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Research Article

Different Approximation Algorithms for Channel Scheduling in Wireless Networks

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We introduce a new two-side approximation method for the channel scheduling problem, which controls the accuracy of approximation in two sides by a pair of parameters (f, g). We present a series of simple and practical-for-implementation greedy algorithms which give constant factor approximation in both sides. First, we propose four approximation algorithms for the weighted channel allocation problem: 1. a greedy algorithm for the multichannel with fixed interference radius scheduling problem is proposed and an one side $O(1)$-IS-approximation is obtained; 2. a greedy $(O(1), O(1))$-approximation algorithm for single channel with fixed interference radius scheduling problem is presented; 3. we improve the existing algorithm for the multichannel scheduling and show an $|E|O^{(d/\epsilon)}$ time $(1-\epsilon)$-approximation algorithm; 4. we speed up the polynomial time approximation scheme for single-channel scheduling through merging two algorithms and show a $(1-\epsilon, O(1))$-approximation algorithm. Next, we study two polynomial time constant factor greedy approximation algorithms for the unweighted channel allocation with variate interference radius. A greedy $O(1)$-approximation algorithm for the multichannel scheduling problem and an $(O(1), O(1))$-approximation algorithm for single-channel scheduling problem are developed. At last, we do some experiments to verify the effectiveness of our proposed methods.

1. Introduction

An important problem in wireless network is to develop efficient algorithms for maximum throughput by scheduling channels among many nodes. Based on the Shannon capacity formula, we can know that there is a minimum SINR required for each user in the network of wireless communication, as the existence of signal propagation loss during the process of data transmission; when the distance of a communication user pair is larger than the threshold, the effect of the received power in the receiver can be neglected because the receiver's SINR is less than the threshold. There is an interference between two users in the same channel when their distance is less than a certain distance. Given a set of users, we find a way which schedules a maximal number of them without introducing interference between any two of them. The weighted version of this problem is to achieve the sum of maximal weights for the users assigned channels for communication. The channel scheduling problem is NP-hard even in the single-channel model [1] and has been studied in a series of papers (for example, [2–4]). According to IEEE standard 802.11a, there are 13 orthogonal channels provided for wireless network [5]. The multichannel or multi-radio scheduling problem is essential to guarantee the performance of wireless networks.

A channel scheduling problem for a given graph $G(V, E)$ is to select an edge subset $E' \in E$ and assign a channel to each edge in E' under the restriction so that all edges in E' are interference-free. The node exclusive interference model has been studied in many articles (for example, [6–12]). The algorithms for the single-channel or single-radio scheduling are reported in [13, 14]. For the model not allowing two communication edges to share the same node, the optimal scheduling problem is converted into matching problem [15–17] which has a polynomial time solution. Some approximation algorithms for single channel were reported in [18] which show the existence of a theoretical polynomial time approximation scheme for the single-channel assignment. A simple greedy algorithm which is easy to implement is shown to have a constant factor approximation [18] for the single-channel assignment.

The theoretical research about the multichannel scheduling has been reported in [19–23]. In [4], they formulate the channel allocation problem as an NP-hard nonlinear integer programming problem and then propose a probabilistic polynomial time $(1-(1/e))$-approximation algorithm based on linear programming. A joint approach between routing and channel scheduling is shown in [24]. Some constant factor approximation algorithms are shown in [25] for assigning a minimum number of channels for a conflict-free communication, which are also NP-hard [26, 27].

A polynomial time approximation scheme was developed in [28] for multi-radio and multichannel wireless network with very high computational complexity $|E|O^{(d/\epsilon^2)}$ for $(1-\epsilon)$-approximation, where d is the number of channels. This theoretical result is not practical for implementation.

A maximal independent set problem based on joint scheduling and routing optimization is studied in [2], which is a mixed integer non-linear programming NP-hard problem. It developed a column generation based ϵ-approximation-bounded approximation algorithm, which can find tight ϵ-bounded approximate solutions and the optimal solutions.

We introduce the notion of two-side approximation for the channel scheduling problem. A pair of parameters (f, g) controls the accuracy of approximation in this paper. A (f, g)-approximation satisfies $\sum_{e\in E'} W(e) \geq \sum_{e\in \mathrm{Opt}} (W(e)/f)$ and $\sum_{e\in E-E'} W(e) \leq g\sum_{e\in E-\mathrm{Opt}^*} W(e)$, where Opt^* is the set of edges assigned channels in an optimal solution and $W(e)$ is the weight of edge e. A f-approximation satisfies $\sum_{e\in E'} W(e) \geq (\sum_{e\in Opt} W(e)/f)$. Our two-side approximation approach combines two complementary problems. The first one is to maximize the total weights of communication edges that have been assigned channels, and the second one is to minimize the total weight of edges that do not receive channel for communication. It brings a more accurate tool for the approximation algorithms for the channel scheduling problem.

As the approximation scheme developed by Cheng et al. [28] has very high computational complexity, it is impossible to implement with software. Finding simple and efficient algorithm for the multichannel scheduling problem is still a challenging research topic. We show that a simple greedy algorithm can obtain an $(O(1), O(1))$-approximation for the single-channel scheduling problem. In many cases, the greedy algorithms give much more accurate results than the worst ratio. Furthermore, we develop an $|E|O^{(1/\epsilon)}$ time $(1-\epsilon, O(1))$-approximation algorithm for the single-channel scheduling problem. We also show that a simple greedy algorithm satisfies $\sum_{e\in E'} W(e) \geq (\sum_{e\in \mathrm{Opt}} W(e)/\Omega(1))$, which can obtain an $O(1)$-approximation for the single-channel scheduling problem. We also develop a $|E|O^{(1/\epsilon)}$ time $(1-\epsilon)$-approximation algorithm for the multi-channel scheduling problem.

Our proposed multi-channel scheduling algorithm can be used in many wireless systems in which the number of users is larger than the number of subchannels; this case is considered in [29] where they study the channel assignment problem in uplink wireless communication system. It also can be used in the distributed data transmission system with limited communication resources, which is shown in [30]. The proposed methods are also suitable for channel assignment problem in the cellular-VANET heterogeneous wireless networks, which is studied in [25]. Our proposed channel scheduling approach has a wide range of applications and can be applied to different wireless resource allocation scenarios.

We also develop a polynomial time constant factor greedy approximation algorithm for the multi-channel scheduling that allows variate interference radius among those nodes. The paper is organized as follows: in Section 2, we present approximation algorithms for the weighted scheduling problems. The interference range is determined by a uniform parameter t, but the edges of communication have different weights. In Section 2.2, we show that a greedy algorithm for multi-channel assignment has a constant factor approximation. In Section 2.3, we show a two-side approximation algorithm for the single-channel assignment problem and show that there does not exist such an approximation for the multi-channel assignment problem. In Section 2.4, we give improved approximation schemes for both single-channel and multi-channel assignment problems. In Section 3, we develop the approximation for the unweighted channel assignment problem. This is the case that all the edges have weight 1, but the interference radius is not fixed. Some simulation results for the greedy approximation algorithm for the multi-channel assignment are given in Section 4. We draw conclusion in Section 5.

