Modelling and improvement of non-standard queuing systems: A gas station case study

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4 authors:

Ebrahim Teimoury
Iran University of Science and Technology

Mohammad Modarres
Sharif University of Technology

Meysam Haddadi

Michel Fathi
University of North Texas

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Ebrahim Teimoury

Department of Industrial Engineering,
Iran University of Science and Technology,
Narmak Street, Tehran, Iran
E-mail: teimoury@iust.ac.ir

Mohammad Modarres Yazdi

Department of Industrial Engineering,
Sharif University of Technology,
Azadi Street, Tehran, Iran
Fax: +98-21-6602-2702
E-mail: modarres@sharif.edu

Meysam Haddadi and Mahdi Fathi*

Department of Industrial Engineering,
Iran University of Science and Technology,
Narmak Street, Tehran, Iran
E-mail: m64haddadi@yahoo.com
E-mail: mfathi@iust.ac.ir
*Corresponding author

Abstract: This research has modelled a queuing system with no standard states. In order to analyse these systems, some parameters such as the mean of waiting time and the length of queue are computed. These situations usually occur when there is delay in the service, for example, in the gas queuing system that cars are unable to leave after receiving the required services due to the presence of other cars in front of them. These kinds of non-standard queuing systems are usually applied to the assembly and production lines. In this paper, a queuing model for the special non-standard state queuing system is developed to be used in gas stations. The proposed model is implemented in fuel stations in Iran and the results are compared with the existing non-standard situations.

Keywords: queuing theory; performance evaluation; layout constraint; gas stations; non-standard states queuing systems.

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Biographical notes: Ebrahim Teimoury received his MS and PhD in Industrial Engineering from Iran University of Science and Technology in 1998. He is currently an Assistant Professor in the Department of Industrial Engineering in Iran University of Science and Technology. His research interests are in supply chain management and statistics. He has published in journals such as *European Journal of Operational Research*, *Production Planning and Control*, *Journal of Manufacturing Systems*, *International Journal of Advanced Manufacturing Technology*, *International Journal of Business and Management*, *Supply Chain Management: An International Journal, International Journal of Industrial Engineering and Production Management*, *Journal of Strategy and Management* and *Journal of Research in Interactive Marketing*.

Mohammad Modarres Yazdi received his BS from the Department of Electrical Engineering, University of Tehran, Iran in 1968. He received his MS and PhD in Systems Engineering and Operations Research from University of California, Los Angles (UCLA), USA in 1973 and 1975, respectively. His main research interests are operations research, mathematical modelling, and stochastic modelling. He has published in journals such as *Naval Research Logistics Quarterly*, *OMEGA, European Journal of Operational Research*, *OR Spectrum*, *Journal of Operational Research Society*, *Fuzzy Sets and Systems*, *Production Planning and Control*, *International Journal of Production Economics*, *Computers and Operations Research*, *Journal of Manufacturing Systems* and *International Journal of Advanced Manufacturing Technology*.

Meysam Haddadi received his BS in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran in 2009. His research interests lie in the area of probability, statistics, and quality control.

Mahdi Fathi received his BS and MS from the Department of Industrial Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran in 2006 and 2008, respectively. His research interests are in the modelling and analysis of stochastic systems, e.g., telecommunications, manufacturing, service, and risk systems; queueing theory, inventory theory, and algorithmic methods in applied probability.

1 Introduction

Waiting in lines is one of the most common annoyances we face in the modern societies and no one likes to wait in queues and lose customers. A queue is created whenever there is more demand than the available service. To respond to the important questions like "what should be the level of service to ensure satisfactory service?", one needs to know the expectations of the customers, also how to calculate performance measures for the corresponding stochastic system, such as how many and how long customers are expected to wait?

Queuing theory is a useful tool for analysing many physical systems, and it is one of the oldest and best-developed techniques used to analyse waiting queues. Manufactures and service providers have to use queuing theory in order to optimise queue management decisions, reduction in customers' waiting time and performance evaluation. The main purpose of manufactures and service providers is to gain customer satisfaction, reflected as customer-oriented characteristics.

One of these characteristics for a customer is to receive goods or services as soon as possible. This will help them to specify resource levels that should be allocated and to accomplish customer satisfaction as much as possible.

Resource allocation and customer satisfaction are very important for companies especially in a highly competitive environment. Analysing and describing the performance of queuing systems in different environments are also essential issues that require detailed attentions. Queuing modelling is a tool for performance evaluation in queuing systems. Queuing systems layout, service allocation, customer allocation, and service type should be concerned in the real condition analysis in order to optimise the productivity of a system.

