Redundancy allocation problem in a bridge system with dependent subsystems

Article *in* Proceedings of the Institution of Mechanical Engineers Part O Journal of Risk and Reliability · November 2018

DOI: 10.1177/1748006X18814627

4 authors:


Kamyar Sabri-Laghaie
Urmia University of Technology


Milad Eshkevari
Urmia University of Technology


Michel Fathi
University of North Texas


Enrico Zio
MINES ParisTech and Politecnico di Milano

Proc IMechE Part O: J Risk and Reliability
XX(X):1–10
©The Author(s) 2016
Reprints and permission:
sagepub.co.uk/journalsPermissions.nav
DOI: 10.1177/ToBeAssigned
www.sagepub.com/
SAGE

Redundancy Allocation Problem in a Bridge System with Dependent Subsystems

Kamyar Sabri-Laghaie[1], Milad Eshkevari[2], Mahdi Fathi[3], and Enrico Zio[4]

Abstract

The Redundancy Allocation Problem (RAP) is an important problem in system reliability design. Many researchers have investigated the RAP under different assumptions and for various system configurations. However, most of the studies have disregarded the dependence among components and subsystems. In real-world applications, the performance of components and subsystems can affect each others. For instance, the heat radiated by a subsystem can accelerate degradation of adjacent components or subsystems. In this paper, a procedure is proposed for solving the RAP of a bridge structure with dependent subsystems. Copula theory is utilized for modeling dependence among subsystems, and artificial neural network (ANN) and particle swarm optimization (PSO) are applied for finding the best redundancy allocation. A numerical example is included to elaborate the proposed procedure and show its applicability.

Keywords

Redundancy allocation problem, Bridge system, dependence, Copula theory, Artificial neural network, Particle swarm optimization

1. Introduction

Today, survival of companies in the competitive markets strongly depends on the capability of effectively assigning to the customer needs of high performance and quality. Reliability is related to the ability of a system to meet the quality requirements. It is one of the most important factors in designing and manufacturing of products. In order to maximize system reliability, three strategies can be adopted: 1) enhancement of component reliability, 2) redundancy and, 3) combination of the two mentioned alternatives[14,23]. These strategies usually increase the demand for resources (cost, volume, weight, etc.). Therefore, at the phase of designing a highly reliable system, an important problem is to get the balance between reliability and other resource constraints.

The problem of maximizing system reliability through redundancy is called "redundancy allocation problem (RAP)". RAP has been vastly studied for different system structures, objective functions and time to failure distributions[5]. It is known that RAP is an NP-hard problem[6]. RAP is usually formulated as a non-linear integer programming problem, which is in general difficult to solve due to the considerable amount of computational effort required to find the exact solution. Hence heuristic and meta-heuristic approaches have been widely used to deal with this problem (e.g. Tabu Search[13], Genetic Algorithm[20], Particle Swarm Optimization[2]).

In real-world applications, system components may share some environmental factors such as temperature, pressure, load, etc. with each other. In other words, factors originated from some components may affect the performance of other components. Moreover, environmental factors may be shared among the subsystems of a system. For example, thermal radiation from a component or subsystem can impact the overall performance of other components or subsystems. Furthermore, the number of components installed in a subsystem can also determine the extent of dependence among subsystems. However, most of the RAP researches typically ignore the dependence among components or subsystems[17]. With regard to the literature, Kotz et al.[12] studied reliability when two components are positively quadrant dependent. For this aim, they used a number of bivariate distributions to model the dependent components and investigate the effect of components correlation on the lifetime of parallel redundant systems. Costa Bueno[7] used the reverse rule of order 2 property between component lifetimes to study the RAP of k-out-of-n systems via a martingale approach. He defined the concept of "minimal standby redundancy" and used it for allocating a redundant spare in a k-out-of-n:F system with dependent components. Belzunce et al.[3] used joint stochastic orders to study optimal allocation of redundant components in series and parallel systems with two dependent components. Belzunce et al.[4] studied optimal allocation of redundant components in series, parallel and k-out-of-n:F systems with more than two components. For this purpose, they extended bivariate joint stochastic orders and used multivariate joint stochastic hazard rate and reversed hazard rate orders to

[1]Faculty of Industrial Engineering, Urmia University of Technology, Urmia, Iran
[2]Faculty of Industrial Engineering Urmia University of Technology, Urmia, Iran
[3]Center for Applied Optimization, Department of Industrial and Systems Engineering, University of Florida, USA
[4]Dipartimento di Energia, Politecnico di Milano, Italy

Corresponding author:
Mahdi Fathi
Email: mahdi.fathi@ufl.edu

select redundant components. You and Li [26] studied RAP in engineering systems with dependent component lifetimes. They considered active and standby policies and built the likelihood ratio order and the hazard rate order for lifetimes in allocating redundancies to k-out-of-n systems. Gupta and Kumar [9] studied the problem of stochastic comparison of component and system redundancies where components are dependent and identically distributed. For this aim, likelihood ratio ordering, reversed failure rate ordering, failure rate ordering and the usual stochastic ordering were considered for carrying out the study. Further, Jeddi and Doostparast [10] studied optimal redundancy allocation problems in engineering systems with dependent component lifetime where no specific assumptions on the dependence structure of lifetimes are considered.

Despite the vast literature on RAP, only few researches have considered dependence among components and subsystems. Therefore, a procedure for evaluating RAP when subsystems are not independent is proposed in the current paper. Then, a bridge system is considered and the proposed procedure is applied to it. In brief, the redundancy allocation problem of a bridge system with dependence among subsystems is considered in this paper. The aim is to propose a procedure for the optimal allocation of components to a bridge system where the subsystems can be mutually dependent on each other. It is supposed that the parameters and characteristics of the components specify the type and extent of dependence among subsystems. A methodology based on Copula theory and Artificial Neural Network (ANN) is applied to model the dependence among subsystems. A Particle Swarm Optimization (PSO) Algorithm is employed to solve the dependent redundancy allocation problem.

