A queueing approach to production-inventory planning for supply chain with uncertain demands: Case study of PAKSHOO Chemicals Company

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Technical paper

A queueing approach to production-inventory planning for supply chain with uncertain demands: Case study of PAKSHOO Chemicals Company

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ABSTRACT

In some industries such as the consumable product industry because of small differences between products made by various companies, customer loyalty is directly related to the availability of products required at that time. In other words, in such industries demand cannot be backlogged but can be totally or partly lost. So companies of this group use make-to-stock (MTS) production policy. Therefore, in these supply chains, final product warehouses play a very important role, which will be highlighted by considering the demand uncertainty as it happens in real world, especially in the consumable product industries in which demand easily varies according to the customer's taste variation, behavioral habits, environmental changes, etc. In this article, an (s, Q) inventory system with lost sales and two types of customers, ordinary and precedence customers and exponentially distributed lead times are analyzed. Each group of demands arrives according to the two independent Poisson processes with different rates. A computationally efficient algorithm for determining the optimal values for safety stock as reorder level and reorder quantity for a multi-item capacitated warehouse is developed. The algorithm also suggests the optimal warehouse capacity. A Multi-item Capacitated Lot-sizing problem with Safety stock and Setup times (MCLSS) production planning model is then developed to determine the optimal production quantities in each period using optimal values computed by the first algorithm as inputs. Finally, the proposed production-inventory-queue model is implemented in a case study in PAKSHOO Chemicals Company and results are obtained and analyzed. Moreover, solving this problem can help to strategic decision making about supply chain decoupling point.

1. Introduction

Consumable products contain groups of standard goods with small volume and value per unit (e.g. foods, juices, detergents, office accessories, etc.). An ordinary consumable product manufacturer makes several products which are all the same in technological aspects. Consumers expect to find their chosen brands on the supermarket shelf anytime they go for shopping and if it is not available they probably will change their minds and buy another brand. It is because of small differences between the consumable products of different brands. In other words, in such industries demand cannot be backlogged but can be totally or partly lost. Therefore, companies of this group use MTS production policy. As the life cycle of such standard products will usually take several years, an efficient database can be prepared for forecasting future demands but about standard consumable products, companies should focus on service level and price, as the market is absolutely competitive.

In consumable product industry, production system is MTS or push policy but in chemical industries according to the competitive nature of market and expiry date of products demand forecasting and pull policies are more and more considerable. Also because of the expensive machinery and factory equipments, the optimal usage of production capacity in these industries is the main managerial worries.

As in these days competition has been extended from companies to supply chains, in this article a production-inventory planning model has been suggested with an objective function of minimizing the supply chain's total cost. Two main factors for replenishing a warehouse are delivery and cost, which should be checked out while ordering the lots. Logistic costs are the biggest

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part of supply chain costs and coordination of supply, transportation, production and inventory control as main logistic processes can cause a significant saving in total costs, on the other hand, will play an important role in achieving higher customer service levels.

The studied supply chain is a real supply chain in chemical detergent industry and we wish to plan its processes under real world's uncertain demand situations which will lead to make decisions of the inventory level, reorder quantity and production amount in every period. The proposed model is a dynamic model made of a combination of two separate models, one is an inventory control system and the other is a production planning model, therefore we had to review different parts of the literature.

An outline of the remainder of this paper is as follows. In Section 2, a literature review of related studies is presented. Section 3, describes the mathematical formulation of model. In Section 4, we state the proposed algorithm of finding optimal solutions for the problem. Finally, the computational results of PAKSHOO Chemicals Company are given in Section 5 to show the effectiveness of the developed method.

2. Literature review

2.1. Supply chain planning under uncertainty

Managing uncertainty is a main challenge within the supply chain management. The complex nature and dynamics of the relationships amongst the different actors imply an important grade of uncertainty in the planning decisions [1]. According to [2], uncertainty is defined as the difference between the amount of information required to execute a task and the information that is actually available. In SC planning decision processes, uncertainty is a key factor that can influence the effectiveness of the configuration and coordination of supply chains [3] and tends to propagate up and down along the SC, appreciably affecting its performance [1].

