution HS image (HSI) [4]. For improving the edge reconstruction of the enhanced image, stationary wavelet transform (SWT) is used to decompose the low resolution image into its sub-bands [5] similar to the work done in [1, 3]. Using dual-tree complex wavelet transform (DT-CWT) and nonlocal means (NLM) filter Iqbal et al proposed an enhancement algorithm using Lanczosinterpolator [6]. Another method proposed by Temizel et al used wavelet coefficient correlation in a local neighbourhood and to estimate the detail coefficients, a linear least-squares regression estimator is used [7]. Also the same wavelet based enhancement is implemented using the cycle-spinning methodology by Temizel et al [8]. By adapting supervised learning techniques, Syrris et al proposed an image enhancement algorithm by reducing the uncertainty to improve the quality of the enhanced image [9].The spatial resolution of hyperspectral imagesare enhanced using fully constrained least squares spectral unmixing and spatial regularization based on modified binary particle swarm optimization [10].Jie Hu et al presented an automatic enhancement method for SAR images based on the mirrorextended curvelettransform and genetic algorithm [12].An investigation on evolutionary computation for image compression shows that it can be used to optimize wavelet coefficients and the transforms are independently trained and tested using three sets of images: digital photographs, fingerprints, and satellite images [13, 14 and 15] and it was concluded that a better evolutionary progress towards an optimized reconstruction transform occurs when both the wavelet and scaling numbers are simultaneously evolved. The spatial frequency based classification of images for image compression using DWT is efficient if separate DWT filter are used for each group of classified images [20].

II. BACKGROUND

The main objective of this paper is to evolve wavelet filter coefficients suitable for image resolution enhancement for various groups of images classified according to their spatial frequency content. A detailed discussion about wavelets and the bio-inspired algorithm would be essential.

2.1. Wavelets and Image Resolution enhancement

The wavelet is a multi-resolution analysis tool widely used in signal and image processing. The analysis of the signal can be carried out at different frequencies and also with different time-resolutions. It should be noted that, there is a trade-off between frequency resolution and time resolution in wavelet. Hence the wavelet can be designed to provide good frequency resolution by giving off the time resolution and vice-versa. Discrete Wavelet Transforms (DWT) is widely used for image resolution enhancement by many researchers recently as it has good response while its coefficients are interpolated. In particular, biorthogonal wavelets prove remarkable capabilities for still images. Perhaps the lifting scheme based DWT converts the high pass and low pass filtering operations into sequence of matrix multiplications and hence it proves to be efficient in terms of computation and memory.

2.1.1. Discrete Wavelet Transform

The wavelet decomposition of the signal into different frequency bands is simply obtained by successive high pass and low pass filtering of the time domain signal. The original input signal x[n] is first passed through a half band high pass filter g[n] and a low pass filter h[n]. After the filtering process, half of the samples can be eliminated according to the Nyquist's rule. The signal now has a highest frequency of π /2 radians instead of π . The signal x[n] can therefore be sub sampled by 2, by discarding every other sample. This constitutes one level of wavelet decomposition as shown in Fig. 1 and can mathematically be expressed as follows:

$$Y_{High}[k] = \sum_n x[n].g[2k-n] \tag{1}$$

$$Y_{Low}[k] = \sum_n x[n].h[2k-n] \tag{2}$$

The above procedure is followed in reverse order for the reconstruction. The signals are up sampled at every level and passed through the synthesis filters $\tilde{g}[n]$ (high pass) and $\tilde{h}[n]$ (low pass) and then added.

$$x'[n] = \sum_{k=-\infty}^{\infty} (Y_{High}[k].\tilde{g}[-n+2k]) + (Y_{Low}[k].\tilde{h}[-n+2k]) \tag{3}$$