2. Weighted Channel Assignment

We develop approximation algorithms for the weighted channel assignment. The interference range is controlled by a fixed parameter t. The polynomial time approximation scheme for multi-channel scheduling problem is based on this model [28].

2.1. Definitions and Models. The network is modeled as an edge-weighted graph $G(V, E)$, where V is the set of nodes and E is the set of edges for traffic flow. The weight of each edge is often represented by the rate of network traffic. For each $e \in E$, let $W(e)$ be the weight of e.

Definition 1. Given an edge-weighted network graph $G(V, E)$, let $d(u, v)$ denote the distance between nodes u and

v. The function $d(u,v)$ can be either the geometric distance between u and v in Euclidean space if V is a set of points in Euclidean space or the hop distance between u and v in the graph G. The edge distance between two edges $e_1 = (u_1, u_2)$ and $e_2 = (v_1, v_2)$ is defined by $ED(e_1, e_2) = \min_{i,j\in\{1,2\}}\{d(u_i, v_j)\}$. Note that the graph $G(V,E)$ only considers those edges E that need to do communication over a wireless network. No silent edge is included in E.

A set E' of edges is t-interference-free matching if $ED(e_1, e_2) > t$ for any two edges e and e' in E', where t is distance threshold for interference.

A (multiple) channel assignment problem has a demand graph $G = (V,E)$ that requests communication for $(u,v) \in E$. A wireless network may not have the resource to satisfy the communications for all edges. A channel assignment algorithm selects a subset $E' \subseteq E$ and assigns a channel to each edge in E' so that the edges in the same channel form a t-interference-free matching, where t is distance threshold for interference. When there is only one channel available for the entire demand graph, the channel assignment problem is called single-channel assignment problem. Otherwise, it is called multi-channel assignment problem.

A channel assignment for an edge $e = (u,v)$ is represented by (e,K), where K is a channel. Assume that M is a set of channel assignments. A channel assignment (e,K) is interfered by M if there is a channel assignment $(e',K) \in M$ such that $ED(e,e') \le t$. Define M^* to be the set of all edges e with $(e,K) \in M$ for some channel K. If $W(.)$ is the weight function for the edges in G, define $W(M^*) = \sum_{e\in M^*} W(e)$. An optimal solution for a channel assignment problem $G = (V,E)$ with weight function $W(.)$ is a set Opt of channel assignments for edges such that $W(\text{Opt}^*)$ is the maximum.

Definition 2. Assume that A is a set of channel assignments for G.

Let $\tau_A(e) = \max_K |\{e': (e',K) \in A \text{ and } ED(e,e') \le t\}|$.
Define $\tau_A(G) = \max_{e\in G} \tau_A(e)$.
Define $\tau(G) = \max_A \tau_A(G)$,

where $\tau_A(e)$ is the interference degree of an edge $e \in E$, $\tau_A(G)$ is the interference degree of the graph $G = (V,E)$, and t is distance threshold for interference.

If the distance is the Euclidean distance and all nodes are on the plane, then $\tau(G) \le 11$, which is shown in Lemma 1. If the distance is the hop distance and all nodes are on the plane, then $\tau(G) \le 49$, which was proved in [18]. Therefore, $\tau(G)$ does not depend on the threshold.

Definition 3. Assume that G is a channel scheduling problem. We define some measures for approximations.

(i) An f-IS-approximation App for the channel scheduling problem satisfies the condition $W(\text{App}^*) \ge (W(\text{Opt}^*)/f)$.
(ii) A g-VC-approximation App for the channel scheduling problem satisfies the condition $W(E - \text{App}^*) \le gW(E - \text{Opt}^*)$.
(iii) A (f,g)-approximation for the channel scheduling problem satisfies the conditions $W(\text{App}^*) \ge (W(\text{Opt}^*)/f)$ and $W(E - \text{App}^*) \le gW(E - \text{Opt}^*)$.

Lemma 1. *Assume that all nodes in the demand graph $G = (V,E)$ are points on a plane. Then, $\tau(G) \le 11$ for the Euclidean distance t as the threshold for the interference.*

Proof. Let $e = (u,v)$ be an edge in the graph G. The distance between u and v is at most t (otherwise, they cannot communicate). Edge e and another edge $e' = (u',v')$ have interference if one of u and v and one of u' and v' have distance at most t. Let each node be a center of a circle of diameter $t/2$ on a plane. Let $C(p,r)$ represent a circle with center at p and radius r. Let A be a channel assignment for G, which gives a list of disjoint circles $C_1, \cdots, C_m$ such that each node in those edges with channel assigned is a circle center. Circle $C(u,t/2)$ touches at most 6 circles among $C_1, \cdots, C_m$, and so does $C(v,t/2)$. If both $C(u,t/2)$ and $C(v,t/2)$ touch 6 circles, there must exist at least one of them touched by both $C(u,t/2)$ and $C(v,t/2)$ because the distance between u and v is at most t.

2.2. Multiple Channel with Fixed Interference Radius Scheduling. In this section, we present a greedy approximation algorithm for the multi-channel scheduling problem shown in Algorithm 1. We can only show a one-side $O(1)$-approximation for the multi-channel scheduling problem.

In the multi-channel scheduling problem, we assume that each node has d channels available for allocation. Two nodes u and v can communicate if their corresponding edge $e = (u,v)$ is assigned a channel (the distance between u and v is at most the threshold t) and has no interference with other edges in the same channel.

We note that when there is a set F of edges that have been assigned channels, it is straightforward to check if a new edge e will be interfered at certain channel K. This can be done by checking $ED(e,e_i) \le t$ for all $e_i \in F_K$, where F_K is the subset of edges in F assigned channel K.

For the case that all nodes are on a plane, and the distance is either Euclidean distance or hop distance, the following theorem gives a constant factor approximation for the multiple channel assignment problem as $\tau(G)$ is bounded by constants in both distances.

Theorem 1. *The Algorithm 1 greedy-multi-channel (.) is a $g(\alpha)$-IS-approximation algorithm for the multi-radio multi-channel scheduling problem and has the computational complexity $O(|E|^2)$, where $g(\alpha) = (1 + (\tau(G)/\alpha) + (2\alpha/(1-\alpha)))$ and $\alpha = \sqrt{\tau(G)}/(\sqrt{2} + \sqrt{\tau(G)})$.*

Proof. Assume that Opt is an optimal solution for the channel scheduling problem. Let App be an approximate solution derived by greedy-multi-channel (.).

For each edge g in Opt^*, assign an edge e, denoted by $H(g)$, in App^* such that $R(e)$ is the least among all edges with interference with g. When the algorithm greedy-multi-

Input: a weighted graph G, and a distance parameter t for the largest distance of interference.
Output: L'
(1) Sort all edges by the increasing order of their weights and put those edges in a list L;
(2) Let $L' = \varnothing$;
(3) Repeat;
(4) Select an edge $e = (u, v)$ with the largest weight from L;
(5) If there is a channel K for e not being interfered by L'
(6) Then, assign the channel K to e and put (e, K) into L';
(7) Remove e from L;
(8) Until L is empty;
(9) Return L';

Algorithm 1: Greedy-multi-channel with fixed interference radius.

channel (.) processes an edge $e = (u, v)$, the available channels for e are those which are not allocated to the edges with end points at either u or v.