The main contributions of the paper are as follows:

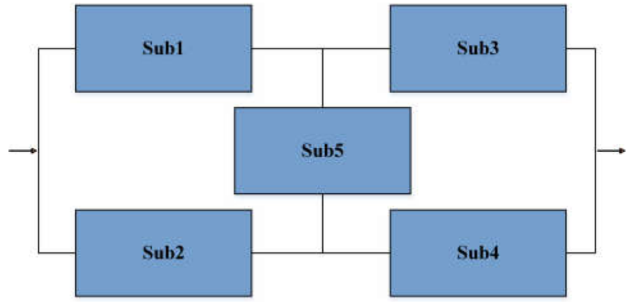
(i) Taking into account the impact of parameters and characteristics of components on the reliability performance of subsystems in RAP.
(ii) Proposing a methodology for modeling the type and extent of dependence among subsystems in RAP.

The rest of the paper is organized as the following. In Section 2 a brief description of bridge system, Copula theory, ANN and PSO is given. In Section 3 the proposed methodology for solving the dependent redundancy allocation problem is illustrated. A numerical example is presented in Section 4 and finally in Section 5 conclusions and suggestions for future research are remarked.

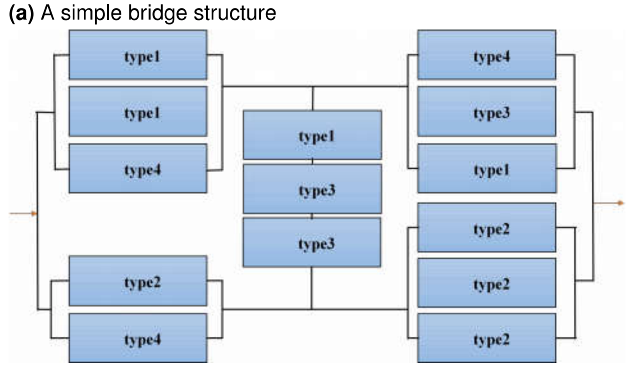
2. Models and methods

2.1. Bridge structure

Bridge topology is a well-known structure commonly used for load balancing and control in various applications such as electric power generation, transmission, computer networks, electronic circuits, etc [24]. Figure 1a Shows a simple bridge structure, which consists of five homogenous subsystems (sub1,...,sub5). To share the imposed load on each subsystem and enhancing the overall reliability of the system, redundant components with different characteristics can be allocated in the subsystems. A redundant bridge structure with nonhomogeneous components is illustrated in Figure 1b. Many researches in the literature have studied the RAP of bridge systems. For more information on this line of research, readers can refer to [1,15,24,25]. The redundancy allocation problem of the bridge system of Figure 1a with constraints on volume, weight, and cost of the system can be formulated as follows:



(a) A simple bridge structure



(b) A typical bridge structure with nonhomogeneous redundant components

Figure 1. Bridge topology

$$\max f(R,N) = R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - R_1R_3R4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \quad (1)$$

subject to:

$$g_1(N) \leq V \quad (2)$$

$$g_2(N) \leq C \quad (3)$$

$$g_3(N) \leq W \quad (4)$$

$$0 \leq r_i \leq 1 \quad (5)$$

$$\mathbf{r_i} \in \mathbb{R} \quad (6)$$

$$i = 1, ..., 5 \quad (7)$$

where, R_i is the reliability of subsystem i for $i = 1, ..., 5$; V, C, and W are the maximum allowed values for volume, cost, and weight of the system, respectively. Also, R and N are, respectively, the reliability and number of components. In addition, $g_j(N)$ for $j = 1, 2, 3$ are functions in terms of the number of components for calculating volume, cost and weight of the system. It should be noted that this formulation is valid for the case of independent subsystems.

2.2. Copula theory

According to Sklar[22], any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a Copula which describes the dependence structure among the variables. Nelsen[17] presented a detailed review of Copula theory and its principles. The Copula is one of the most popular methods for modeling the dependence of data[8], including components life time data. According to Noorossana and Sabri-Laghaie[18], utilizing the Copula in comparison to using traditional multivariate distributions for modeling dependency is very advantageous. Some of the advantages of the Copula method with respect to traditional multivariate distributions are: 1- By the Copula method, one can determine the degree and structure of dependence,2- dependence structure and marginal performance can be specified separately, 3- Copulas are robust to strictly increasing and continuous transformations, 4-univariate marginal functions can be easily derived from different distributions.

Consider H as a joint cumulative distribution function of a vector of continuous random variables $(T_1, ..., T_n)$ with univariate marginals $F_1, ...F_n$. Based on Copula theory a C_θ can be found where:

$$H(T_1, ..., T_n) = C_\theta(F_1(T_1), ..., F_n(T_n)) \quad (8)$$

In which, θ is the vector of Copula parameters expressing dependence among $T_1, ..., T_n$ and can be estimated by means of correlation coefficients.

In order to model the dependence among the lifetimes of components, many Copulas can be used. Choosing an appropriate Copula to model the dependence structure is a critical issue. In reliability problems the dependence among component lifetime is positive and this should be considered in the Copula selection process[8]. One of the most common Copulas used for modeling the dependence of component lifetimes is the Archimedean family. The Archimedean family can be defined as:

$$C(u_1, ..., n_n) = \varphi(\varphi^{-1}(u_1), ..., \varphi^{-1}(u_n)) \quad (9)$$

in which φ is a continuous and non-increasing function $\varphi : [0, \infty] \to [0, 1]$ and is called Archimedean generator. In the present study, Frank, Gumbel and clayton Copulas from the Archimedean family are utilized. Table 1 gives the mathematical definition of these Copulas.

2.3. Artificial neural networks

Artificial neural networks are composed of simple computational elements operating in parallel. These elements are inspired by biological nervous systems. The main applications of ANNs are function approximation by obtaining regression and transformations from input space to feature space by means of nonlinear mapping. An ANN is trained by some data examples to learn the function or transformation mapping and, then, it is used to provide the outputs to the new input data. The ANN has been widely used in various fields, such as pattern recognition[19], classification[16] and prediction[21]. As shown in Figure 2, ANN consists of three main parts: 1-input layer, 2- hidden layer, and 3- output layer. In the input layer, the value of each input is multiplied in by a weight and sent to the nodes of the next layer (called neurons). In the neurons of the hidden layer, a function called activation function is applied to the weighted sum of the inputs to the neuron. The most commonly used activation function is the sigmoid function that is defined as:

$$f(x) = \frac{1}{1 + \exp(-x)} \quad (10)$$

The weights of the output neurons of the network are obtained by applying an activation function, often linear, on the weighted sum of the hidden layer neuron values.