Consider a simple queuing system with a number of servers with layout constraint. In order to analyse such a system, one needs to obtain and calculate its performance measures such as the average waiting time and the length of queue in the system. The aim of this paper is to model queuing systems with non-standard states. Non-standard states in queuing systems occur whenever there are layout constraints to get service. We can see an example of non-standard states in a gas station queuing systems where cars, after the filling service, are not able to leave the station because of front cars that are getting service or front cars that leave the station but the rear car is still receiving service. Non-standard states in queuing systems occur whenever there are layout constraints to get services. The non-standard states usually happen because of having layout constraints in queuing systems. Application of these kinds of non-standard queuing systems is common in assembly and production lines. In this study, a queuing model for special non-standard state queuing systems used in gas stations is proposed.

A queuing system with non-optimised layout creates long queues and high waiting time. An obvious example of such systems is filling stations whose performance measures (and station's utilisation) are low because of layout constraint problems in their design. In this paper, we analyse the output of such systems by queuing modelling where the results can be used for demand modelling. The proposed model of this paper is one of the main concerns for the National Iranian Oil Products Distribution Company (NIOPDC) and the study of this paper is considered for gas stations in Iran. Moreover, this study can help the gas station strategic decision makers for demand modelling.

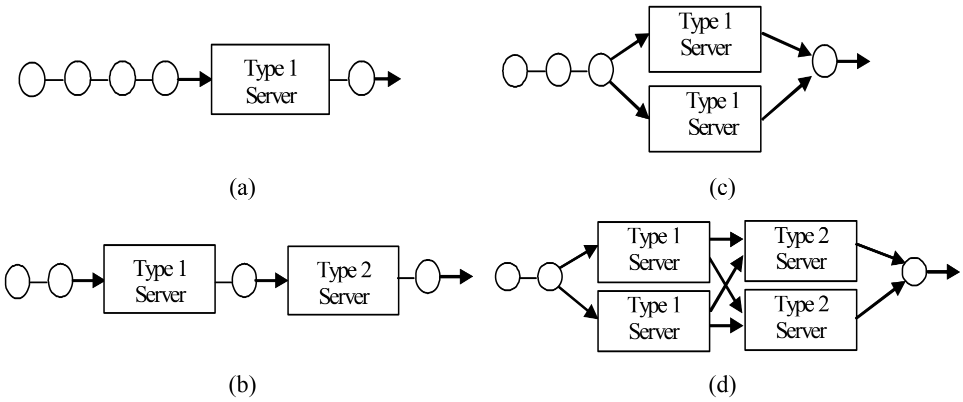
The outline of the paper is as follow. In Section 2, the problem is defined more precisely and non-standard queuing systems and its features are studied. Section 3 is dedicated to queuing modelling and the mathematical formulation of our model. In addition, the proposed queuing system is based on one row with two servers is analysed and then the approach is generalised to more than two rows by expanding the process. In Section 4, the proposed model is implemented for a real filling station and the results of queuing system are discussed. Moreover, we have recommended some basic rules to obviate defects and improve the non-standard queuing system's performance. Then our suggestions are stimulated for the queuing systems and compared with the existing non-standard situation. Finally, some concluding remarks are provided in Section 5.

2 Problem statement

2.1 Types of queueing systems based on its layouts

In this section, the basics of queuing systems based on different channels are explained. Figure 1 depicts different queuing systems. Figure 1(a) represents the simplest form of queuing system where customers are lined up in one queue and there is only one server. In Figure 1(b), there are more than one service to be received. In Figure 1(c), customers are able to use more than one server and finally, Figure 1(d) represents a more complicated queuing system where there are more than one station and a server.

Figure 1 Types of queuing systems, (a) single channel and single server system (b) serial single channel and multi server (c) parallel multi channel and single server (d) parallel multi channel and multi server system

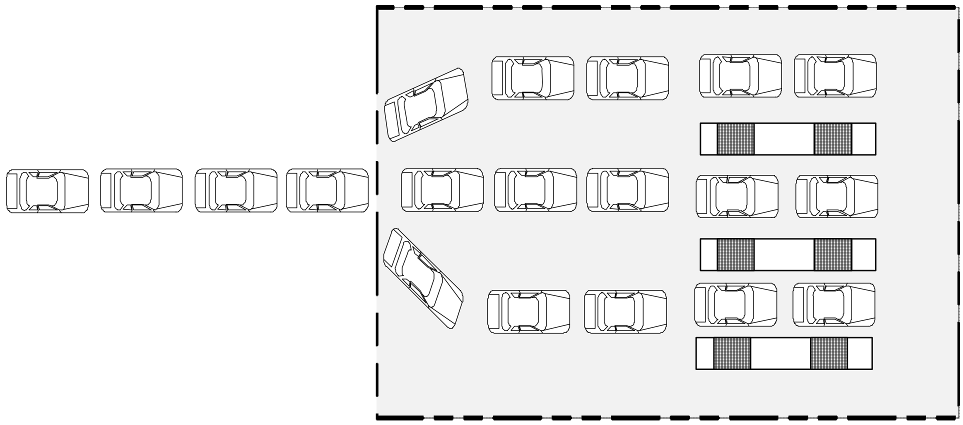


2.2 Problem description

During the past decade, there has been a tremendous increase in the number of gas stations since the domestic auto industries could significantly increase their production level. Therefore, modelling fuelling station is an essential issue in a systems analysis. Investing in the building of new gas stations is an expensive process as the equipment and price of land especially in the large cities are significant.