By the majority of studies the source of uncertainty is classified into three groups: demand, process/manufacturing and supply (e.g. [4,5]). Uncertainty in supply is caused by the variability brought about by how the supplier operates because of the faults or delays in the supplier's deliveries. Uncertainty in the process is a result of the poorly reliable production process due to, for example, machine holdups. Finally, demand uncertainty, according to Davis [6], is the most important of the three and is presented as a volatility demand or as inexact forecasting demands. In this context, it is important to highlight the works by Dejonckheere et al. [7], Disney et al. [8] and Gaalman and Disney [9] in order to measure and avoid the bullwhip effect in supply chains. The analytical models are robust optimization, stochastic programming, games theory, linear programming and parametric programming [10]. McDonald and Karimi [11] devised a mixed integer linear programming model (multi-site, multi-product, multi-period) for a mid-term SC production planning. The model they developed is of deterministic nature and adopts safety stocks to face demand uncertainties. Gupta and Maranas [12] devised a stochastic two-stage programming model based on the mixed integer linear programming model proposed by McDonald and Karimi [11] for the tactical planning of a multi-site SC with demand uncertainty. Subsequently, Gupta et al. [13] incorporated constraints to measure customer satisfaction. Gupta and Maranas [14] generalized their approach to consider the tactical planning of a multi-site, multi-product and multi-period SC with demand uncertainty. They compared the objective of finding the equilibrium between the level of customer service and the costs associated with the planning. Yu and Li [15] presented a robust optimization model based on Mulvey et al. [16] and on Mulvey and Ruszczynski [17]. It included the classic programming techniques per objective and considered scenarios to solve stochastic logistic problems. Lario et al. [18] described the generation and scenario analysis process as a tool for SC management with uncertainty in the following settings: manufacturing, assembly, distribution and service in the car manufacturing and assembly sector, within a European project framework. Lucas et al. [19] addressed the problem of planning capacities within the SC through stochastic programming with scenarios to deal with demand uncertainty. The authors applied the Lagrangian relaxation to obtain feasible integer solutions.

Nagar and Jain [20] presented a multi-stage planning model consists of supply, production and distribution under uncertain demand. They have developed a multi-period model for new products so that the demand is uncertain. The proposed model determines optimum order quantity, production quantity, distribution system and the needed outsourcings in the case of demand shortages.

Queueing theory has been extensively adopted to analyze a variety of performance analysis problems of manufacturing systems (see [21,22]). Queueing models, in turn, can be categorized as descriptive (provide values for performance measures of interest for a given configuration) or prescriptive (provide guidelines for running the system most effectively). Govil and Fu [21] conducted a comprehensive survey on queueing models for manufacturing applications. To our knowledge no supply chain planning model has been designed using queueing theory approaches and as mentioned before analytical models are usually based on robust optimization, stochastic programming, games theory, linear programming and parametric programming. In all these methods although inventory quantities are considered, warehouses have inferior roles and stochastic parameters affect all parts of planning such as supply, inventory, production, transportation, etc. but in this article using continuous review inventory system we will focus on warehouses to overcome demand uncertainty. We will use queueing theory techniques in this case.