In this study, the ANN is used to model the structure of dependence among the subsystems of bridge system. To do so, it is trained to relate the failure times of the subsystems in the bridge system topology to parameters of the subsystems. In detail, for given input parameters of subsystems, the output will be the type and parameters of the Copula that is suitable for modeling the dependence among the intended subsystems. In this case, the back propagation algorithm is used to train the network. The relationships between the parameters of different subsystems and their corresponding Copula type and parameters are characterized as:

$$CupulaType = F_1(P_1, P_2, ..., P_n) \quad (11)$$

$$\theta = F_2(P_1, P_2, ..., P_n) \quad (12)$$

where $P_1, P_2, ..., P_n$ are vectors of effective parameters of subsystem 1 to subsystem n, θ is the vector of Copula parameters, and F_1 and F_2 are mapping functions from subsystem parameters to Copula type and Copula parameters, respectively.

2.4. Particle Swarm Optimization

Particle Swarm Optimization is a well-known optimization algorithm for the optimization of continuous nonlinear functions, introduced by Kennedy and Eberhart[11]. This algorithm has been inspired by collective behaviors, such as bird flocking and fish schooling. In this method, random solutions, as initial particles, are scattered in solution space and through an iterative procedure, all particles are converged to global optima. In the iterative procedure, the position of each particle is updated by means of its velocity vector, which takes into account the past direction of the particle, best position of the particle in past iterations and best-observed position of all particles in the iterations already elaborated. Then, the position of each particle is updated by its corresponding velocity vector. The mathematical expression of the aforementioned process is stated as follows:

$$v_i^{t+1} = wv_i^t + r_1c_1(pbest_i^t - x_i^t) + r_2c_2(gbest_i^t - x_i^t) \quad (13)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (14)$$

where v_i^{t+1} is the velocity vector of particle i in the iteration t, x_i^t is the position of particle i in the iteration t, and w is an inertia coefficient that expresses the tendency of the particle of keeping its position and takes values between 0 and 1. $pbest_i^t$ and $gbest_i^t$ are the best position of particle$i$

Table 1. Summary of the multivariate Copula functions in this study

Type	Formula	Parameter
Frank	$C_\theta = \frac{-1}{\theta} log\left(1+\frac{\prod_{i=1}^{d}(e^{-\theta u_i}-1)}{(e^{-\theta}-1)^{d-1}}\right)$	$\theta > 0$
Gumbel	$C_\theta = \exp\left(-\left(\sum_{i=1}^{d}\left(-\log(u_i)\right)^{\theta}\right)^{1/\theta}\right)$	$\theta \geq 1$
Clayton	$C_\theta = \max\left\{\left(\sum_{i=1}^{d}(u_i)^{-\theta}-(d-1)\right)^{-1/\theta}, 0\right\}$	$\theta \geq \frac{-1}{d-1}, \theta \neq 0$

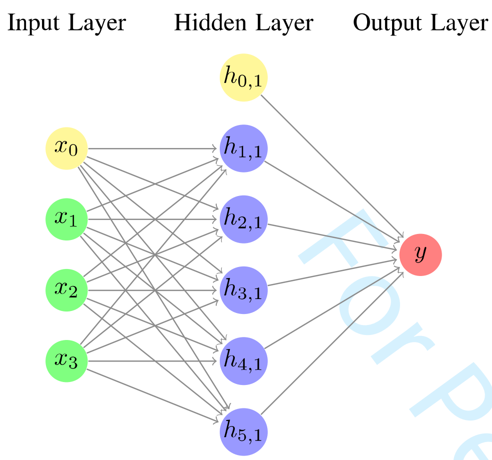


Figure 2. Neural Network structure

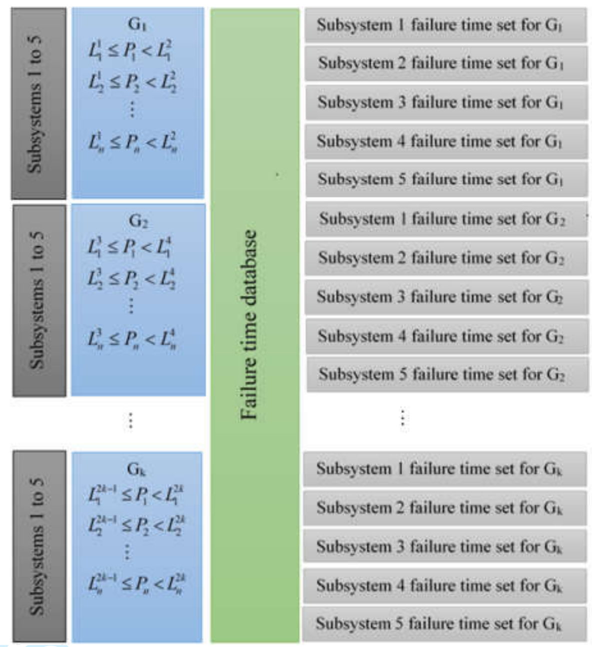


Figure 3. Data gathering process

in the past iterations and of all particles in the already elaborated iterations, respectively. r_1 and r_2 are random numbers between 0 and 1 and c_1 and c_2 are learning factors, respectively. The steps of this algorithm are as the following:

(i) define algorithm parameters such as number of iterations and population size.
(ii) generate initial population and evaluate the fitness functions.
(iii) update the position of each particle according to Relations 13 and 14 , and then evaluate the fitness function of the new particles.
(iv) stop if the termination condition of the iterative process is met, else go to Step (iii) .

3. Methodology

In this section, a methodology for considering dependence among subsystems in a redundancy allocation problem is proposed with respect to a bridge system configuration. A historical database of subsystem failure times, parameters and configurations of a bridge system are required. In this system, all components work under cold standby strategy and are either operating or failed at any given moment in time. As in the redundancy allocation problem, a combination of components can be used in each subsystem of the system. For the bridge structure, when the subsystems are independent, the reliability of the system can be calculated as Relation 1.

In this study, the Copula theory is utilized to take into account dependence among subsystems. Specifically, an ANN and Copula-based approach are proposed to model the dependence among subsystems. According to this approach, a relationship between parameters and characteristics of the subsystems and the impact that subsystems may have on each other is established. Subsystems with potentially dependent failure times are chosen and a database of their characteristics and failure history is built. To form this database, parameters and characteristics of components which may be affect the failure times should be included to be considered in the model.