The layout of all Iranian gas stations is designed in a way that, except for the front cars, cars cannot leave until the other cars receive their services and this could increase both mean service and mean waiting times. Therefore, the present queuing system model has a main waiting queue that is formed out of a gas station. There is normally more than one pump in every gas station and once a driver arrives at the gas station, he/she normally selects the shorter. In this system, there are usually two pumps in each stage and if both of them are unoccupied, the customer chooses the first one and if it is occupied, then the second pump is selected and so on. A schematic concept of a gas station is shown in Figure 2.

Figure 2 Schematic plan of a the fuel station



Due to the variable service rates and physical limitations, it is not feasible to use any of the available models to analyse such systems. We assume that customers' arrival is based on a Poisson process with the rate of λ and two servers in each row as series, in special conditions, whose service times have exponential random variables with rates of μ_i, $i = 1, 2$. Since the performance of each row is independent, the existing queuing system is modelled based on two servers in a row with the rates of μ_1 and μ_2, respectively. By developing *M*/*M*/*C* model as well as using Markov chain concepts, a mathematical model is developed and proposed (see Asmussen, 2003; Ching and Ng, 2006; Cooper, 1980; Gross and Harris, 1985; Lipschutz, 1968; Little, 1961; Medhi, 1991; Ross, 1997; Solomon, 1983).

3 Problem formulation

3.1 Modelling the non-standard queueing system based on one row with two servers

In order to model the queuing system (as shown in Figure 3), certain assumptions, parameters, and variables are defined as follow:

- customer arrival in the queue is based on Poisson process with the rate of λ
- customer service time for the first one is t_1 and service time for the second one is t_2 and so on
- just a one-row queue is plausible
- there are two servers in as a series in the system
- if both servers are idle, the customer will use the first one and if it is busy then the customer will use the second one
- while serving the first customer, the second customer should wait.

Figure 3 Plan of a two server with one row system

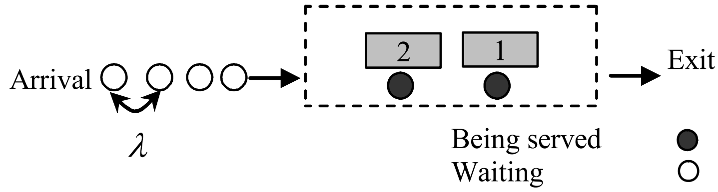


Figure 4 Service system for the first situation

Period	0		ΔT		T
1st Server's output	1	2	3	$n-1$	n
Customer's output rate for the 1st server and the 2nd server	t_1 t_2	$2t_1$ t_1+t_2	$3t_1$ $2t_1+t_2$	$(n-1)t_1$ $(n-2)t_1+t_2$	$(n)t_1$ $(n-1)t_1+t_2$
2nd Server's output	1	2	3	$n-1$	n

Note that in Figure 4, the grey areas represent the idle time for the second server.

- In this system if server 2 is busy and server 1 is idle, and then the customer in queue cannot go to server 1.

Based on service time there are two possible cases for the modelling system which come as follow,

1 Case I: the first server service time is equal to or more than that of the second one. It means that the first customer is served in a time that is equal to or larger than the second customer time.

2 Case II: the service time of server 2 is more than that of server 1.

3.2 Case I

In this case $t_1 \geq t_2$ and $\mu_1 \leq \mu_2$. In fact, the service system and the service rates of the servers are not equal. If there are no limitations for departure from this system, then in ΔT period, server 1 will serve $[\mu_1]$ number of customers and server 2 will serve $[\mu_2]$ customers. It means that in ΔT we expect $[\mu_1] + [\mu_2]$ customers to leave the system. Nevertheless, because of limitation in departure, customer in server 2 cannot leave the system until the first customer leaves. In Figure 4, we observe that in ΔT period server 1 can rarely serve n customers as $nt_1 \leq \Delta T$. The maximum number of customers served by the second server in ΔT period is n, whereas $(n - 1)t_1 + t_2 \leq \Delta T$ (concerning layout constraint). Therefore, n is μ_2' or real second service rate. By considering these two equations, if $X'(\Delta T)$ is the maximum number of customers served in ΔT period and leaves the system, then we have $X'(\Delta T) = 2\mu_1$.

At this time server 2 can serve more customers (see Figure 4), but the next customer cannot enter the system until the first customer's service time is passed and the total idle time of the second server in ΔT period is $I(\Delta T) = (t_1 - t_2)\mu_1$.