2.2. Inventory control models with queueing theory approaches

In recent years customer service has attracted more and more attention and product availability, especially in consumable product industry, has been the most important aspect of service. This can be controlled by inventory systems; a way to present a higher service level is to classify the customers. In fact all customers of a single product do not require the same service level or do not have the same stock out fine. This inventory controlling approach was of no interest in inventory management systems and does not appear in reviewing articles [23,24], nor in basic references of supply chain and inventory control (e.g. [25,26]). The first study of demand classifications in inventory control models was done by Veinott [27]. He presented a periodic review inventory system with several demand groups and no lead time. After him a few more research were carried out by considering various customer classes in periodic inventory systems. Kleijn and Dekker [28] provided a review of inventory systems with different customer classes. By the improvement of inventory systems to continuous review policies, analyzing the usage of different demand classes approach in new systems opened new doors to researchers which were a result of information technology advances. The first work on this new approach was by Nahmias and Demmy [29]. They modeled an (s, Q) inventory system with two customer groups. Each group's demand comes under a Poisson process, unsatisfied demand is postponed. Ha [30] studied a lot-for-lot model with previous assumptions but they had considered a fixed lead time while in this model an exponentially distributed lead time was modeled; he also showed that this problem can be formulated as a queueing model. The results of Nahmias and Demmy [29] were extended by Moon and Kang [31] to a compound Poisson demand while other assumptions

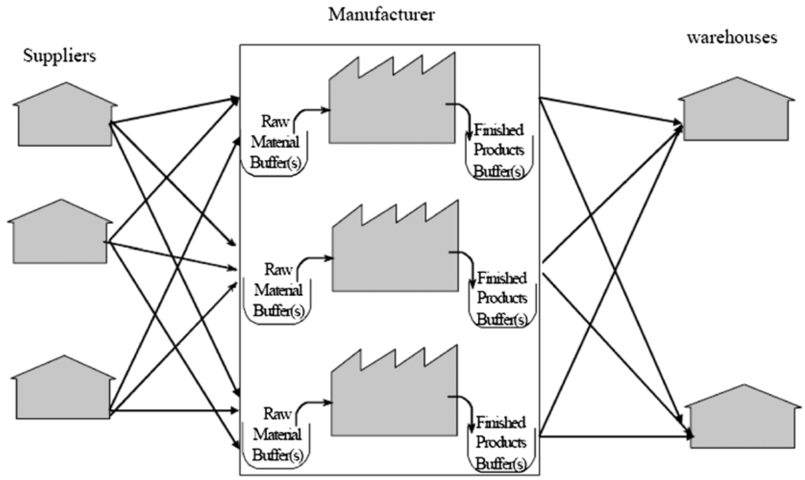

Fig. 1. Supply chain construction.

were constant. Dekker et al. [32] also presented a lot-for-lot model with the same characteristics as [29] and approximated the service level. Deshpande et al. [33] studied critical level rationing policy for demand classes which are classified by required service levels. Four control approaches have been analyzed in their study: priority fulfilling of backlogs, threshold fulfilling of backlogs, hybrid policy (combination of using threshold fulfilling parameters and the priority policy for backlog fulfilling) and optimal rationing. By their examples it is proved that, the hybrid policy works better than the threshold fulfilling and priority fulfilling policies. For small setup costs and different penalty costs for demand classes the hybrid policy cannot give significant results but for high setup costs both hybrid and the optimal rationing policy perform same as each other.

Against all these assumptions while considering lost sales, we can mention Ha [30] who modeled a single product MTS production policy with different demands coming under the Poisson process and exponentially distributed lead times. He formulated the model as an $M/M/1/S$ queue. Dekker et al. [32] as explained before presented the same model but as an $M/M/S/S$ queue. Melchiors et al. [34] provided an (s, Q) inventory system with two customer groups considering lost sales and deterministic lead times. They expressed the total expected cost formula and suggested an optimization algorithm which used some convexity characteristics to calculate optimum reorder level. Isotupa [35] developed a model with the same assumptions but exponentially distributed lead times which makes the expected cost function pseudo-convex in both parameters s and Q. In fact it is a single-item continuous review inventory system with two independent Poisson demands of different priorities to minimize replenishing, lost sales and inventory costs. She also described a computationally efficient algorithm to determine s, Q, and the optimal cost. In current article we have extended her model to multi-item supply chain with capacitated warehouse.