In order to relate parameter values and corresponding failure times, parameter values are classified into specified categories. Each category consists of a combination of parameters with different ranges. By this categorization, each system can be assigned to a specific category according to its parameters. As mentioned, the database contains failure times of systems with different parameter values. Further, the subsystems which have failed and caused the system to fail are recorded in the database. The structure of the database is given in Table 2. In this table, failed subsystem, failure times of subsystems (T), and parameters $(P_1, P_2, ..., P_n)$ of the system are given. Based on this database one can explore the parameter values that may have an effect on the failure of a specific subsystem. Moreover, categories are built for parameters and subsystems according to failed subsystems. This results in a set of parameter values and failure times for each subsystem. Then, the corresponding set of each

Table 2. Database structure

No.	Subsystem 1				Subsystem 2				Subsystem 3				Subsystem 4				Subsystem 5				T	Failed subsystem
	P_1	P_2	...	P_n	P_1	P_2	...	P_n	P_1	P_2	...	P_n	P_1	P_2	...	P_n	P_1	P_2	...	P_n		
1																						
2																						
⋮																						
N																						

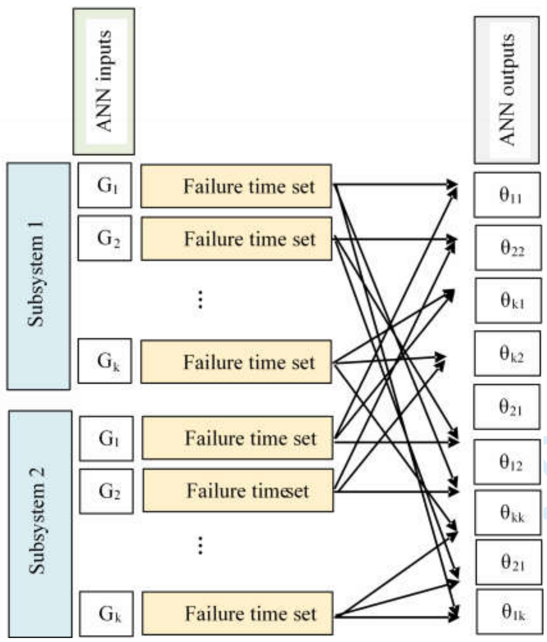


Figure 4. A typical example of ANN inputs and outputs

subsystem is categorized with regard to parameter values in specific ranges. Number and limits of the ranges are chosen according to the database size and domain of the parameters. Next step is to find the set of subsystem failure times corresponding to each parameter's category. Relation between parameter categories and failure time sets is shown in Figure 3. In this figure, G_i for $i = 1, ..., k$ is representative of the ith category of parameters, and $L_j^{(2k-1)}$ and L_j^{2k} for $j = 1, ...n$ are respectively the lower and upper bounds of the jth parameter in category k.

As mentioned before, the reliability of the bridge system in the case when all subsystems are independent is calculated according to Relation 1. When the subsystems are dependent on each other, the reliability function becomes:

$$\begin{aligned} R =& R_{12} + R_{34} + R_{145} \\ &+ R_{235} - R_{1234} - R_{1235} \\ &- R_{1245} - R_{1345} - R_{2345} \\ &+ 2R_{12345} \end{aligned} \tag{15}$$

where, R is the reliability of the system, R_{12}, R_{34}, R_{145}, R_{235}, R_{1234}, R_{1235}, R_{1245}, R_{1345}, R_{2345}, and R_{12345} are, respectively, the joint reliability functions of subsystems (1,2), (3,4), (1,4,5), (2,3,5), (1,2,3,4), (1,2,3,5), (1,2,4,5), (1,3,4,5), (2,3,4,5), and (1,2,3,4,5). In this paper, Copula theory is utilized to model the joint reliability functions of the subsystems. To obtain the Copula model, the following optimisation model is considered:

$$\begin{aligned} \mathbf{max} f(N) =& \mathbf{C}_\theta^\mathbf{2}(R_1, R_2) + \mathbf{C}_\theta^\mathbf{2}(R_3, R_4) + \mathbf{C}_\theta^\mathbf{3}(R_1, R_4, R_5) \\ &+ \mathbf{C}_\theta^\mathbf{3}(R_2, R_3, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_3, R_4) \\ &- \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_3, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_4, R_5) \\ &- \mathbf{C}_\theta^\mathbf{4}(R_1, R_3, R4, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_2, R_3, R_4, R_5) \\ &+ 2\mathbf{C}_\theta^\mathbf{5}(R_1, R_2, R_3, R_4, R_5) \end{aligned} \tag{16}$$

subject to:

$$g_1(N) \le V \tag{17}$$

$$g_2(N) \le C \tag{18}$$

$$g_3(N) \le W \tag{19}$$

$$0 \le r_i \le 1, \mathbf{r_i} \in real\ number \tag{20}$$

where C_θ^d for $d = 2, 3, 4, 5$ is a d-dimensional Copula function with parameter vector θ for modeling the joint reliability function of the corresponding subsystems. Here, it is supposed that the parameters or characteristics of the subsystems affect the parameter vector of the Copula functions. Therefore, the relation between the parameter vector of the Copula functions and the parameters of the subsystems is obtained. By means of an ANN. The subsystem parameters are the inputs and the Copula parameters are the outputs of the ANN. Suppose that we want to model the joint reliability function between subsystems 1 and 2. To do so, a Copula function is fitted for every combination of parameter categories. Then, an ANN is trained between the fitted Copula parameters and subsystem parameter categories. By means of the trained ANN, one can find the Copula parameters of the joint reliability function between subsystems 1 and 2. This procedure is followed for all combinations of subsystems a joint reliability function is required based on Relation 15. Inputs and outputs of the ANN for building the joint reliability function between subsystems 1 and 2 are depicted in Figure 4, where, θ_{ij} for $i = 1, ..., k$ and $j = 1, ..., k$ is the Copula parameter vector corresponding to the failure time sets of categories i and j.