Claim 1. For each edge g in Opt, $W(g) \leq W(g')$ for each g' that is already processed before g.

Proof. It follows from the greedy algorithm which processes the edges according the decreasing order of their weights.

Let d be the total number of channels. For each edge $e = (u, v) \in \text{Opt}^* - \text{App}^*$, we consider two cases. The constant α used in the two cases will be assigned later.

Case 1. There are at least αd channels available for the edge e when e is processed. Such an edge e is of type 1.

In this case, since e is not assigned for a channel, there must be at least αd edges e' that are already assigned channels and have $ED(e, e') \leq t$. For such an edge e' with $ED(e, e') \leq t$, let $b(e, e') = W(e)/\alpha d$. Since e' is processed before e in the algorithm, we have $W(e) \leq W(e')$. Thus, $b(e, e') \leq W(e')/\alpha d$. Let $b(e, e') = 0$ for all the other edges e'. For each edge e of type 1, we have inequality

$$\sum_{e'} b(e, e') \geq W(e). \tag{1}$$

For each edge e' in App^*, we have

$$\sum_{e \text{ is of type } 1} b(e, e') \leq d \cdot \tau(G) \cdot \frac{W(e')}{\alpha d} = \frac{\tau(G) W(e')}{\alpha} = b_1 W(e'), \tag{2}$$

where $b_1 = \tau(G)/\alpha$.

Case 2. There are fewer than αd channels available for the edge $e = (u, v)$ when e is processed in greedy-multi-channel (.). Such an edge e is of *type 2.*

For at least one of u and v, say u, there are at least $((1 - \alpha)d)/2$ channels that are already assigned. For each edge e' with an end point in u already assigned channels, let $b(e, e') = (W(e)/ \quad ((1 - \alpha)d)/2) = (2W(e)/(1 - \alpha)d) \leq (2W(e')/(1 - \alpha)d)$. Let $b(e, e') = 0$ for all the other edges e'. For each edge $e' \in \text{App}^*$, we have

$$\sum_{e \text{ is of type } 2} b(e, e') \leq (\alpha d) \cdot \frac{2W(e')}{(1 - \alpha)d} = \frac{2W(e')}{(1 - \alpha)} = b2W(e'), \tag{3}$$

where $b_2 = 2\alpha/(1 - \alpha)$. For each edge e of type 2,

$$\sum_{e'} b(e, e') \geq W(e). \tag{4}$$

Each edge in $\text{Opt}^* - \text{App}^*$ is of either type 1 or type 2. We have

$$\begin{aligned} W(\text{Opt}^* - \text{App}^*) &= \sum_{e \text{ is of type } 1} W(e) + \sum_{e \text{ is of type } 2} W(e) \\ &\leq \sum_{e \text{ is of type } 1} \sum_{e' \in \text{App}^*} b(e, e') \\ &\quad + \sum_{e \text{ is of type } 2} \sum_{e' \in \text{App}^*} b(e, e') (by\ (1) \text{ and } (1)) \\ &\leq \sum_{e' \in \text{App}^*} \sum_{e \text{ is of type } 1} b(e, e') \\ &\quad + \sum_{e' \in \text{App}^*} \sum_{e \text{ is of type } 2} b(e, e') \\ &\leq \sum_{e' \in \text{App}^*} b_1 W(e') \\ &\quad + \sum_{e' \in \text{App}^*} b_2 W(e') (by\ (1) \text{ and } (1)) \\ &\leq (b_1 + b_2) W(\text{App}^*). \end{aligned} \tag{5}$$

We have

$$\begin{aligned} W(\text{Opt}^*) &= W(\text{Opt}^* \cap \text{App}^*) + W(\text{Opt}^* - \text{App}^*) \\ &\leq W(\text{App}^*) + W(\text{Opt}^* - \text{App}^*) \\ &\leq W(\text{App}^*) + (b_1 + b_2) W(\text{App}^*) (by\ (1)) \\ &\leq (1 + b_1 + b_2) W(\text{App}^*) \\ &\leq \left(1 + \frac{\tau(G)}{\alpha} + \frac{2\alpha}{(1 - \alpha)}\right) W(\text{App}^*). \end{aligned} \tag{6}$$

Select the constant α so that $(1 + (\tau(G)/\alpha) + (2\alpha/(1-\alpha)))$ is minimal. Let $g(\alpha) = (1 + (\tau(G)/\alpha) + (2\alpha/(1-\alpha))) = (-1 + (\tau(G)/\alpha) + (2/(1-\alpha)))$. Let $g(\alpha)' = (2/(1-\alpha)^2) - (\tau(G)/\alpha^2) = 0$. We have $\alpha = \sqrt{\tau(G)}/(\sqrt{2} + \sqrt{\tau(G)})$. Thus, the ratio of approximation is $g(\alpha)$.

2.3. Single Channel with Fixed Interference Radius Scheduling. In this section, we present a greedy $(O(1), O(1))$ approximation algorithm for the channel scheduling problem shown in Algorithm 2. Our two-side approximation bound for the greedy algorithm improves the one-side approximation bound in [18].

We note that when there is a set F of edges that have been assigned channels, it is straightforward to check if a new edge e will be interfered at certain channel K. This can be done by checking $ED(e, e_i) \le t$ for all $e_i \in F_K$, where F_K is the subset of edges in F in channel K.

Theorem 2. *The Algorithm 2 greedy-single-channel (.) is a $(\tau(G), \tau(G))$-approximation algorithm for the single-channel scheduling problem and has the computational complexity $O(|E|^2)$.*

Proof. Assume that Opt is an optimal solution for the channel scheduling problem. Let App is an approximate solution derived by greedy-single-channel (.).

For each edge g in Opt*, assign a edge e, denoted by $H(g)$, in App* such that e has the largest weight among all edges with interference with g.

Consider an edge $e = (u, v)$ selected in App*. Let $A(e) = e_1, \cdots, e_m$ be the list of edges with $H(e_i) = e$.

Claim 2. For each edge g in Opt, $W(g) \le W(H(g))$.

Proof. If g is in App*, we consider an edge has interference with itself. Thus, it is trivial. By the definition of $H(g)$, $H(g)$ has the largest of weight among all edges in Opt* with interference with g.

Assume that g' is the first edge in App* and has the same channel with $g \in$ Opt*. Before selecting g' for assigning a channel, there is no interference between g and other edges in App. Therefore, $W(g') \ge W(g)$ (otherwise, g' should not be selected for channel assignment). Since $H(g)$ has the largest weight among all edges with interference with g in App, we have $W(H(g)) \ge W(g') \ge W(g)$.

Assume that App contains channel assignments for edges $e_1, \cdots, e_m$. Partition Opt* into $A(e_1), \cdots, A(e_m)$. By Claim 2, we have that

$$\tau(G)W(e_i) \ge \tau(e_i)W(e_i) \ge \sum_{e' \in A(e_i)} W(e'). \quad (7)$$

Therefore, we have $\tau(G)W(\text{App}^*) \ge W(\text{Opt}^*)$. Thus, App is a $\tau(G)$-IS approximation for G.