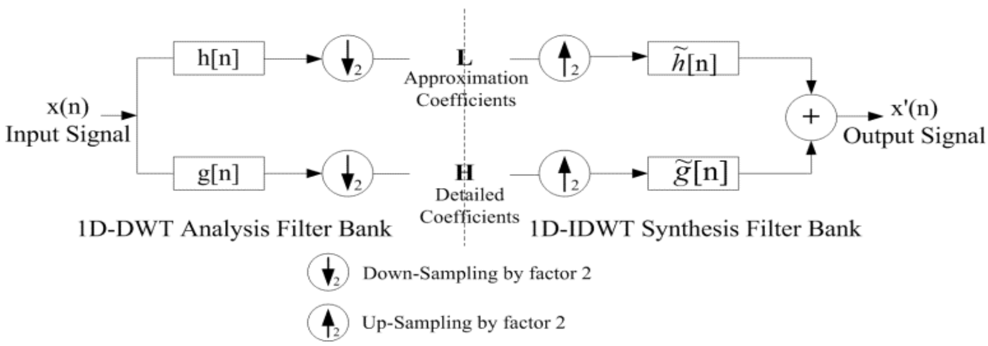


Fig. 1. Single level wavelet transforms using convolution scheme

Fast Wavelet Transform (FWT) and/or Mallat's herringbone algorithm [19] which is a computationally efficient implementation of the DWT is used here to compute the wavelet coefficients. Table 1 shows the CDF 9/7 filter coefficients for both forward and inverse DWT.

Wavelets are described by four sets of coefficients:

1. LOW is the set of wavelet numbers for the forward DWT.
2. HIGH is the set of scaling numbers for the DWT.
3. LOWR is the set of wavelet numbers for IDWT.
4. HIGHR is the set of scaling numbers for the IDWT.

Table 1- CDF 9/7 filter coefficients

n	**Analysis filter coefficients**		**Synthesis filter coefficients**	
	LOW	**HIGH**	**LOWR**	**HIGHR**
1	0.02674875741080976	0.09127176311424948	0.09127176311424948	0.02674875741080976
2	-0.01686411844287495	-0.05754352622849957	-0.05754352622849957	0.01686411844287495
3	-0.07822326652898785	-0.5912717631142470	0.5912717631142470	-0.07822326652898785
4	0.2668641184428723	1.115087052456994	1.115087052456994	-0.2668641184428723
5	0.6029490182363579	-0.5912717631142470	0.5912717631142470	0.6029490182363579
6	0.2668641184428723	-0.05754352622849957	-0.05754352622849957	-0.2668641184428723
7	-0.07822326652898785	0.09127176311424948	-0.09127176311424948	-0.07822326652898785
8	-0.01686411844287495			0.01686411844287495
9	0.02674875741080976			0.02674875741080976

2.2. Genetic Algorithm

Genetic algorithms (*GA*s) (first proposed by Holland), have frequently been used to solve a number of difficult optimization problems. *GA* work by first creating a population of randomly generated chromosomes. Over a number of generations, new chromosomes are created by mutating and recombining chromosomes from the previous generation. Among the total population, the best chromosomes (solutions) are then selected for survival to the next generation based on some fitness criteria that is used to compare chromosomes. The flow diagram of the GA for evolving wavelet filter coefficients is shown in Fig. 4.

Types of Genetic Algorithm

- Binary coded *GA*
- Real Coded Genetic Algorithm(*RCGA*)

2.2.1. Real Coded Genetic Algorithm

In *RCGA* the chromosomes are represented as real valued coefficients. The evolution of filters for image processing requires the simultaneous optimization of many real valued coefficients.

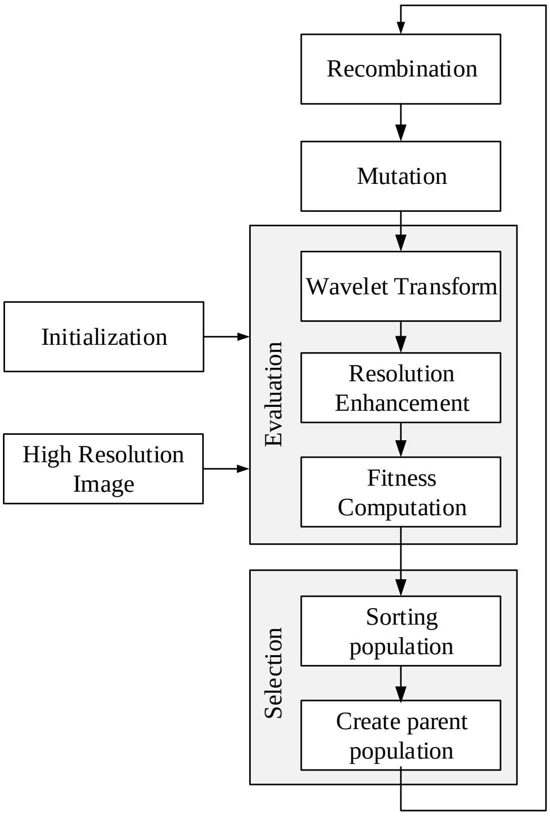


Fig. 2. GA for evolving wavelet filter coefficients (Ruben Salvador et al., [16])