It can be concluded that if the first server spends time, t_1, to serve each customer then the second server spends the same time for the same reason. Therefore, their real service times are not equal. t_1 and t_2 represent the service time for the first and the second customers, respectively. Since the second customer needs to wait for the amount of β till the first customer leaves the station we have,

$$t_2 = t_1 - \beta \tag{1}$$

The service rate for both of servers are,

$$\mu_1 : \frac{\Delta T}{t_1}, \ \mu_2 : \frac{\Delta T}{t_2 + \beta} = \frac{\Delta T}{t_1} = \mu_1 \tag{2}$$

This makes the explained model as a *M*/*M*/2 model in which service rate of both servers is μ_1. Therefore, we have,

$$\lambda_n = \lambda \quad : n = 0,1,2,\ldots$$
$$\mu_n = \begin{cases} \mu_1 & n = 1 \\ 2\mu_1 & n = 2,3,\ldots \end{cases} \quad \text{There is no waiting line} \tag{3}$$

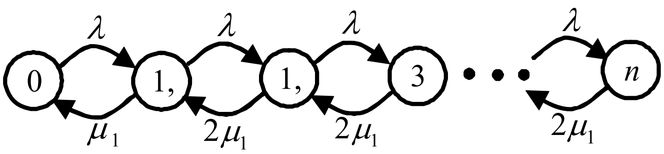
Table 1 demonstrates different possible conditions for a queuing system. Since we have $t_1 > t_2$, the (0, 1) condition will never happen.

Table 1 Possible situations for system in the first condition

State	*State description*
0	System is empty
(1, 0)	There is a customer in the system and this customer is being served by server 1
(0, 1)	There is a customer in the system and this customer is being served by server 2
(1, 1)	There are two customers in the system and they are being served by server 1 and 2
$n \geq 2$	There are *n* customers in the system

Figure 5 shows the transition rate diagram based on model assumptions and limitations.

Figure 5 The transition rate diagram for the first situation



Moreover, by using this diagram and equality of arrival rate and departure rate for each state, the equilibrium equation is written as follow,

$$
\begin{array}{ll}
\text{State} & \text{Equilibrium equation set} \\
0 & \lambda P_0 = \mu_1 P_{1C} \\
(1,0) & (\lambda + \mu_1) P_{10} = \lambda P_0 + (2\mu_1) P_{11} \\
(1,1) & (\lambda + 2\mu_1) P_{11} = \lambda P_{10} + (2\mu_1) P_3 \\
\dots & \dots \\
n > 2 & (\lambda + 2\mu_1) P_n = \lambda P_{n-1} + (2\mu_1) P_{n+1}
\end{array}
\tag{4}
$$

As it can be seen in Figure 5, except for the state 0 of each n state, there are two possible transition states: $(n + 1)$ and $(n - 1)$. Therefore, it is easy to use one of the variables to solve the equations. Here, P_0 is a good base to solve equations. We also have equation (11) as below:

$$
C_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1}\right) & n = 1 \quad \text{There is no waiting line} \\ \left(\dfrac{\lambda}{2\mu_1}\right)^{n-1} \dfrac{\lambda}{\mu_1} & n = 2, 3, \dots \end{cases}
\tag{5}
$$

For $n > 1$, we will have $P_n = C_n P_0$.

Since $\sum_{n=0}^{\infty} P_n = 1,$ therefore:

$$
P_0 = \left(1 + \frac{\lambda}{\mu_1} + \frac{\lambda^2}{\mu_1(2\mu_1 - \lambda)}\right)^{-1}.
\tag{6}
$$

In order to calculate the probability of the existence of n customers in a system for a long period of long time we have,

$$
P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1}\right)^n P_0 & n = 0, 1, 2 \\ \left(\dfrac{\lambda}{2\mu_1}\right)^{n-1} \dfrac{\lambda}{\mu_1} P_0 & n \geq 2 \end{cases}
\tag{7}
$$

To calculate integrative measures like expected number of customers in the queue (L_q) and expected number of customers in the system (L), by applying Little's relations and steady state condition of system $\left(\rho = \dfrac{\lambda}{2\mu_1} < 1\right),$ we have,

$$
L_q = \frac{P_0}{2}\left(\frac{\lambda}{\mu_1}\right)^2 \frac{\rho}{(1-\rho)^2}
\tag{8}
$$

$$L = L_q + \frac{\lambda}{\mu_1} = \frac{P_0}{2}\left(\frac{\lambda}{\mu_1}\right)^2 \frac{\rho}{(1-\rho)^2} + \frac{\lambda}{\mu_1} \tag{9}$$

Also to calculate time measures such as expected waiting time in the queue (W_q) and expected waiting time in the system (W), by applying Little's relations, we get:

$$W_q = \frac{L_q}{\lambda} = \frac{\pi_0}{2}\left(\frac{\lambda}{\mu_1^2}\right)\frac{\rho}{(1-\rho)^2} \tag{10}$$

$$W = W_q + \frac{1}{\mu_1} = \frac{\pi_0}{2}\left(\frac{\lambda}{\mu_1^2}\right)\frac{\rho}{(1-\rho)^2} + \frac{1}{\mu_1} \tag{11}$$