On the other hand, for each $e_i \in (E - \text{Opt}^*) \cap \text{App}^*$, we always have $\tau(e_i)W(e_i) \ge \sum_{e' \in A(e_i)} W(e')$ (by Claim 2). For each edge e' in $(E - \text{App}^*) \cap \text{Opt}^*$, there is an edge $e \in \text{App}^*$ such that e and e' have the interference at the same channel (otherwise, e' would be assigned some channel by Greed1 (.)). Furthermore, $e \notin \text{Opt}^*$ since e has interference with e'. Therefore, there is an edge $e^* \in (E - \text{Opt}^*) \cap \text{App}^*$ such that $e' \in A(e^*)$. By inequality (1), we have

$$W((E - \text{App}^*) \cap \text{Opt}^*) \le \tau(G)W((E - \text{Opt}^*) \cap \text{App}^*). \quad (8)$$

We have the inequalities:

$$\begin{aligned} W(E - \text{App}^*) &= W((E - \text{App}^*) \cap (E - \text{Opt}^*)) \\ &\quad + W((E - \text{App}^*) \cap \text{Opt}^*) \\ &\le W((E - \text{Opt}^*) \cap (E - \text{App}^*)) \\ &\quad + \tau(G)W((E - \text{Opt}^*) \cap \text{App}^*)\,(by\,(1)) \\ &\le \tau(G)W((E - \text{Opt}^*) \cap (E - \text{App}^*)) \\ &\quad + W((E - \text{Opt}^*) \cap \text{App}^*) \\ &= \tau(G)W(E - \text{Opt}^*). \end{aligned} \quad (9)$$

This gives $W(E - \text{App}^*) \le \tau(G)W(E - \text{Opt}^*)$. Therefore, greedy-single-channel (.) gives a $(\tau(G), \tau(G))$-approximation.

Theorem 3. *Let $d(u, v)$ be the Euclidean distance and the input has the geometric position of all nodes in a Euclidean plane. The Algorithm 2 greedy-single-channel (.) gives a $(11, 11)$-approximation for the single-channel scheduling problem and runs in $O(|E|)$ time.*

Proof. For the implementation, we partition the plane into grid of size $t \times t$. When an edge is assigned a channel, put the assignment into the corresponding grid for one of its two nodes in the edge. When assigning a channel a new edge, check the assigned edges in the nearby $O(1)$ grids. Thus, each edge only costs an $O(1)$ time.

If $d(u, v)$ is defined to be the hop distance, then $\tau(G)$ is at most 49, which is shown in [18]. If $d(u, v)$ is the Euclidean distance, we show that $\tau(G)$ is at most 11.

Theorem 4. *Let $d(u, v)$ be the hop distance in network graph $G(V, E)$. The Algorithm 2 greedy-single-channel (.) gives a $(49, 49)$-approximation for the single-channel scheduling problem and runs in $O(|E|^2)$ time.*

Proof. It follows from Theorem 2 and the fact $\tau(G) \le 49$, which is shown in [18]. A brute force implementation takes $O(|E|^2)$ time.

For the multi-channel scheduling problem, we show that it does not have two-side approximation unless P=NP.

Theorem 5. *Assume that $f(n)$ is a function from N to N with $f(n) > 0$. Then, there is no polynomial time $f(n)$-VC-approximation for the multiple channel scheduling problem unless P=NP.*

Proof. Let G be the input graph of the multi-channel assignment problem. Assume that there exists a polynomial time $f(n)$-VC-approximation algorithm, the multiple

Require: a weighted graph G, and a distance parameter t for the largest distance of interference.
Ensure: L';
(1) Sort all edges by the decreasing order of their weights and put those edges in a list L;
(2) Let $L' = \varnothing$;
(3) Repeat;
(4) Select an edge $e = (u, v)$ with the largest weight from L;
(5) If e is not being interfered by L';
(6) Then, assign the channel K to e and put (e, K) into L';
(7) Remove e from L;
(8) Until L is empty;
(9) Return L';

Algorithm 2: Greedy-single-channel with fixed interference radius (G, t).

channel scheduling problem. Let k be the least number of channels that can support the communications of all edges in G. When the number of total available channels is equal to k, an optimal solution Opt assigns channels to all edges in G. This makes $W(E - \text{Opt}^*) = 0$. When the approximate solution App satisfies $W(E - \text{App}*) \leq f(n) W(E - \text{Opt}^*) = 0$, it becomes an optimal solution. Thus, we can search the least number of channels from 1 to k to support all edges in G. This brings a polynomial time solution for the conflict-free channel assignment problem, which was proved to be NP-hard [26]. Therefore, P=NP.

We have the following corollary that shows we cannot have a two-side approximation for the multi-channel scheduling problem.

Corollary 1. *Assume that $f(n)$ is a function from N to N with $f(n) > 0$. Then, there is no polynomial time $(f(n), f(n))$-approximation for the multiple channel scheduling problem unless P=NP.*

In Section 2.4, we show a faster approximation for $(1-\epsilon)$-IS-approximation than that in [28]. Part of the algorithm is based on the shifting technology which has originated from [31] and has been widely used in developing approximation for networking problem (for example, [18, 28]).

In Section 3.1, we present a greedy $O(1)$-approximation algorithm for the unweighted multi-channel scheduling problem with variate interference radii.

In Section 3.2, we present a greedy $(O(1), O(1))$ approximation algorithm for the unweighted single channel with variate interference radii scheduling problem.

In Section 4, we did some simulation for the multi-channel assignment problems for the greedy algorithm. Greedy algorithm is easy to implement and fast to output the result. Its experimental results show much better performance than the theoretical approximation ratio, which is derived under the worst case analysis.

2.4. Improving the Existing Algorithm for the Multichannel Scheduling. We show a faster approximation for $(1-\epsilon)$-IS-approximation than that in [28]. Let PTAS (.) (polynomial-time approximation scheme) represent the algorithm described below. Part of the algorithm is based on the shifting technology which has originated from [31] and has been widely used in developing approximation for networking problem (for example, [18, 28]).

Assume that the maximal distance of two nodes for communication is one. Let t be the distance of interference. The grid size is $D = t + 2$. The plane is partitioned into grids of size $D \times D$.

Let ϵ be a constant in $(0, 1)$. Define $m = \lceil 1/(1 - \sqrt{1-\epsilon}) \rceil$.

For two integers $a, b \in [0, m-1]$, define $P_{a,b}$ to be a partition such that the plane is partitioned into the disjoint union of squares of size $mD \times mD$, and the left bottom point of each square of size $mD \times mD$ in $P_{a,b}$ has coordinates $(imD + a, imD + b)$ for some integers i and j.

Lemma 2. *There is a $m^{O(dm)}$ time algorithm to find an optimal solution for the multi-channel scheduling problem in a $m \times m$ square with at most d channels in each node.*

Proof. We apply a division and conquest method to find an optimal solution. Partition a $m \times m$ square into four squares by one strip in the vertical middle and one strip in the horizontal middle. The width of the two stripes is equal to the width of the grid $(t + 2)$. We can only select at most $O(m)$ edges in the two strips. The number of cases of choices is $|E|^{O(m)}$ for a single channel. The number of cases of choices is $|E|^{O(dm)}$ for d channels. The four sub-problems in the four sub-squares can be solved independently.