Population Initialization

The initial population includes one chromosome consisting of CDF9/7 filter coefficients. The remaining individuals are copies of the original wavelet filter coefficients multiplied by a small random factor. Additionally, 5% of the filter coefficients are negated. Each chromosome is composed of: low pass filter coefficients, high pass filter coefficients, low pass filter reconstruction coefficients and high pass filter reconstruction coefficients.

Evaluation

The fitness of initial population is evaluated by first performing two dimensional (2-D) *DWT* on the test images and then the conventional decomposition and reconstruction is performed on the transformed coefficients and finally 2-D *IDWT* is carried out to get the reconstructed image and the population is sorted according to the average fitness value.

Image quality (*PSNR*) and distortion (*MSE*) metrics are calculated between the original and the reconstructed image and the *PSNR* value is taken as the fitness measure. *PSNR* and *MSE* between the original (X) and reconstructed ($\hat{X}$) image of size MxN can be calculated using the equations 4 and 5 respectively. Here B represents bits per pixel (bpp).

$$\mathrm{PSNR(dB)} = 20\log_{10}\left(\frac{2^{B}-1}{\sqrt{\mathrm{MSE}}}\right) \quad (4)$$

$$\mathrm{MSE} = \frac{\|X-\hat{X}\|^2}{MN} \quad (5)$$

An MSE=0 in a reconstructed image indicates that $\hat{X}$ is a perfect reconstruction of X. Increasing values of MSE corresponds to increasing error.

New Population Creation

Once the population is evaluated for its performance, the new population is created from the parent population by:

- *Sorting* the population according to the evaluated fitness measure
- *Selecting* the parents for reproduction by random/stochastic uniform selection methods.
- *Reproducing* the population for the next generation.

Reproduction (Recombination and Mutation)

The new population for next generation is created by crossover and mutation.

- *Elite* - Ne, represents the number of best individuals which is copied from the parent population to the new population, Ne is elite count number.
- *Heuristic Crossover* - by which a child is created from two parents P_i^1 and P_i^2 biased in the direction of the parent with better fitness. Assuming P_i^1 has better fitness than P_i^2, then a child gene C_i is created as;

  $$C_i = r(P_i^1 - P_i^2) + P_i^1, \qquad (21)$$

  where r is randomly chosen in the interval [0, 1].
- *Gaussian Mutation* **-** Mutation is required to avoid the premature population convergence in RCGA. Given a parent vector P, a new child vector C is created by $C = P + M$, M is based on Gaussian mutation**,** where the mutation shrinks in successive generations. Mutation Shrink rate controls the rate at which the average amount of mutation decreases. In early generations, the large variance permits quick exploration of the search space. Towards the end of the run, the variance is quite small, and the mutation operator makes very small refinements. If k is the current generation, "*gens"* is the total number of generations in the GA run. Thus the variance is calculated as;

  $$var_k = var_{k-1}\left(1 - 0.75 * (k/gens)\right)$$

III. Methodology for Resolution enhancement

Initially the input images are classified according the SFM values calculated and the corresponding evolved DWT filters are used in the enhancement algorithm. Using edge detection algorithm, the image pixels are grouped into near edge and far edge pixels, so that the DWT filters can be applied separately to obtain the near edge and far edge coefficients. The DWT based image enhancement algorithm is further applied to the two sets of coefficients and the two images are enhanced individually. Finally the enhanced two images are combined using the binary mask created during edge detection of the original low resolution image.