3.3 Case II

In this part, we assume that the service time of server 2, t_2, is greater than that of server 1, t_1, which can be represented as $t_1 \le t_2$ and $\mu_1 \ge \mu_2$. If there is no departure limitation, then in ΔT period, the first and second servers will serve $[\mu_1]$ and $[\mu_2]$ customers, respectively. It means that in ΔT we expect $[\mu_1] + [\mu_2]$ customers to be served and leave the system. But because of space limitation for departure, if the first customer should leave the system, the customer who is waiting in the line cannot be replaced. This is because the second server is busy; thereby the whole system is busy.

As can be seen from Figure 6 in ΔT period, the first server can serve at most n customers whereas $(n-1)t_2 + t_1 \le \Delta T$. Therefore, n is μ_1' or real service rate of the first server. Also, in ΔT period, the second server can serve at most n customers with $nt_1 \le \Delta T$. Here, n is the same as μ_2, the service rate of the second server. By considering these two relations, it can be concluded that if the maximum number of customers that can be served and leave the system is $X'(\Delta T)$ in ΔT period, then we have, $X'(\Delta T) = 2\mu_2$.

Although the first server can serve more customers (in Figure 6), because of the limitation of space the next customer cannot enter the system until the second customer service time is finished and therefore server 2 should be idle for a while. Based on the following equation the total idle time of server 1 in ΔT period is $I(\Delta T) = (t_2 - t_1)\mu_2$.

Figure 6 Service system for the second case

Period	0		ΔT		T
2nd Server's output	1	2	3	$n-1$	n
Customer's output rate for the 1st server and the 2nd server	t_1 t_2	t_1+t_2 $2t_2$	t_1+2t_2 $3t_2$	$t_1+(n-2)t_2$ $(n-1)t_2$	$t_1+(n-1)t_2$ $(n)t_2$
1st Server's output	1	2	3	$n-1$	n

Now if the service time for the first and the second customers is t_2 and since the second customer has to wait for a time, say α, until leaves the station we have,

$$\mu_1 : \frac{\Delta T}{t_1 + \alpha} = \frac{\Delta T}{t_2} = \mu_2 \tag{12}$$

Note that grey areas in Figure 6 represent the first server's idle time. Therefore, we have $\lambda_n = \lambda$, $n = 0, 1, \cdots$ and

$$\mu_n = \begin{cases} \mu_1 & n = 1 \\ 2\mu_2 & n = 2,3,\ldots \end{cases} \quad \text{There is no waiting line} \tag{13}$$

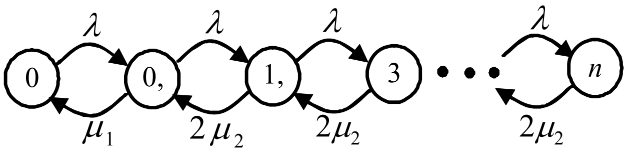
There are again different possibilities as shown all in Table 2. Since we have $t_1 < t_2$, the condition (1, 0) will never happen.

Table 2 Possible situations in the second condition

State	*State description*
0	System is empty
(1, 0)	There is a customer in the system and this customer is being served by server 1
(0, 1)	There is a customer in the system and this customer is being served by server 2
(1, 1)	There are two customers in the system and they are being served by server 1 and 2
$n > 2$	There are *n* customers in the system

Figure 7 shows the transition rate diagram based on the assumption of Table 2.

Figure 7 The transition rate diagram for the second situation



Moreover, using diagram shown in Figure 7, equality of arrival rate and departure rate for each state, the equilibrium equation are summarised as follows,

State	Equilibrium equation set
0	$\lambda P_0 = \mu_1 P_{10}$
(1,0)	$(\lambda + \mu_1) P_{10} = \lambda P_0 + (2\mu_2) P$
(1,1)	$(\lambda + 2\mu_2) P_{11} = \lambda P_{10} + (2\mu_2) P_3$
…	…
$n > 2$	$(\lambda + 2\mu_2) P_n = \lambda P_{n-1} + (2\mu_2) P_{n+1}$

(14)

Let $\sum_{n=0}^{\infty} P_n = 1$ then we have,

$$C_n = \begin{cases} \dfrac{\lambda}{\mu_1} & n = 1 \quad \text{No waiting line} \\ \left(\dfrac{\lambda}{2\mu_2}\right)^{n-1} \dfrac{\lambda}{\mu_1} & n = 2,3,\ldots \end{cases} \tag{15}$$

and

$$P_n = C_n P_0. \tag{16}$$

In order to calculate P_0 we have,

$$P_0 = \left(1 + \frac{\lambda}{\mu_1} + \frac{\lambda^2}{\mu_1(2\mu_2 - \lambda)}\right)^{-1} \tag{17}$$