Three classes of Copulas, Clayton, Gumbel and Frank, are here considered for reliability modeling. Maximum Likelihood Estimation (MLE) method is applied for fitting the Copulas to the failure time data and finding the most appropriate Copula function. A list of ten Copulas should, then, be recorded for every system in the database. Choosing the appropriate Copula class for a new system, a classification process is performed. In order to determine the Copula type, a classifier ANN is trained for each joint reliability term in Relation 15. The ANN classifier relates subsystem parameters and Copula types. So, according to the values of the subsystem parameters, an appropriate Copula is proposed. Then, as mentioned earlier, an ANN is trained to find the relation between subsystem parameters and the parameter vector of the chosen Copula. By following this procedure one can approximate the type and amount of dependence in a new bridge system just by evaluating the parameters of its subsystems.

A PSO algorithm is utilized to find the optimal configuration in the redundancy allocation problem of the bridge system. During each iteration of the PSO algorithm when solutions are updated to new ones, trained ANNs are applied to find the dependence structure of the new solutions. By knowing the dependence structure, the reliability of the solutions can be calculated based on Relation 16. The procedure of the proposed algorithm is as the following:

(i) Collect data, categorize them and fit the best Copula to failure time data of all required combinations of parameter categories and subsystems.
(ii) Train classifier ANNs for classifying type of Copulas and, then, train ANNs for approximating parameters of the Copulas based on outputs of step 1.
(iii) Generate initial solutions of the ANN algorithm.
(iv) Determine effective parameters of each subsystem and apply trained ANNs to find the dependence structure among subsystems.
(v) Use Relation 16 to calculate the reliability of the solutions (particles).
(vi) Update the position of each particle and calculate fitness of the new particles based on Relation 16.
(vii) Stop if the termination condition is met, else go to Step (vi) .

This procedure is also illustrated in Figure 5.

4. Numerical Example

In order to validate the proposed model, a numerical example is included to show model applicability. As mentioned in Section 3 , all components work under cold standby strategy and are either operating or failed at any moment in time. Also, each subsystem contains nonhomogeneous components. In this regard, a bridge structure with five subsystems is considered where a different number of redundant components can be allocated to each subsystem. Redundant components can be selected from four types of components as, type 1, type 2, type 3 and type 4. All types may fail according to Weibull probability distribution function with parameters as detailed in Table 3. Therefore, reliability of a component at a given time t is:

$$r_i(t) = \exp(\frac{-t}{\alpha_i})^{\gamma_i} \tag{21}$$

where $r_i(t)$ for $i = 1, 2, 3, 4$, is the reliability of component i at time t, and α_i and γ_i are respectively the scale and shape parameters of the ith component Weibull distribution. Three parameters of volume, weight, and cost are recorded for the components. In this regard, volume, weight, and cost of the system should not exceed specified values. It is supposed that parameters P_1 and P_2 are parameters of the components that

Table 3. Component characteristics

Component type	Weibull parameters		Volume	Weight	Cost
	shape (γ)	scale (α)			
1	0.001	17	40	7	3.5
2	0.02	21	30	5	2
3	0.03	32	30	2	3.5
4	0.004	44	10	1	5

may affect the reliability performance of other components. For example, P_1 and P_2 can be considered as thermal and radiation coefficients of components. Since there are only constraints on volume, weight, and cost of the system, the maximum allowed values of these parameters are given in Table 3. Also, the maximum number of components in each subsystem is set to be 10. On the other hand, parameters P_1 and P_2 affect the Copula parameters among different subsystems. Hence, the reliability optimization model at a predetermined time (e.g. $t = 100$) can be proposed as follows:

$$\begin{aligned} \mathbf{max} f(N) = & \mathbf{C}_\theta^\mathbf{2}(R_1, R_2) + \mathbf{C}_\theta^\mathbf{2}(R_3, R_4) + \mathbf{C}_\theta^\mathbf{3}(R_1, R_4, R_5) \\ & + \mathbf{C}_\theta^\mathbf{3}(R_2, R_3, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_3, R_4) \\ & - \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_3, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_1, R_2, R_4, R_5) \\ & - \mathbf{C}_\theta^\mathbf{4}(R_1, R_3, R4, R_5) - \mathbf{C}_\theta^\mathbf{4}(R_2, R_3, R_4, R_5) \\ & + 2\mathbf{C}_\theta^\mathbf{5}(R_1, R_2, R_3, R_4, R_5) \end{aligned} \tag{22}$$

subject to:

$$\sum_{i=1}^{5}\sum_{j=1}^{4} n_{ij} v_j \leq 50 \tag{23}$$

$$\sum_{i=1}^{5}\sum_{j=1}^{4} n_{ij} c_j \leq 25 \tag{24}$$

$$\sum_{i=1}^{5}\sum_{j=1}^{4} n_{ij} w_j \leq 150 \tag{25}$$

$$\sum_{j=1}^{4} n_{ij} \leq 10 \tag{26}$$

$$0 \leq r_i \leq 1, \mathbf{r_i} \in real\ number \tag{27}$$

where, R_i for $i = 1, ..., 5$ is the reliability of subsystem i, r_j, v_j,c_j, and w_j for $j = 1, ..., 4$ are respectively the reliability, volume, cost, and weight of component j. $N = [n_{11}, ..., n_{54}]$ is the vector of component numbers in the subsystems.

Based on the proposed procedure in Section 3 , first a historical database is required. In this regard, a database of failure times and parameter values is generated. A sample of the database and ranges of the parameters are presented in Table 4 and Table 5. According to this database, parameters P_1 and P_2 are used for determining the Copula parameters and volume, cost and weight are constraining parameters. Then, the failure times are categorized based on the failed subsystems. This results in a set of parameter values and failure times for each subsystem. The ranges of parameters are, then, categorized into some sub-ranges. In this example, five sub-ranges are considered for each of the parameters P_1 and P_2. These sub-ranges are given in Table 5. Afterward, the failed subsystems with their corresponding failure times are assigned to the combination of proposed sub-ranges.