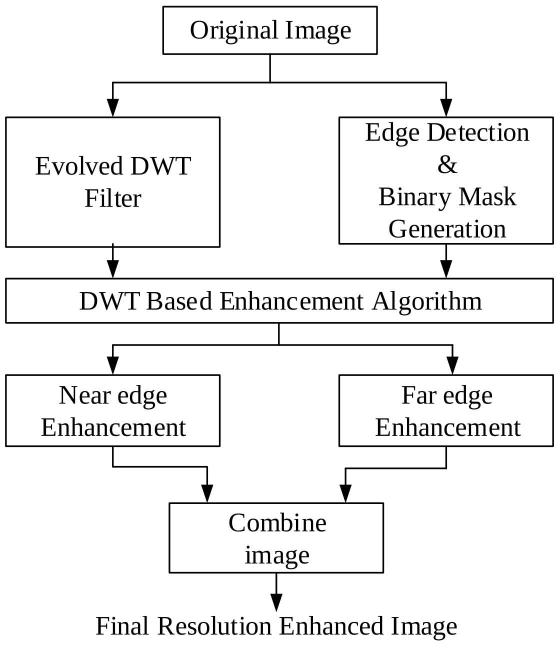


Fig. 3. Image decomposition and reconstruction with evolved filters targeting edge-adjacent and non-edge-adjacent portions of image (Michael R. Peterson et al., [17]).

3.1 Image Classification Based on spatial Frequency

The DWT filter coefficients evolved for images with smooth regions might not suit well for edge and texture rich images. Also, it is not practical to construct the optimal wavelet for each image as an online process in spite of the best results with the evolved filter coefficients. Hence all the test images are classified according to the complexity of the images (edges and textures) and optimal wavelets are evolved for each class to build a wavelet library offline. The quality of the DWT-based compression method for remote sensing images is effectively assessed using a gradient based approach by classifying image pixels according to the gradient magnitude and texture complexity thus proving the importance of the edges and textures in an image [18]. Hence we propose a systematic approach to find the edges and textures of the image by using the DWT itself. The high frequency sub-bands of transformed image will depict the edge and texture content in an image. Texture rich images will have more coefficients in the high frequency sub-bands as depicted in Figure 6 (3 cases are considered). This implies that the images can be classified by looking in to the high frequency sub-bands. Thus the Spatial Frequency Mean (SFM, summation of absolute averages of all the high frequency sub-bands) in frequency domain of an image is taken as measure to classify the images.

3.1.1 Test Images

We have taken 6 satellite images as shown in Figure 3 for each run and those 6 images are classified into six groups (G1, G2... G6) and the wavelets are evolved separately for each group and all of them are classified according to spatial Frequency Mean (SFM).

Satellite Images (named S1, S2, S3, S4, S5, S6 serially)

Fig. 4. Test Images

3.2Calculation of SFM

The SFM is calculated using the steps followed in the Figure 5 and the calculated *SFM* are shown in Table 2 for the considered 6 test images.

Table 2 - SFM values for 6 test images

Image	**SFM**
S1	6.8748
S2	13.8322
S3	18.5637
S4	24.5577
S5	29.3111
S6	35.6246

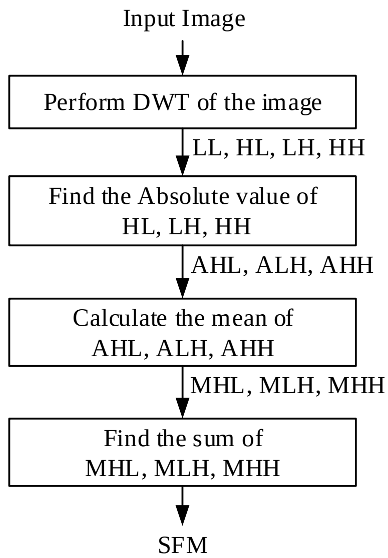


Fig. 5. Calculation of SFM

3.3 Classification of Images

The images are classified into one of the six groups (G1, G2... G6) according to the SFM value and the corresponding classification rule is shown in Table 3. For more clarity, the classified images are categorized in Figure 5 according to their groups. Finally, a library is build offline by evolving wavelets for each group separately using RCGA with PSNR as the fitness function.