3. Statement of the problem

The supply chain studied is made of several suppliers, several manufacturers and several distributors which are shown in Fig. 1. The main objective is to find optimum production lot and inventory level so that the total cost of supply chain is minimized. Solving this problem can help to strategic decision making about SC decoupling point. In the studied supply chain each product can just be produced in their respective plant because of the technological availability and hygienic standards that need to be followed, so the assignment is not in our scope. Finished products will be stored in warehouses and demands fulfilled from there. Although market demands are uncertain it will be calculated and announced by distributors as production forecasts which are the Poisson processes and certain demand will be known at the beginning of each production period. Planning horizon contains $\boldsymbol{N}$ periods. Unsatisfied demand in each period is lost. In this supply chain two demand classes are considered, one is the domestic market and the other is the international market (export) which is called as higher priority market by managerial decisions. Each of these demands comes as a Poisson process with different independent rates and the lead time for the production is exponentially distributed with parameter μ (>0).

The problem will be followed in two phases: first phase is the distributor's inventory management by queueing techniques which will lead to finding safety stock and reorder quantity of each product by minimizing total cost which is structured of inventory cost, production cost and lost sales loss. The production variable cost for each product is independent of others because of BOM differences. Using different formulation causes constant production cost as well. In the second phase, a production planning problem is constructed to calculate the production quantity of each period. The scope of this study is only the finished product warehouses and manufacturing plants, not the product distribution and raw material supply.

Warehouse capacity is restricted to number of products and is not partitioned according to customers nor product group so that the whole warehouse capacity can be assigned to one kind of product. Production capacity is also restricted by available time, which is allocated to different products considering setup times and variable production times.

4. Problem formulation

This section is dedicated to mathematical formulation of model. The model is developed in two phases: inventory-queue model (Section 4.1) and production planning model (Section 4.2).

4.1. Inventory-queue model

Two demand classes are considered, one is the domestic market (distribution companies) and the other is the international market (export department) which has priority over the first one. Each of these demands comes as a Poisson process with different independent rates. Priority demand (export department) of product p arrives according to the Poisson Process with rate for $\lambda_{1p}(>0)$. Domestic demand (distribution companies) of product p arrives according to the Poisson Process with rate for $\lambda_{2p}(>0)$. As soon as the inventory level of product p reaches the safety level s_p, an order for Q_p units is placed. Therefore the maximum inventory level is $Q_p + s_p$. The condition $Q_p > s_p$ ensures that there are no perpetual shortages. If $Q_p \leq s_p$ and the inventory level reaches zero then the system will be in shortage forever. The production lead time follows an exponentially distributed function with parameter μ_p (>0). As orders are usually placed when stock levels get low, domestic demands which arrive when an order is pending are not served and hence these demands are lost. Also, when the inventory level is zero, demands due to both types of customers are assumed to be lost.

Let $I_p(t)$ denote the on-hand inventory level at time t. Since $Q_p > s_p$, at any given point of time there is at most one order pending, and as such from our assumptions it is clear that the inventory level process $\{I_p(t);\ t \geq 0\}$ with state space $E_p = \{0, 1, 2, \ldots, Q_p + s_p\}$ is a Markov process. Let

$$P_p(i, j, t) = Pr[I_p(t) = j | I_p(0) = i] \quad i, j \in E_p \tag{1}$$

$$P_p(j) = \lim_{t \longrightarrow \infty} P_p(i, j, t). \tag{2}$$

By the Markov process properties we have the following balance equations:

$$(\lambda_{1p} + \lambda_{2p}) P_p(Q_p + s_p) = \mu_p P_p(s_p) \tag{3}$$

$$(\lambda_{1p} + \lambda_{2p}) P_p(j) = (\lambda_{1p} + \lambda_{2p}) P_p(j + 1) + \mu_p P_p(j - Q_p) \quad Q_p \leq j \leq Q_p + s_p - 1 \tag{4}$$

$$(\lambda_{1p} + \lambda_{2p}) P_p(j) = (\lambda_{1p} + \lambda_{2p}) P_p(j + 1) \quad s_p + 1 \leq j \leq Q_p - 1 \tag{5}$$