Therefore, we have the probability of existence of n customers in the system for a long time as follows,

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1}\right)^n P_0 & : n = 0,1 \\ \left(\dfrac{\lambda}{2\mu_2}\right)^{n-1} \dfrac{\lambda}{\mu_1} P_0 & : 2 \le n \end{cases} \tag{18}$$

In order to calculate integrative criteria like expected number of customers in queue (L_q) and expected number of customers in system (L), by applying Little's relations and steady state condition of system, we have,

$$L_q = \frac{\pi_0}{2}\left(\frac{\lambda^2}{\mu_1\mu_2}\right)\frac{\rho}{(1-\rho)^2} \tag{19}$$

$$L = L_q + \frac{\lambda}{\mu_2} = \frac{P_0}{2}\left(\frac{\lambda^2}{\mu_1\mu_2}\right)\frac{\rho}{(1-\rho)^2} + \frac{\lambda}{\mu_2} \tag{20}$$

Also in order to calculate time criteria like expected waiting time in queue (W_q) and expected waiting time in system (W), by applying Little's relations and definitions, we have,

$$W_q = \frac{L_q}{\lambda} = \frac{P_0}{2}\left(\frac{\lambda}{\mu_1\mu_2}\right)\frac{\rho}{(1-\rho)^2} \tag{21}$$

$$W = W_q + \frac{1}{\mu_2} = \frac{P_0}{2}\left(\frac{\lambda}{\mu_1\mu_2}\right)\frac{\rho}{(1-\rho)^2} + \frac{1}{\mu_2} \tag{22}$$

3.4 The proposed hybrid model by assuming two servers

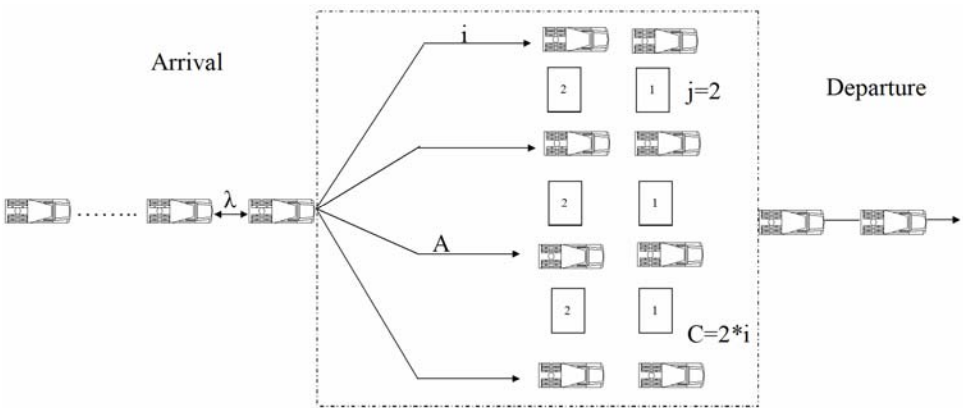
So far, the queuing system is modelled with two servers in one row with rate μ_1 and μ_2, respectively which yields to $M/M/C$ model. However, in reality, these conditions do not occur, but almost a combination of both states is possible. To have a hybrid model we need two new variables θ_1 and θ_2, where θ_1 and θ_2 are the probabilities of the occurrence of state (X) and state (Y) in ΔT period, respectively. To calculate variables in hybrid conditions, $Z_{(c=2)}$ we assume that both conditions will happen. Using a weighted average yields,

$$Z_{(c=2)} = \frac{\theta_1 X + \theta_2 . Y}{\theta_1 + \theta_2} \tag{23}$$

3.5 Extensions to more than two servers

The last part of the results for two servers is used to analyse the whole system's performance. As it can be seen in Figure 8 when a customer arrives at *A* point, against *M*/*M*/*C* mode, she/he does not have the possibility of choosing one of the *C* servers. Here, she/he should choose one of the service rows *i*. Therefore, after arriving into one of the rows, the customer has nothing to do with other branches and just has the option to choose one of two servers in his row. The *C* server model will then become a special kind of hybrid service system with two servers.

Figure 8 Schematic plan of queuing system



Let's assume *A* as the number of customers in the main queue and n_i as the number of people in each branch. In order to calculate probabilities we need the number of people in system *N*. Table 3 demonstrates different scenarios.