Table 3. Classification Rule

Complexity level	SFM
G1	0 - 6.9
G2	7 - 13.9
G3	14 - 20.9
G4	21 - 27.9
G5	28 - 34.9
G6	35 and above

Group **Images**

G1

G2

G3

G4

G5

G6

Fig. 6. Group of classified images

IV. IMAGE ENHANCEMENT ALGORITHMS

The proposed image resolution enhancement algorithm adapts the two DWT based image resolution enhancement technique by Hasan Demirel et al [11, 5] as the fundamental method. Employing the GA for optimizing DWT coefficients, we have evolved six sets of DWT filters for each group for both near edge and far edge reconstruction named local and global filters. As shown in Fig. 7 shows the enhancement algorithm convolution scheme in which DWT filter is used to enhance the low resolution image from its coefficients. The evolved DWT near edge and far edge filters are used for this.

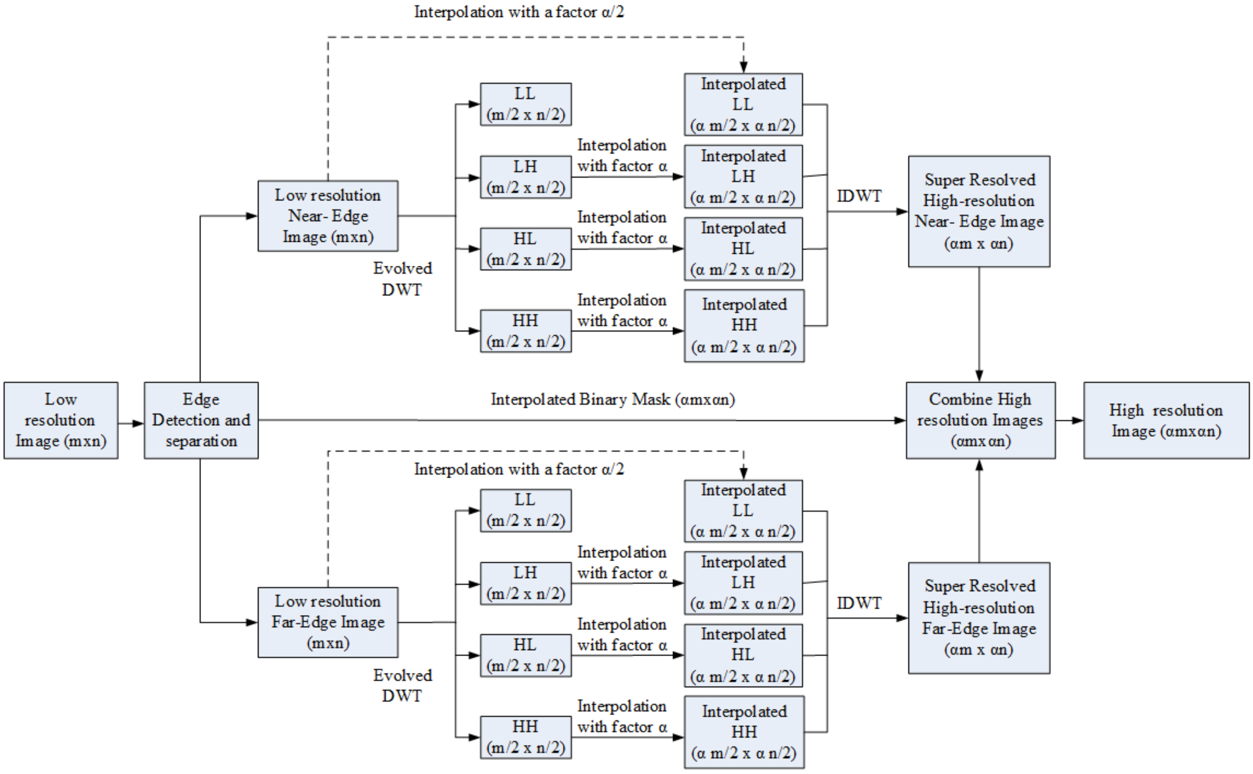


Fig. 7. Equalization Technique -1 [11]

V. EXPERIMENTAL RESULTS AND DISCUSSION

Initially the images are classified according to the edges and textures using the algorithm discussed in the section 3.2. The initial classification step provides six groups of images with different texture details. The idea is to evolve wavelet filter coefficients for each individual group both for near-edge and far-edge pixels in an image. Based on the output of the edge detection algorithm, a binary mask is created for the considered image and the binary mask separates the near-edge and far-edge pixels. The next step is to evolve wavelet filter coefficient for the near-edge pixels followed by far-edge pixels. The experiment is repeated for all the images which fall in the same group and the corresponding evolved filter coefficient are stored in the library. The experiment continues with the next group and concludes after evolving filter coefficients for all the six groups. Fifty GA runs are considered for convolution scheme and GA run would consider one of the test images shown in the figure 2.