Let $T(m)$ be the computational time for solving the channel scheduling problem in a $m \times m$ square. We have the recursive equation: $T(m) = |E|^{O(dm)} 4T(m/2)$. Select a constant c_0 so that $T(m) = |E|^{c_0 m} 4T(m/2)$. Expanding the recursion, we have

$$\begin{aligned} T(m) &= |E|^{c0\,dm} 4T\left(\frac{m}{2}\right) = 4^{O(\log m)} |E|^{c0\,d\frac{m}{2} + c0\,d\frac{m}{2^2} + c0\,d\frac{m}{2^3} + \ldots} \\ &= |E|^{O(dm)} m^{O(1)}. \end{aligned} \tag{10}$$

Theorem 6. *Assume that ϵ is an arbitrary constant in $(0, 1)$. Then, there is an $|E|^{O(d/\epsilon)}$ time algorithm to give a $(1-\epsilon)$-IS-approximation for the multi-channel scheduling with d channels.*

Proof. A point is in the boundary of $G_{i,j}$ if it is in a $D \times D$ grid in the boundary of a $mD \times mD$ big grid of $P_{i,j}$. Let $E_{i,j}^{B,O}$ be the set of all edges that connect the points in the boundary of $G_{i,j}$ in an optimal solution.

Assume $P_{i,j}$ is the disjoint union of $mD \times mD$ grids $Q_1 \cup Q_2 \cup, \ldots, Q_t$. Let $O(Q_i)$ be the optimal solution for the set of edges connecting at least one node not in any $D \times D$ boundary grid of Q_i. Let $E_{i,j}^{I} = O(Q_1) \cup O(Q_2) \cup, \ldots, \cup O(Q_t)$. We have

$$W\left(E_{i,j}^{B,O}\right) + W\left(E_{i,j}^{I}\right) \geq W(\mathrm{Opt}^*). \quad (11)$$

Each boundary grid is in at most $(2m-1)\, P_{i,j}$. We also have $\sum_{i,j} W(E_{i,j}^{B,O}) \leq (2m-1) W(\mathrm{Opt}^*)$. Therefore, there are i_0 and j_0 such that

$$W\left(E_{i_0,j_0}^{B,O}\right) \leq \frac{(2m-1)}{m^2} W(\mathrm{Opt}^*). \quad (12)$$

By inequalities (11) and (12), we have

$$\begin{aligned} W\left(E_{i_0,j_0}^{I}\right) &\geq W(\mathrm{Opt}^*) - W\left(E_{i_0,j_0}^{B,O}\right) \\ &\geq W(\mathrm{Opt}^*) - \frac{(2m-1)}{m^2} W(\mathrm{Opt}^*) \\ &\geq \frac{(m-1)^2}{m^2} W(\mathrm{Opt}^*) \\ &\geq (1-\epsilon) W(\mathrm{Opt}^*). \end{aligned} \quad (13)$$

The computational time is reduced to $|E|^{O(1)} T(m)$, where $T(m)$ is the time for finding the optimal solution in a $m \times m$ area. By Lemma 2, the algorithm runs in $|E|^{O(d/\epsilon)}$ time and gives an $(1-\epsilon)$-approximation.

2.5. Merging Two Algorithms in Single-Channel Scheduling. In this section, we show that merging Algorithm 1 and PTAS (.) can speed up the polynomial time approximation scheme in many cases for the single-channel scheduling. We propose Algorithm 3 which shows a polynomial time $(1-\epsilon, O(1))$-approximation scheme for the single-channel scheduling problem.

Lemma 3. *Assume that a channel scheduling problem G satisfies the condition $W(Opt) \geq cW(E)$. Then, a g-VC-approximation for the channel scheduling implies a $(1 - g(1-c))$-IS-approximation.*

Proof. Assume an approximation App satisfies $W(E - \mathrm{App}^*) \leq gW(E - \mathrm{Opt}^*)$.

$$\begin{aligned} W(\mathrm{App}^*) &= W(E) - W(E - \mathrm{App}^*) \\ &\geq W(E) - gW(E - \mathrm{Opt}^*) \\ &\geq W(E) - g(1-c) W(E) \\ &\geq (1 - g(1-c)) W(E) \\ &\geq (1 - g(1-c)) W(\mathrm{Opt}^*). \end{aligned} \quad (14)$$

Theorem 7. *Assume that ϵ is an arbitrary constant in $(0,1)$. Then, there is an $|E|^{O(1/\epsilon)}$ time algorithm to give a $(1-\epsilon, O(1))$-approximation for the single-channel scheduling. Furthermore, Algorithm 3 runs in $O(|E|^2)$ time if $\tau(G)(1 - (W(Opt^*)/W(E))) \leq \epsilon$, where G is the graph of wireless network.*

Proof. By Theorem 2, we have that App_1 is a $(\tau(G), \tau(G))$-approximation for the single-channel scheduling problem with $W(E - \mathrm{App}_1^*) \leq \tau(G) W(E - \mathrm{Opt}^*)$ and $W(\mathrm{App}_1^*) \leq \tau(G) W(\mathrm{Opt}^*)$. By Lemma 3, if $g(1 - (W(\mathrm{Opt}^*)/W(E))) \leq \epsilon$, then App_1 is a $(1-\epsilon)$-IS-approximation, where $g = \tau(G)$.

Since App_2 is a $(1-\epsilon)$-IS-approximation for the channel scheduling problem and App_2 is also an approximation for the channeling problem, we have $W(\mathrm{App}_i^*)$ with $W(\mathrm{App}_i^*) = \max(W(\mathrm{App}_1^*), W(\mathrm{App}_2^*))$ being at most $W(\mathrm{Opt}^*)$ and at least $W(\mathrm{Opt}^*)/(1-\epsilon)$. Therefore, App_i is a $(1-\epsilon)$-IS-approximation for the channel scheduling problem.

Since $W(E - \mathrm{App}_i^*) \leq W(E - \mathrm{App}_1^*) \leq O(1) W(E - \mathrm{Opt}^*)$, we have that App_i is a $(1-\epsilon, O(1))$-approximation for the channel scheduling problem.

3. Unweighted Channel Assignment

In this section, we develop the approximations for the unweighted channel assignment problem. This is the case that all the edges have weight 1. The radii of interference are not fixed.

In the unweighted model, each node u has a radius $r(u)$ for the range that u has the interference. Any node v with distance at most $r(u)$ to u is interfered by u when u is active. To an edge $e = (u, v)$ in an active communication, we assume $\mathrm{dist}(u, v) \leq \min(r(u), r(v))$.

For edges $e = (u_1, u_2)$ and $e' = (v_1, v_i)$, there is an interference between them if $\mathrm{dist}(u_i, v_j) \leq \max(r(u_i), r(v_j))$ for some u_i and v_j in the nodes of the two edges.

Definition 4. Assume that G is a channel scheduling problem. We define some measures for approximations.

An optimal solution for G is a set Opt^* of edges without interference with largest $|\mathrm{Opt}^*|$.

A f-IS-approximation App for the channel scheduling problem satisfies the condition $|\mathrm{App}^*| \geq (|\mathrm{Opt}^*|/f)$.