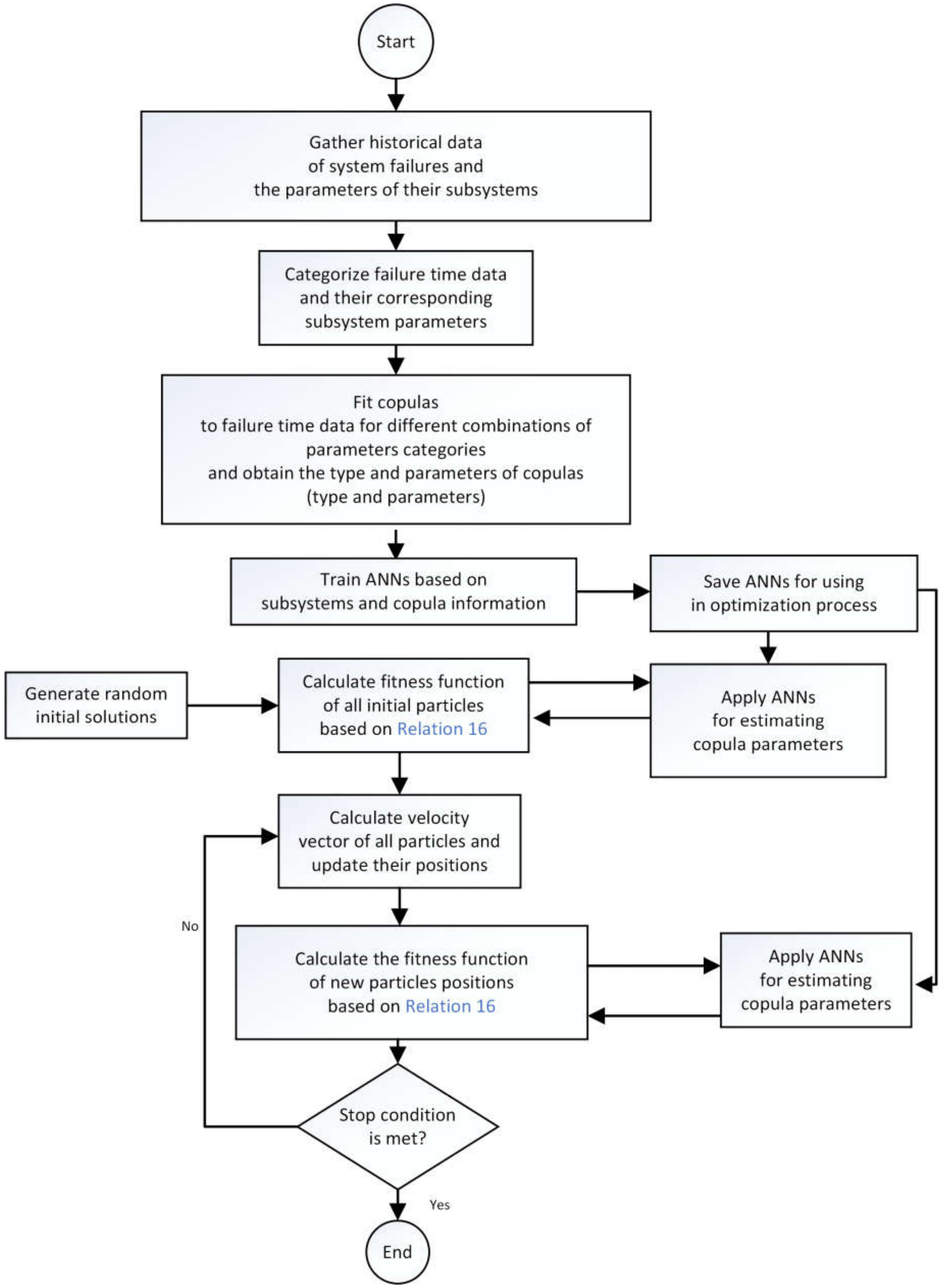


Figure 5. Flowchart of the proposed procedure

This results in 25 sets of failure times for each subsystem, $k = 125$ sets in total. Combinations of sub-ranges which do not contain enough failure times for model fitting can be disregarded. For modeling the joint reliability functions, the most appropriate Copulas among Clayton, Gumbel and frank Copulas are fit to the failure time data. In this regard, for each combination of subsystems in Relation 22 a Copula is required. Therefore, 125 sets of failure times are used for

Table 4. A sample of the database

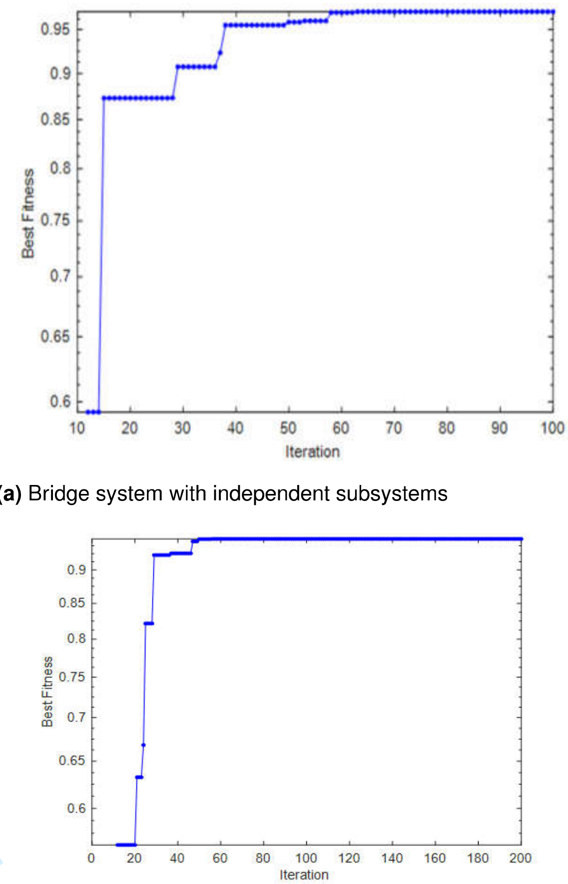
No.	Subsystem 1		Subsystem 2		T	Failed subsystem
	P_1	P_2	P_1	P_2		
1	35	30	15	50	0.4310	1
2	35	15	8	60	0.4876	2
3	7	15	17	120	0.5061	1
4	14	30	27	120	0.5895	2
5	42	180	17	120	0.7988	2
6	42	45	27	90	0.0401	2
7	63	45	180	150	0.0399	1
8	70	50	190	150	0.2974	1
9	14	60	50	60	0.1861	1
10	14	45	45	50	0.2796	1

Table 5. Sub-ranges of parameters (in arbitrary units)

Sub-range number	P_1	P_2
1	1-15	1-40
2	16-30	41-80
3	31-45	81-120
4	46-60	121-160
5	61-75	161-200