Thus we have created an optimal wavelet library suitable for image resolution enhancement for each class of images. For enhancing an arbitrary image, its optimal wavelet filter coefficients need to be selected from the pre-stored library based on its SFM value which serves as an index for the selection of wavelets. We have evolved 9 filter coefficients for convolution scheme as the 2 variable evolver failed in most situations to produce a better wavelet than the CDF 9/7. For lifting scheme single variable is evolved as the 5 variable evolver failed because of its NIL constraint situation. The evolved wavelet libraries for both global and local filter are shown in Table 4.

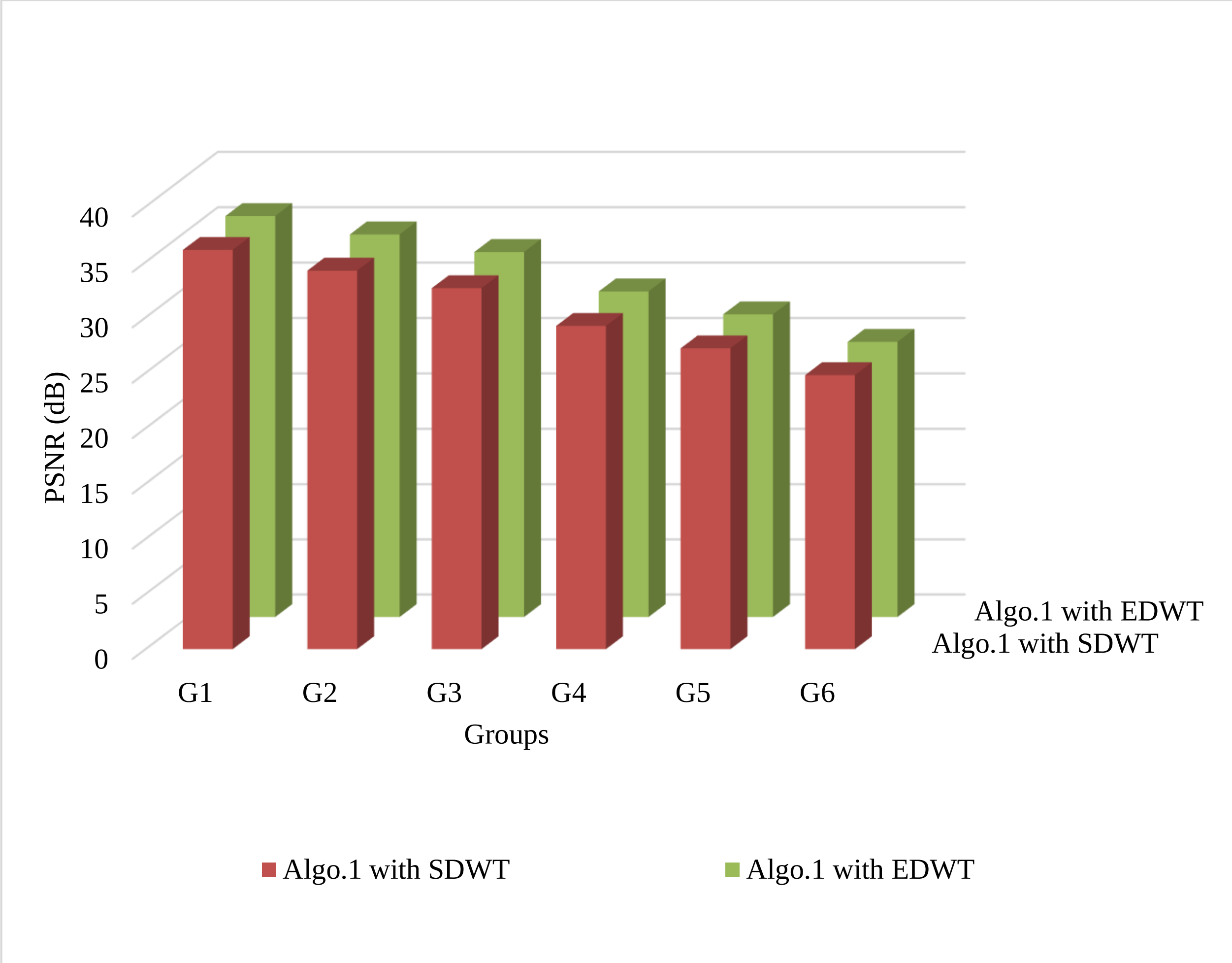
Table 4 – wavelet libraries for individual groups