A g-VC-approximation App for the channel scheduling problem satisfies the condition $|E - \mathrm{App}^*| \leq g|E - \mathrm{Opt}^*|$.

A (f, g)-approximation for the channel scheduling problem satisfies the conditions $|\mathrm{App}^*| \geq (|\mathrm{Opt}^*|/f)$ and $|E - \mathrm{App}^*| \leq g|E - \mathrm{Opt}^*|$.

By Definition 3, a (f, g)-approximation for the channel scheduling problem is both f-IS-approximation and g-VC-approximation for it. Many existing papers used the f-IS-approximation to measure the accuracy.

Require: constant $\epsilon \in (0,1)$ and graph $G(V,E)$.
Ensure: App_i
(1) Let $\text{App}_1 = \text{App}_2 = \varnothing$;
(2) Run greedy-single-channel (.) to find an $(O(1), O(1))$-approximation App_1;
(3) Let $c = W(\text{Opt})/W(E)$;
(4) If $(1 - \tau(G)(1-c)) \geq 1 - \epsilon$;
(5) Then, run PTAS (.) to find an $(1-\epsilon)$-IS-approximation App_2;
(6) ReturnApp$_i$ with $W(\text{App}_i^*) = \max(W(\text{App}_1^*), W(\text{App}_2^*))$;

Algorithm 3: Optimized algorithm in single-channel $M(G,\epsilon)$.

Definition 5. Assume that A is a set of channel assignments for G.

For each edge $e = (u,v)$, define $R(e) = \max(r(u), r(v))$, where $r(u)$ is the distance of interference from u.

Let $d_A^*(e) = \max_K |\{e' = (s,t)$: $(e', K) \in A$; there is an interference between e and $e'\}|$.

Define $d_A^*(G) = \max_{e \in G} d_A^*(e)$.

Lemma 4. *Assume that all nodes in the demand graph $G = (V,E)$ are points on a plane. Then, $d_A^*(G) \leq 98$ for the Euclidean distance for the interference.*

Proof. Let $e = (u,v)$ be an edge in the graph G. We only consider a fixed channel K. Let $C(p,r)$ represent a circle with center at p and radius r. Define $I(u)$ to be the set of edges $e' = (u', v') \in A$ assigned with channel K such that u has interference with u' or v' (in other words, $\text{dist}(u,u') \leq \min(r(u), r(u'))$ or $\text{dist}(u,v') \leq \min(r(u), r(v'))$. Define $J_{\leq}(u) = \{x$: x is a node in $e' \in I(u)$ with $r(u) \leq r(x)\}$. Define $J_{>}(u) = \{x$: x is a node in $e' \in I(u)$ with $r(u) > r(x)\}$.

Assume that $d_A^*(G)$ is large. Consider the case that node u has interference with other edges. For the case of v, we have similar conclusions.

Case 1. $J_{\leq}(u) > 24$. Let $h = 12$. Using u as the center, evenly partition the plane into the h fan areas with angle $(2\pi/h)$ each. By the pigeon hole principle, there is a fan area F_i that has at least $(d_A^*(G)/4h) \geq 2$. In this case, the node x_L with the largest radius interferes all other nodes of F_i. This is a contradiction since all nodes F_i are from the solution A and have no interference each other. $J_{\leq}(u) \leq 24$.

Case 2. $J_{>}(u) > 25$. We have more than 25 nodes y within the distance at most $2r(u)$ to u, and $r(y) \geq r(u)$. The number of circles of radius at least $r(u)/2$ without overlap inside a big circle (with center at u) of radius $2r(u) + r(u)/2$ is at most $((\pi \times 2.5^2)/(\pi \times 0.5^2)) \leq 25$. This gives a contradiction. Thus, we have $J_{>}(u) \leq 25$.

Similarly, we also have $J_{\leq}(v) \leq 24$ and $J_{>}(v) \leq 25$. Therefore, $d_A^* \leq J_{\leq}(u) + J_{>}(u) + J_{\leq}(v) + J_{>}(v) \leq 24 + 25 + 24 + 25 = 98$.

3.1. Multiple Channels with Variate Interference Radii. In this section, we present a greedy $O(1)$-approximation algorithm for the unweighted multi-channel scheduling problem with variate interference radii, which is shown in Algorithm 4.

We note that when there is a set F of edges that have been assigned channels, it is straightforward to check if a new edge e will be interfered at certain channel K. This can be done by checking $ED(e, e_i) \leq t$ for all $e_i \in F_K$, where F_K is the subset of edges in F in channel K.

Theorem 8. *The Algorithm 4 greedy (.) is a $g(\alpha)$-IS-approximation algorithm for the multi-radio multi-channel scheduling problem and has the computational complexity $O(|E|^2)$, where $g(\alpha) = (1 + (d_A^*(G)/\alpha) + (2\alpha/(1-\alpha)))$ and $\alpha = \sqrt{d_A^*(G)}/(\sqrt{2} + \sqrt{d_A^*(G)})$.*

Proof. Assume that Opt is an optimal solution for the channel scheduling problem. Let App be an approximate solution derived by greedy (.).

For each edge g in Opt^*, assign an edge e, denoted by $H(g)$, in App^* such that $R(e)$ is the least among all edges with interference with g.

Claim 3. For each edge g in Opt, $R(g) \geq R(H(g))$.

Proof. If g is in App^*, we consider an edge has interference with itself. Thus, it is trivial. By the definition of $H(g)$, $R(H(g))$ is the least among all edges in Opt^* with interference with g.

Assume that g' is the first edge in App^* and has the same channel with $g \in \text{Opt}^*$ and has interference with g. Before selecting g' for assigning a channel, there is no interference between g and other edges in App. Therefore, $R(g') \leq R(g)$ (otherwise, g' should not be selected for channel assignment). Since $H(g)$ has the least $R(H(g))$ among all edges with interference with g in App, we have $R(H(g)) \leq R(g') \leq R(g)$.

Let d be the total number of channels. For each edge $e = (u,v) \in (\text{Opt}^* - \text{App}^*)$, we consider two cases.

Case 1. There are at least αd channels available for the edge e when e is processed. Each of such an edge is called type A.

In this case, since e is not assigned for a channel, e must have interference with at least αd edges already assigned channels. For each edge e' with interference with e, let

Require: a weighted graph G, and a distance parameter t for the largest distance of interference.
Ensure: L'
(1) Sort all edges by the increasing order of their sum of radii and put those edges in a list L.
(2) Let $L' = \varnothing$;
(3) Repeat;
(4) Select an edge $e = (u, v)$ with the least $R(e)$ from L;
(5) If there is a channel K for e not being interfered by L'
(6) Then, assign the channel K to e and put (e, K) into L';
(7) Remove e from L;
(8) Until L is empty;
(9) Return L';

Algorithm 4: Greedy-multi-channel with variate interference radii (G, t).

$b(e, e') = 1/\alpha d$, and let $b(e, e') = 0$ otherwise. For each e of type A, we have

$$\sum b(e, e') \geq 1. \tag{15}$$

Let $b_1 = d_A^*(G)/\alpha$. For each edge in $e' \in \text{App}^*$, let $h_1(e') = \sum_{e \text{ is of type } A} b(e, e')$. We have

$$h_1(e) = \sum_{e \text{ is of type } A} b(e, e') \leq d.d_A^*(G) \cdot \frac{1}{\alpha d} = \frac{d_A^*(G)}{\alpha}. \tag{16}$$

Case 2. There are less than αd channels available for the edge e when e is processed. Each of such an edge is called type B.