Table 6. Structure and performance criteria of classifier ANNs

Subsystems	Performace criteria (MSE)			Hidden layer size
	train	validation	test	
1,2	0.006	0.007	0.01	[50 50]
3,4	0.05	0.05	0.05	[12 2]
1,4,5	0.01	0.01	0.02	[16 8]
2,3,5	0.03	0.04	0.05	[9 7]
1,2,3,4	0.01	0.02	0.01	[12 10]
1,2,3,5	0.02	0.02	0.02	[9 10]
1,2,4,5	0.03	0.04	0.05	[6 6]
1,3,4,5	0.02	0.04	0.04	[12 5]
2,3,4,5	0.03	0.04	0.05	[9 3]
1,2,3,4,5	0.007	0.004	0.02	[24 12]



(a) Bridge system with independent subsystems

(b) Bridge system with dependent subsystems

Figure 6. Trend of the best fitness versus iteration number of PSO algorithm

determining the type and parameters of the most appropriate Copulas in modeling the joint reliability functions among different combinations of subsystems. As a result, a list of ten Copulas is recorded for every system in the database.

Now, ANNs are trained with input parameter values and output dependence structure (type and parameters of Copulas). Hereby, for every Copula term in Relation 22 a classifier ANN for determining the type of the Copula and an ANN for specifying the Copula parameters are trained. For example, structure and performance criteria (Mean Square Error or MSE) of classifier ANNs trained by the database are represented in Table 6.

After constructing ANNs for classifying the type of Copulas, a similar procedure is performed for modeling the relation between Copula parameters and parameters of subsystems. For example, structure and performance criteria (MSE) of ANNs trained for predicting different Copula parameter values are given in Table 7. All trained ANN structures are saved for estimating the dependence structure of a typical system based on its effective parameter values. For each subsystems combination, the ANN that has the best MSE is chosen for estimating the dependence structure.

After training the ANNs and based on the proposed procedure, the optimum number of redundant components in each subsystem is found by the PSO algorithm. During each iteration of the PSO algorithm and based on Relation 22, the trained ANNs are applied to find the dependence structure of new solutions. In more details, when a new solution is generated during different steps of the PSO algorithm, according to the P_1 and P_2 parameters of that solution and by means of trained ANNs, the type and parameters of the required Copulas are predicted.

The trend of the best fitness versus iteration number of the PSO algorithm in case when subsystems are independent and dependent is represented in Figure 6a and Figure 6b respectively. It can be observed from Figure 6a and Figure 6b that the outputs of the PSO algorithm for dependent subsystems are generally less than the outputs for subsystems that are independent. Also, optimal system configuration for both cases of independent and dependent subsystems is given in Table 8. According to Table 8, reliability of the best configuration when subsystems are dependent is less than the reliability when subsystems are independent.

5. Conclusion

In this paper, the redundancy allocation problem of a bridge system with dependent subsystems was studied.

Table 7. Structure and performance criteria (MSE) of ANNs trained for estimating Copula parameters

Copula types	subsystems	Performace criteria (MSE)			R^2			Hidden layer size
		train	validation	test	train	validation	test	
Clayton	1,2	0.003	0.004	0.008	0.99	0.98	0.97	[12 8]
Gumbel		0.07	0.06	0.07	0.99	0.98	0.98	[4 2]
Frank		0.11	0.12	0.12	0.98	0.98	0.98	[4 2]
Clayton	3,4	0.05	0.1	0.1	0.99	0.97	0.89	[14 3]
Gumbel		0.01	0.04	0.02	0.99	0.98	0.98	[9 4]
Frank		0.02	0.04	0.05	0.99	0.98	0.96	[9 4]
Clayton	1,4,5	0.02	0.07	0.06	0.99	0.98	0.95	[11 3]
Gumbel		0.02	0.03	0.03	0.99	0.99	0.99	[11 2]
Frank		0.03	0.02	0.01	0.99	0.99	0.99	[12 5]
Clayton	2,3,5	0.004	0.007	0.01	0.99	0.97	0.97	[7 6]
Gumbel		0.006	0.01	0.01	0.99	0.99	0.99	[7 6]
Frank		0.007	0.03	0.03	0.99	0.99	0.99	[8 7]
Clayton	1,2,3,4	0.002	0.02	0.01	0.99	0.98	0.95	[7 6]
Gumbel		0.02	0.01	0.02	0.99	0.99	0.98	[7 7]
Frank		0.005	0.02	0.08	0.99	0.99	0.98	[8 9]
Clayton	1,2,3,5	0.002	0.005	0.04	0.99	0.98	0.98	[11 6]
Gumbel		0.006	0.01	0.01	0.99	0.99	0.99	[10 5]
Frank		0.002	0.003	0.003	0.99	0.98	0.97	[11 7
Clayton	1,2,4,5	0.002	0.003	0.003	0.99	0.98	0.97	[13 5]
Gumbel		0.005	0.01	0.02	0.99	0.98	0.98	[10 7]
Frank		0.01	0.1	0.03	0.99	0.98	0.97	[7 5]
Clayton	1,3,4,5	0.08	0.03	0.03	0.99	0.99	0.98	[10 8]
Gumbel		0.002	0.01	0.06	0.99	0.99	0.98	[11 9]
Frank		0.002	0.001	0.003	0.99	0.99	0.99	[17 9]
Clayton	2,3,4,5	0.002	0.001	0.002	0.99	0.99	0.99	[19 8]
Gumbel		0.001	0.003	0.01	0.99	0.98	0.98	[10 7]
Frank		0.01	0.03	0.06	0.99	0.98	0.98	[8 8]
Clayton	1,2,3,4,5	0.04	0.05	0.2	0.99	0.98	0.93	[16 5]
Gumbel		0.001	0.02	0.01	0.99	0.99	0.98	[17 9]
Frank		0.006	0.01	0.02	0.99	0.99	0.98	[12 4]

Table 8. Optimal system configuration

	Subsystem	Type 1	Type 2	Type 3	Type 4	Best fitness
Independent	1	1	0	3	7	0.9637
	2	0	0	0	2	
	3	0	0	9	3	
	4	0	0	0	1	
	5	0	0	0	8	
Dependent	1	0	0	0	5	0.9485
	2	0	0	0	3	
	3	0	4	2	10	
	4	0	0	0	1	
	5	0	0	0	5	

It is supposed that some parameters of components and subsystems can affect the reliability performance of others. In this context, Copula theory was used for modeling the dependence structure among subsystems. The ANNs were applied for modeling the relationship between the parameters of the subsystems and the dependence structure. To do so, a historical database of system parameters and their failure times was used. Then, a particle swarm optimization algorithm was applied for finding the best redundancy structure. Numerical examples show that disregarding dependence can underestimate system reliability.