$$(\lambda_{1p} + \mu_p) P_p(s_p) = (\lambda_{1p} + \lambda_{2p}) P_p(s_p + 1) \tag{6}$$

$$(\lambda_{1p} + \mu_p) P_p(j) = \lambda_{1p} P_p(j + 1) \quad 1 \leq j \leq s_p - 1 \tag{7}$$

$$\mu_p P_p(0) = \lambda_{1p} P_p(1). \tag{8}$$

Solving balance equations (3)–(8) results:

$$P_p(j) = \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{j-1} \frac{\mu_p}{\lambda_{1p}} P_p(0) \quad 1 \leq j \leq s_p \tag{9}$$

$$P_p(j) = \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} \frac{\mu_p}{\lambda_{1p} + \lambda_{2p}} P_p(0) \quad s_p + 1 \leq j \leq Q_p \tag{10}$$

$$P_p(j) = \left[\left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} - \left(1 + \frac{\mu_p}{\lambda_{2p}}\right)^{j - Q_p - 1}\right] \frac{\mu_p}{\lambda_{1p} + \lambda_{2p}} P_p(0) \quad Q_p + 1 \leq j \leq Q_p + s_p. \tag{11}$$

As $\sum_{j=0}^{Q_p + s_p} P_p(j) = 1$ from Eqs. (9)–(11) we have

$$P_p(0) = \frac{\lambda_{1p} + \lambda_{2p}}{\lambda_{1p} + (\lambda_{2p} + Q_p \mu_p)\left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p}}. \tag{12}$$

Now we calculate the inventory level of each product $\bar{I}_p$ as below:

$$\begin{aligned}\bar{I}_p = {} & \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} \left[\frac{s_p \lambda_{2p}}{\lambda_{1p} + \lambda_{2p}} + \frac{\mu_p Q_p (Q_p + 2 s_p + 1)}{2(\lambda_{1p} + \lambda_{2p})} \right. \\ & \left. - \frac{Q_p \lambda_{1p}}{\lambda_{1p} + \lambda_{2p}} - \frac{\lambda_{1p} \lambda_{2p}}{\mu_p (\lambda_{1p} + \lambda_{2p})}\right] P_p(0) \\ & + \left(Q_p + \frac{\lambda_{2p}}{\mu_p}\right)\left(\frac{\lambda_{1p}}{\lambda_{1p} + \lambda_{2p}}\right) P_p(0). \end{aligned} \tag{13}$$

The mean reorder rate R_p, and the mean shortage rates for the export and domestic customers per product Γ_{1p} and Γ_{2p}, are given by

$$R_p = (\lambda_{1p} + \lambda_{2p}) P_p(s_p + 1) = \mu_p \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} P_p(0) \tag{14}$$

$$\Gamma_{1p} = \lambda_{1p} P_p(0) \tag{15}$$

$$\Gamma_{2p} = \lambda_{2p} \sum_{j=0}^{s_p} P_p(j) = \lambda_{2p} \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} P_p(0). \tag{16}$$

We will use the notation below as well as the parameters defined above:

K_p: Setup cost of product p.
C_p: Variable production cost per unit of product p.
g_{1p}: Cost per unit shortage of product p for priority demand.
g_{2p}: Cost per unit shortage of product p for domestic demand.
h_p: Inventory holding cost of product p per unit time.

Also $C_p < g_{1p}, g_{2p}$ as by definition g_{1p}, g_{2p} are lost sales losses which mean the decrease of total revenue of supply chain. In fact this problem can be used when a fine should be paid for shortage.