For at least one of u and v, say u, there are at least $(1-\alpha)d/2$ channels that are already assigned. For each edge e' with an end point in u already assigned channels, let $b(e, e') = 1/(1-\alpha)d/2 = 2/(1-\alpha)d$. For each edge in $e \in \text{App}^*$, let $h_2(e') = \sum_{e \text{ is of type } B} b(e, e')$. For each e of type B, we have

$$\sum b(e, e') \geq 1. \tag{17}$$

Let $b_2 = 2\alpha/(1-\alpha)$.

$$h_2(e') = \sum_{e \text{ is of type } B} b(e, e') \leq (\alpha d) \cdot \frac{2}{(1-\alpha)d} = \frac{2}{(1-\alpha)} = b_2. \tag{18}$$

We have

$$\begin{aligned} \left|\{e\colon e \in \text{Opt}^* - \text{App}^*\}\right| &\leq \sum_{e_1 \in \text{Opt}^* - \text{App}^*, e_2 \in \text{App}^*} b(e_1, e_2) \\ &\leq (b_1 + b_2)\left|\text{App}^*\right|. \end{aligned} \tag{19}$$

Thus, we have

$$\begin{aligned} \left|\text{Opt}^*\right| &= \left|\text{Opt}^* \cap \text{App}^*\right| + \left|\text{Opt}^* - \text{App}^*\right| \\ &\leq \left|\text{App}^*\right| + \left|\text{Opt}^* - \text{App}^*\right| \\ &\leq \left|\text{App}^*\right| + (b_1 + b_2)\left|\text{App}^*\right| \\ &\leq (1 + b_1 + b_2)\left|\text{App}^*\right| \\ &\leq \left(1 + \frac{d_A^*(G)}{\alpha} + \frac{2\alpha}{(1-\alpha)}\right)\left|\text{App}^*\right|. \end{aligned} \tag{20}$$

We select the constant α so that $(1 + d_A^*(G)/\alpha + 2\alpha/(1-\alpha))$ to be minimal. Let $g(\alpha) = (1 + d_A^*(G)/\alpha + 2\alpha/\ (1-\alpha)) = (-1 + d_A^*(G)/\alpha + 2/(1-\alpha))$. Let $g(\alpha)' = 2/(1-\alpha)^2 - d_A^*(G)/\alpha^2 = 0$. We have $\alpha = \sqrt{d_A^*(G)}/(\sqrt{2} + \sqrt{d_A^*(G)})$. Thus, the ratio of approximation is $g(\alpha)$.

3.2. Single Channel with Variate Interference Radii. In this section, we present a greedy $(O(1), O(1))$ approximation algorithm for the unweighted single channel with variate interference radii scheduling problem; the pseudocode is shown in Algorithm 5 as follows.

We note that when there is a set F of edges that have been assigned channels, it is straightforward to check if a new edge e will be interfered at certain channel K. This can be done by checking if e has interference with any $e_i \in F_K$, where F_K is the subset of edges in F in channel K.

Theorem 9. *The Algorithm 5 greedy (.) is an $(d_A^*(G), d_A^*(G))$-approximation algorithm for the multi-radio multi-channel scheduling problem and has the computational complexity $O(|E|^2)$.*

Proof. Assume that Opt is an optimal solution for the channel scheduling problem. Let App be an approximate solution derived by greedy (.).

For each edge g in $\text{Opt}^*\infty$, assign a edge e, denoted by $H(g)$, in App^* such that $R(e)$ is the least among all edges with interference with g.

Consider an edge $e = (u, v)$ selected in App^*. Let $A(e) = e_1, \ldots, e_m$ be the list of edges with $H(e_i) = e$.

Claim 4. For each edge g in Opt, $R(g) \geq R(H(g))$.

Proof. If g is in App^*, we consider an edge has interference with itself. Thus, it is trivial. By the definition of $H(g)$, $R(H(g))$ is the least among all edges in Opt^* with interference with g.

Assume g' is the first edge in App^* and has the same channel with $g \in \text{Opt}^*$ and has interference with g. Before selecting g' for assigning a channel, there is no interference between g and other edges in App. Therefore, $R(g') \leq R(g)$ (otherwise, g' should not be selected for channel assignment). Since $H(g)$ has the least $R(H(g))$ among all edges

Input: a weighted graph G, and a distance parameter t for the largest distance of interference.
Output: L'
(1) Sort all edges by the increasing order of their sum of radii and put those edges in a list L.
(2) Let $L' = \varnothing$;
(3) Repeat;
(4) Select an edge $e = (u, v)$ with the least sum of radii from L;
(5) If there is a channel K for e not being interfered by L'
(6) Then, assign the channel K to e and put (e, K) into L';
(7) Remove e from L;
(8) Until L is empty;
(9) Return L';

Algorithm 5: Greedy-signal-channel with variate interference radii (G, t).

with interference with g in App, we have $R(H(g)) \leq R(g') \leq R(g)$.

Assume that App contains channel assignments for edges $e_1, \ldots, e_m$. Partition Opt* into $A(e_1), \ldots, A(e_m)$. By Claim 1, we have that

$$d_A^*(G) \geq d_A^*(e_i) \geq |A(e_i)|. \qquad (21)$$

Therefore, we have $d_A^*(G)|\text{App}^*| \geq |\text{Opt}^*|$. Thus, App is a $d_A^*(G)$-IS approximation for G.

On the other hand, for each $e_i \in (E - \text{Opt}^*) \cap \text{App}^*$, we always have $d_A^*(e_i) \geq |A(e_i)|$ (by Claim 1). For each edge e' in $(E - \text{App}^*) \cap \text{Opt}^*$, there is an edge $e \in \text{App}^*$ such that e and e' have the interference at the same channel (otherwise, e' would be assigned some channel by Greed (.)). Furthermore, $e' \notin \text{Opt}^*$ since it has interference with e. Therefore, there is an edge $e^* \in (E - \text{Opt}^*) \cap \text{App}^*$ such that $e' \in A(e^*)$. By inequality (1), we have

$$|(E - \text{App}^*) \cap \text{Opt}^*| \leq d_A^*(G)|(E - \text{Opt}^*) \cap \text{App}^*|. \qquad (22)$$

We have the inequalities

$$\begin{aligned} W(E - \text{App}^*) &= W((E - \text{App}^*) \cap (E - \text{Opt}^*)) \\ &\quad + W((E - \text{App}^*) \cap (\text{Opt}^*)) \\ &\leq W((E - \text{Opt}^*) \cap (E - \text{App}^*)) \\ &\quad + \tau(G) W((E - \text{Opt}^*) \cap \text{App}^*)\,(by\,(2)) \\ &\leq \tau(G) W((E - \text{Opt}^*) \cap (E - \text{App}^*)) \\ &\quad + \tau W((E - \text{Opt}^*) \cap \text{App}^*) \\ &= \tau(G) W(E - \text{Opt}^*). \end{aligned} \qquad (23)$$

This gives $|E - \text{App}^*| \leq d_A^*(G)|E - \text{Opt}^*|$. Therefore, greedy (.) gives a $(d_A^*(G), d_A^*(G))$-approximation.