According to the size of the available database, different approaches can be utilized. The goal of this paper was to propose a methodology which can be useful in modeling dependency in the RAP. In this research, we supposed that enough failure data is available. When enough data for training ANN is not at hand, other approached such as ANFIS or other types of ANNs can be utilized to increase database size or build the predicting model. In addition, experts can be helpful in compensating lack of data. For future research, we aim to propose a methodology which can be useful when enough data is not available.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

1. Mostafa Abouei Ardakan and Mohammad Taghi Rezvan. Multi-objective optimization of reliability-redundancy allocation problem with cold-standby strategy using nsga-ii. *Reliability Engineering & System Safety*, 2017.
2. Noura Beji, Bassem Jarboui, Mansour Eddaly, and Habib Chabchoub. A hybrid particle swarm optimization algorithm for the redundancy allocation problem. *Journal of Computational Science*, 1(3):159–167, 2010.
3. Flix Belzunce, Helena Martnez-Puertas, and Jos M. Ruiz. On optimal allocation of redundant components for series and parallel systems of two dependent components. *Journal of Statistical Planning and Inference*, 141(9):3094 – 3104, 2011. ISSN 0378-3758.
4. Flix Belzunce, Helena Martnez-Puertas, and Jos M. Ruiz. On allocation of redundant components for systems with dependent components. *European Journal of Operational Research*, 230(3):573 – 580, 2013. ISSN 0377-2217.
5. Maw-Sheng Chern. On the computational complexity of reliability redundancy allocation in a series system. *Operations research letters*, 11(5):309–315, 1992.
6. David W Coit. Maximization of system reliability with a choice of redundancy strategies. *IIE transactions*, 35(6):535–543, 2003.

7. Vanderlei da Costa Bueno. Minimal standby redundancy allocation in a k-out-of-n: F system of dependent components. *European Journal of Operational Research*, 165(3):786–793, 2005.
8. Serkan Eryilmaz. Multivariate copula based dynamic reliability modeling with application to weighted-k-out-of-n systems of dependent components. *Structural Safety*, 51:23–28, 2014.
9. Nitin Gupta and Somesh Kumar. Stochastic comparisons of component and system redundancies with dependent components. *Operations Research Letters*, 42(4):284 – 289, 2014. ISSN 0167-6377.
10. Hamideh Jeddi and Mahdi Doostparast. Optimal redundancy allocation problems in engineering systems with dependent component lifetimes. *Applied Stochastic Models in Business and Industry*, 32(2):199–208, 2016.
11. J Kennedy and RC Eberhart. (1995). particle swarm optimization. In *IEEE International Conference on Neural Networks (Perth, Australia), IEEE Service Center, Piscataway, NJ*, pages 1942–1948, 1942.
12. Samuel Kotz, Chin Diew Lai, and Min Xie. On the effect of redundancy for systems with dependent components. *IIE Transactions*, 35(12):1103–1110, 2003.
13. Sadan Kulturel-Konak, Alice E Smith, and David W Coit. Efficiently solving the redundancy allocation problem using tabu search. *IIE transactions*, 35(6):515–526, 2003.
14. Way Kuo and V Rajendra Prasad. An annotated overview of system-reliability optimization. *IEEE Transactions on reliability*, 49(2):176–187, 2000.
15. Chyh-Ming Lai and Wei-Chang Yeh. Two-stage simplified swarm optimization for the redundancy allocation problem in a multi-state bridge system. *Reliability Engineering & System Safety*, 156:148–158, 2016.
16. Yang Lu, Nianyin Zeng, Yurong Liu, and Nan Zhang. A hybrid wavelet neural network and switching particle swarm optimization algorithm for face direction recognition. *Neurocomputing*, 155:219–224, 2015.
17. RB Nelsen. An introduction to copulas, 2nd. *New York: SpringerScience Business Media*, 2006.
18. Rassoul Noorossana and Kamyar Sabri-Laghaie. Reliability and maintenance models for a dependent competing-risk system with multiple time-scales. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 229(2):131–142, 2015.
19. Amarjit Roy, Salam Shuleenda Devi, and RH Laskar. Impulse noise removal from gray scale images based on ann classification based fuzzy filter. In *Computational Intelligence and Networks (CINE), 2016 2nd International Conference on*, pages 97–101. IEEE, 2016.
20. Seyed Jafar Sadjadi and Roya Soltani. An efficient heuristic versus a robust hybrid meta-heuristic for general framework of serial–parallel redundancy problem. *Reliability Engineering & System Safety*, 94(11):1703–1710, 2009.
21. Nitin Singh, Soumya Ranjan Mohanty, and Rishabh Dev Shukla. Short term electricity price forecast based on environmentally adapted generalized neuron. *Energy*, 125: 127–139, 2017.
22. Abe Sklar. Random variables, distribution functions, and copulas: a personal look backward and forward. *Lecture notes-monograph series*, pages 1–14, 1996.
23. Roya Soltani. Reliability optimization of binary state non-repairable systems: A state of the art survey. *International Journal of Industrial Engineering Computations*, 5(3):339–364, 2014.
24. Yong Wang and Lin Li. A pso algorithm for constrained redundancy allocation in multi-state systems with bridge topology. *Computers & Industrial Engineering*, 68:13–22, 2014.
25. Wei-Chang Yeh, Siang-Tai Wang, Chyh-Ming Lai, Yen-Cheng Huang, Yuk Ying Chung, and Jsen-Shung Lin. Simplified swarm optimization for repairable redundancy allocation problem in multi-state systems with bridge topology. In *Neural Networks (IJCNN), 2016 International Joint Conference on*, pages 3935–3941. IEEE, 2016.
26. Yinping You and Xiaohu Li. On allocating redundancies to koutof n reliability systems. *Applied Stochastic Models in Business and Industry*, 30(3):361–371, 2014.