Table 3 Number of people in system

Number of people in system (N)	*System state*
$n + n_1$	The customer chooses pump station number 1
$n + n_2$	The customer chooses pump station number 2
$n + n_i - 1$	The customer chooses pump station number $i - 1$
$n + n_i$	The customer chooses pump station number i

In order to compute measures like $\hat{L}$ and $\hat{W}$ for queuing system, because of the similarity amongst service conditions in all rows, we can modify customer arrival rate (λ) and introduce $\hat{\lambda}$ as follow and generalise the results to a *C* server system as follow,

$$\hat{\lambda} = \frac{\lambda}{i}. \tag{24}$$

4 Case study

The proposed model was implemented to a real-world case and it was examined in an Iranian fuel station and the results of queuing system were recorded. We used a simulation technique to study the behaviour of the new system and the results are compared with the existing situation.

4.1 Model outputs

To study the relationships between waiting time in queues and the number of servers (pumps), we used a gas station data as shown in Table 4. In order to have a suitable sample size to calculate the waiting queue we have used design of experiment (DOE) technique. The case study has three rows and two servers shown in Figure 9.

Table 4 Gathered data of gas station

Description	*Parameter*	*Value*		*Unit*
		State 1	*State 2*	
Mean service rate of server 1	μ_1	26	27	Customer per hour
Mean service rate of server 2	μ_2	28	27	Customer per hour
Mean interracial time	λ	50		Customer per hour
Probability of events	θ_1, θ_2	48	52	Percent
Number of pumps (servers)	C	6		Numbers
Number of branches	I	3		Row
Maximum capacity of each branch	n_i	3		Numbers
Station area	S	1,000		Square metres

Figure 9 The existing filling station

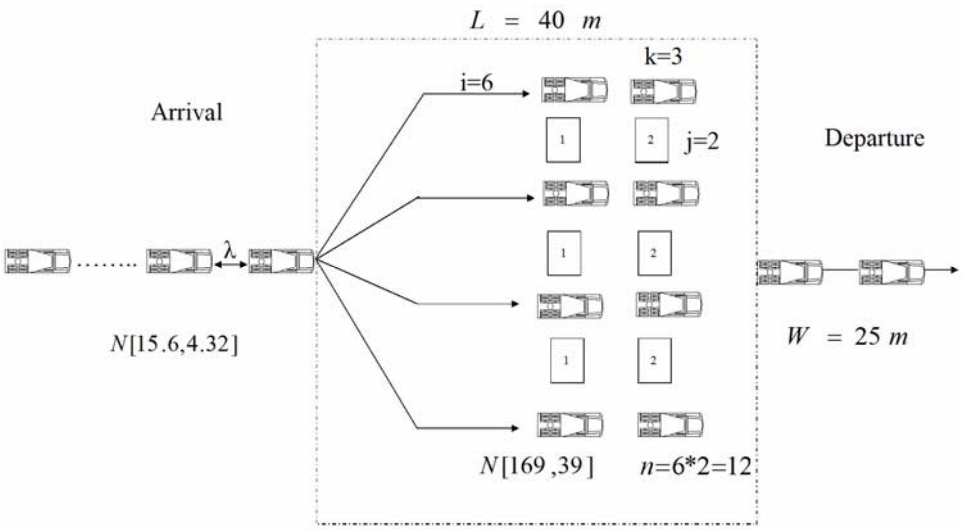


Table 5 demonstrates the details of the steady-state performance analysis.

Table 5 Information of gas station

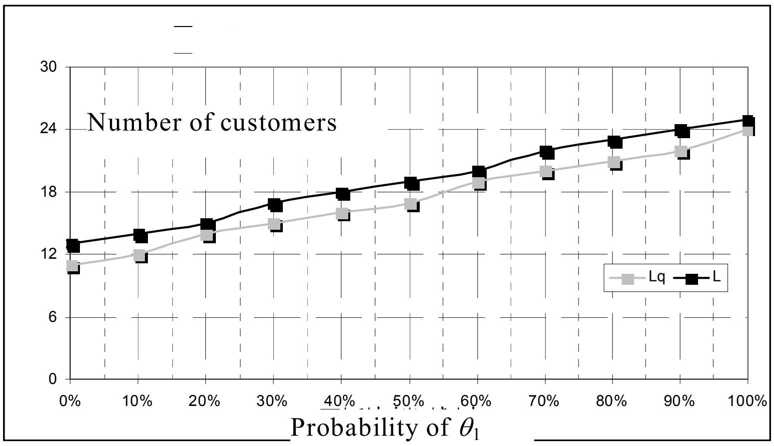
Description	*Measures*	*State 1*	*State 2*	*Unit*
Utilisation coefficient	ρ	96	92	Percent
Probability of existence of nobody in system	P_0	1.96	3.98	Percent
Expected number of customers in queue	L_q	23	11	Customer
Expected number of customers in system	L	25	12	Customer
Expected waiting time in queue	W_q	28	13	Minutes
Expected waiting time in system	W	30	15	Minutes

Using the information of Table 5 and multiplying probability indexes θ_1, θ_2 and the modified arrival rate, $\hat{\lambda}$ is calculated and it is summarised in Table 6.