4. Experimental Results

Greedy algorithm is easy to implement and fast to output the result. We did some simulation for the multi-channel assignment problems for the greedy algorithm. The parameter T represents a distance, which is either Euclidean distance or hop distance. In a tuple (a, b), the first item a is the total weight of an optimal solution ($W(\text{Opt}^*)$), and the second item b is the total weight of the approximation solution ($W(\text{App}^*)$).

We also evaluate the effectiveness of the proposed greedy algorithm; the algorithm performance is compared with the baseline random assignment algorithm, the greedy coloring algorithm which is proposed in [32], and the conventional graph coloring algorithm (GC) in [33]. These three methods are briefly described as follows:

Random: we allocate channels randomly for each node.

Greedy coloring: we allocate the best channel to each node greedily and then transfer it to robust graph coloring problem which aims at minimizing the interference of system.

GC: we use an interference negligible distance to find which pairs of users can share channels with the cellular users.

4.1. Multichannel with Hop Distance. Our experimental results for the hop distance show that the results from simulation are much better than the theoretical analysis for the greedy algorithm. In this experiment, we randomly generate a network with 30 nodes and 50 edges. For each edge, we generate a random number as a weight. Replacing with the optimal solution of total weight, we get the following simulation.

$T = 1$: (560, 438), (575, 428), (606, 486), (598, 460), (576, 457), (590, 437), (545, 431), (501, 394), (589, 424), (556, 411).

$T = 2$: (570, 443), (586, 421), (607, 456), (605, 446), (582, 453), (558, 413), (579, 443), (607, 464), (666, 546), (538, 426).

$T = 3$: (525, 407), (580, 441), (569, 423), (566, 434), (558, 409), (537, 370), (486, 389), (658, 485), (486, 389), (485, 347).

The performance of the algorithm is shown in Figure 1. We note that the performance of the greedy algorithm goes down as the threshold of the distance parameter T goes up. We can also observe that the performance of our greedy algorithm is superior to the greedy coloring, GC, and random algorithms.

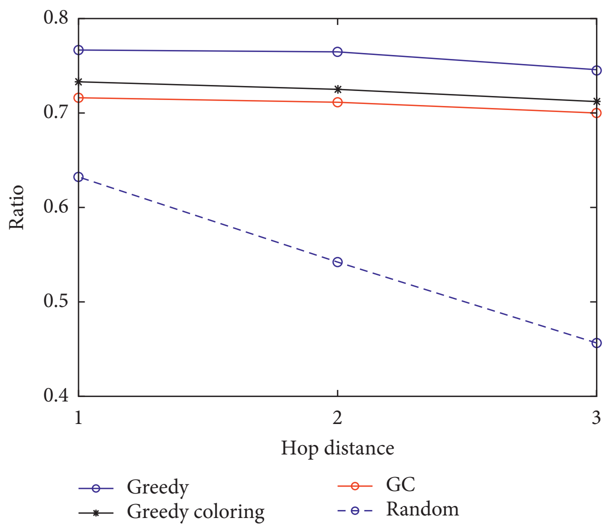

Figure 1: Multi-channel with hop distance.

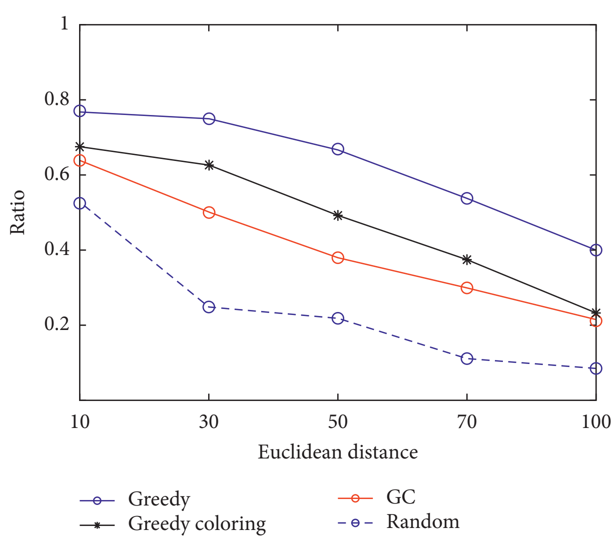

Figure 2: Multichannel with Euclidean distance.

4.2. Multichannel with Euclidean Distance. Our experimental results for the Euclidean distance show that the results from simulation are much better than the theoretical analysis for the greedy algorithm. In this experiment, we randomly generate a network with 30 nodes and 50 edges in the space of 500×500. For each edge, we generate a random number as a weight. Replaced with the optimal solution of total weight, the following simulation has been achieved.

$T = 10$: (411, 300), (467, 372), (443, 336), (463, 379), (475, 377), (407, 317), (471, 385), (416, 339), (494, 330), (456, 354).

$T = 30$: (421, 321), (445, 349), (466, 356), (436, 333), (436, 325), (425, 302), (418, 341), (418, 355), (486, 375), (454, 359).

$T = 50$: (467, 352), (499, 315), (459, 301), (488, 309), (502, 399), (484, 313), (403, 314), (481, 348), (466, 325), (390, 282).

$T = 70$: (426, 260), (476, 253), (415, 232), (432, 259), (459, 302), (449, 284), (454, 241), (488, 296), (507, 319), (445, 210)..

$T = 100$: (444, 190), (459, 197), (531, 195), (435, 173), (478, 228), (441, 216), (442, 208), (488, 257), (458, 265), (490, 198).

The performance of the algorithm is shown in Figure 2. From Figure 2, we can also know that the performance of the greedy algorithm degrades as the threshold of the distance parameter T increases from 10 to 100. The performance of the proposed greedy algorithm is better than the other three comparison algorithms.

5. Conclusion

We develop a new measure, which controls the ratios in two sides, for the approximation algorithms for channel scheduling problem. We first study the weighted multi-channel and single-channel assignment with fixed interference range problems and propose corresponding algorithms for these two problems; furthermore, we improve the existing algorithms for the multi-channel scheduling and propose a faster and more accurate $(1-\epsilon, O(1))$-approximation algorithm for signal channel scheduling problem. Next, we study the unweighted channel assignment with variate interference radius problem and present a multi-channel and a single-channel allocation algorithm, respectively, for this situation. All of the algorithms are given detail theoretical proofs and got constant approximation guarantees. Finally, we verified our algorithms with simulations.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

An earlier version of this paper was presented at the "Wireless Algorithms, Systems, and Applications 6th International Conference WASA 2011 Chengdu China August 11-13 2011." We have added Section 2.4 "Improving the Existing Algorithm for the Multi-Channel Scheduling," Section 2.5 "Merging Two Algorithms in Single-Channel Scheduling," Section 3 "Unweighted Channel Assignment," and Section 4 "Experimental Results."

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Algorithm for the Multi-Channel Scheduling," Section 2.5 "Merging Two Algorithms in Single-Channel Scheduling," Section 3 "Unweighted Channel Assignment," and Section 4 "Experimental Results."

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