DWT		G1	G 2	G 3	G 4	G 5	G 6
CDF9/7	Global	0.058198695613	0.016609715037	0.036443748802	0.033069962512	0.040066752610	0.039796724002
		-0.021365016384	0.002700932505	-0.019385726850	-0.015625536019	-0.017366838812	-0.018796743257
		-0.061760884596	-0.070591529540	-0.075154425890	-0.075047444925	-0.071867521201	-0.083009626283
		0.338461578278	0.265731923284	0.282518508866	0.284190116507	0.313355821643	0.299294585058
		0.635605791508	0.696617511175	0.717882899372	0.710365088556	0.630980190810	0.687174535848
		0.146451402940	0.112814653882	0.036224608360	0.099230746454	0.151313208650	0.093363682372
		-0.090044080936	-0.039193255965	0.093516314953	-0.053786046195	-0.053580756598	-0.051606396854
		-0.545606294968	-0.563888730378	-0.054644923172	-0.535696353004	-0.585852174365	-0.530682641852
		1.003787384725	0.990845240177	-0.527609977035	1.001429153545	1.003441876755	1.000807350546
	Local	-0.138143244415	-0.112757113088	-0.089828219685	-0.089901009522	-0.159754234880	-0.088555197954
		-0.086829074698	-0.060766772815	-0.056659948412	-0.056177974642	-0.072662714836	-0.055057061440
		0.554742739844	0.558900884132	0.541495660727	0.541772608462	0.589642038365	0.541926934583
		0.976249044912	1.003715760516	0.941773919334	0.926920011898	1.003176146333	0.967143085771
		0.554667735134	0.559065382769	0.541713059331	0.541474071276	0.588972152293	0.541825981229
		-0.064209699396	-0.076685036828	-0.076280702296	-0.075849377833	-0.084681826641	-0.084546810206
		-0.326207799500	-0.253328657932	-0.278599825732	-0.278823004981	-0.316159476576	-0.303300411358
		0.630341750519	0.706350498588	0.707638859684	0.707458354417	0.625120895482	0.674026299024
		-0.325996785979	-0.253307537045	-0.278817039853	-0.279258049911	-0.316656242131	-0.303243291907

The comparison of the quality measures in convolution scheme are shown in Fig. 8. And Fig. 7 shows the images reconstructed using global and local evolved filters and Fig. 8 shows the comparison between images reconstructed using CDF 9/7 and evolved filters coefficients.

Table 5 – PSNR (dB) for individual groups

Group	SDWT (dB)	EDWT (dB)
G1	35.91966	36.37597
G2	34.25851	34.46936
G3	32.81179	32.93358
G4	29.04727	29.35297
G5	27.43379	27.41209
G6	24.90072	25.05069



Average PSNR of the enhanced image using convolution scheme with standard and evolved DWT filters

Fig. 8. Comparison of quality metrics of the reconstructed image using CDF 9/7 and evolved filter coefficients (a) Average and maximum PSNR for convolution scheme

VI. CONCLUSION AND FUTURE WORK

The proposed image resolution enhancement algorithm for enhancing the resolution of satellite images using DWT based techniques are proven to be good both qualitatively and quantitatively. The experimental simulation results provide an average improvement of 0.5 dB using the existing algorithms. The DWT filters for various groups of satellite images helps to identify proposed DWT filters while applying the enhancement algorithms. Thus applying the enhancement algorithm for the near edge and far edge images separately using local and global DWT filters further improves the PSNR of the resolution enhanced image compared to the already existing DWT based resolution enhancement algorithms. Comparison of the PSNR of the enhanced image using the SDWT filters and EDWT filters shows a significant improvement in the image quality in both the algorithms.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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