Table 6 Total analysis results

Description	*Measures*	*Value*	*Unit*
Modified arrival rate	$\hat{\lambda}$	17	Customer per hour
Total expected number of customers in queue	$\hat{L}_q$	4	Customer
Total expected number of customers in system	$\hat{L}$	6	Customer
Total expected waiting time in queue	$\hat{W}_q$	50	Seconds
Total expected waiting time in system	$\hat{W}$	180	Seconds

Figure 10 Sensitivity analysis of number of customers in system based on θ_1



As we can see from Figure 10, θ_1 > θ_2. We discussed the details of our results with industry experts to improve the system.

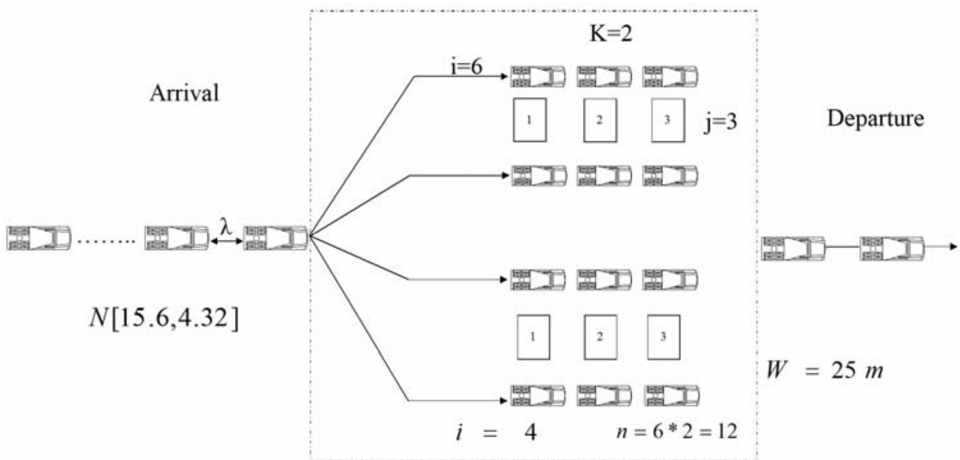
4.2 Managements' suggestions

The proposed model has many applications in a wide range of queuing systems. This queuing model with non-standard state leads to the large queue length and large waiting time. We want to present a scenario in which the system remains empty immediately for the existing cars in the queue after each service accomplishment. Therefore, we could remove the wasting time between each two consecutive services, which is the main defect of non-standard queuing systems. It seems that we cannot increase the productivity of a gas station by simply adding more pumps in the system. In order to study the behaviour of our system, we have used *showflow* simulation tool to simulate a scenario in standard state with two rows and three servers in each row (as shown in Figure 9).

4.3 Scenario simulation and comparison with proposed model

This scenario is proposed with the same number of servers (six pumps) and the same station space constraint (1,000 square metres), which includes two rows with three servers in each row (as shown in Figure 9). Schematic scenario simulation is shown in Figure 11.

Figure 11 Schematic of the proposed filling station



The simulation programme runs for one working day. The output of management proposal simulation and their comparisons with the existing non-standard queuing model for the gas station are shown in Table 7.

The number of pumps in the station is 12 for both the proposed scenario and the existing situation and the number of queue branches in the proposed scenario is four and for the existing situation is six. The evaluation measures for fuel queuing system are described in Table 7 are W_q, L_q, L_q max, ρ, $X'(t)$.

As we can see in Table 7, the systems' performance proposed by the model is better than that the existing non-standard situation, which is modelled in the paper. Queue mean waiting time in the proposed scenario is approximately 20 times less than the existing situation and the maximum queuing length in the proposed scenario is also reduced significantly.

Management scenario has better utilisation but the gas stations are still built based on the non-standard format.

Table 7 Comparison between the simulation and the non-standard queuing systems

Measures	*Notation*	*Value*		*Unit*
		Proposed scenario	*Existing situation*	
Mean waiting time in queue	W_q	590	30	Second
Mean queue length	L_q	38	1	Car
Maximum queue length	L_q max	68	6	Car
Utilisation	ρ	80	95	Percent
Number of served cars until time *t*	$X'(t)$	1,700	1,825	Car
Number of pumps in the station	N	12	12	Number
Number of queue branches	i	6	4	Number
Number of pump in each rows	j	2	3	Number
Number of rows	k	3	2	Number
Station space	S	40 * 25	40 * 25	Square metres

5 Conclusions

In this paper, we have developed a model for queuing systems in non-standard states. In addition, we calculated the key performance measures. These kinds of non-standard queuing systems are applicable in some assembly and production lines, too. We verified the proposed model by applying it to the real-world case (see Govil and Fu, 1999; Rao et al., 1998). Moreover, management proposals are recommended in order to prevent defects and improve the non-standard queuing systems' performance. We have shown, using a simulation technique, that the performance of the gas stations could significantly improved if the proposed scenario of this paper is used.

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