The expected cost structure for product p is

$$C_p(S_p, Q_p) = h_p \bar{I}_p + (K_p + C_p Q_p) R_p + g_{1p} \Gamma_{1p} + g_{2p} \Gamma_{2p}. \tag{17}$$

Therefore the total expected cost for all products is

$$C(s, Q) = \sum_p C_p(s_p, Q_p). \tag{18}$$

As we know if $f(x_i) \geq 0$ then

$$Minimize \left(\sum_i f(X_i)\right) = \sum_i Minimize\, f(X_i). \tag{19}$$

Considering the product's independence we can solve the problem for each product independently. By substituting $\Gamma_{2p}, \Gamma_{1p}, R_p, \bar{I}_p$ in $C_p(S_p, Q_p)$, we have

$$\begin{aligned} C_p(s_p, Q_p) = {} & h_p \left(Q_p + \frac{\lambda_{2p}}{\mu_p}\right)\left(\frac{\lambda_{1p}}{\lambda_{1p} + \lambda_{2p}}\right) P_p(0) \\ & + (K_p + C_p Q_p) \mu_p \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} P_p(0) \\ & + h_p \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} \left[\frac{s_p \lambda_{2p}}{\lambda_{1p} + \lambda_{2p}} - \frac{\lambda_{1p} \lambda_{2p}}{\mu_p (\lambda_{1p} + \lambda_{2p})} \right. \\ & \left. - \frac{Q_p \lambda_{1p}}{\lambda_{1p} + \lambda_{2p}} + \frac{\mu_p Q_p (Q_p + 2 s_p + 1)}{2(\lambda_{1p} + \lambda_{2p})}\right] P_p(0) \\ & + g_{2p} \lambda_{2p} \left(1 + \frac{\mu_p}{\lambda_{1p}}\right)^{s_p} P_p(0) + g_{1p} \lambda_{1p} P_p(0). \end{aligned} \tag{20}$$

4.2. Production planning model

A mixed integer MCLSS model is constructed here to plan the rest of the processes in supply chain. The used notation is defined below:

Sets and Indices

t : periods' index $t \in \{1, \ldots, T\}$
p : products' index $p \in \{1, \ldots, N\}$.

Decision variables

x_{pt}: The quantity of product p produced at period t
y_{pt}: $\begin{cases} 1 & \text{if product } p \text{ is produced at period } t \\ 0 & \text{otherwise} \end{cases}$
r_{pt}: The shortage for product p at period t
S^+_{pt}: Overstock of product p at period t
S^-_{pt}: Safety stock deficit of product p at period t.

Parameters

υ_{pt}: Time consumption per unit of product p at period t
f_{pt}: Setup time of product p at period t
B_t: The total available capacity at period t
β_p: Price per unit of product p
D_{pt}: Demand of product p at period t.

In this model as the order quantity and rate at each period is constant – calculated by the previous model – the total demand at each period is $D_{pt} = R_p Q_p$. The inventory level of product p at period t is also defined as

$$S_{pt}^{+} + S_p - S_{pt}^{-}.$$

We assume that planning horizon contains T periods and the total number of products is N The MCLSS model formulation is represented below:

$$\text{Min} \sum_p \sum_t (c_p x_{pt} + k_p y_{pt} + g_{1p} r_{pt} + h_p (S_{pt}^{+} + s_p - S_{pt}^{-}) + g_{2p} S_{pt}^{-}) \tag{21}$$

St :

$$S_{p,t-1}^{+} - S_{p,t-1}^{-} + r_{pt} + x_{pt} = D_{pt} + S_{pt}^{+} - S_{pt}^{-}, \quad \forall p, t \tag{22}$$

$$\sum_p (v_{pt} x_{pt} + f_{pt} y_{pt}) \le B_t, \quad \forall t \tag{23}$$

$$x_{pt} \le M_{pt} y_{pt}, \quad \forall p, t \tag{24}$$

$$r_{pt} \le D_{pt}, \quad \forall p, t \tag{25}$$

$$S_{pt}^{-} \le s_p, \quad \forall p, t \tag{26}$$

$$x_{pt}, r_{pt}, S_{pt}^{+}, S_{pt}^{-} \ge 0, \quad \forall p, t \tag{27}$$

$$y_{pt} \in \{0, 1\}, \quad \forall p, t. \tag{28}$$

The objective function minimizes the total cost of production plan, that is, production costs, inventory costs, shortage costs, safety stock deficit costs and setup costs. Constraints (22) are the inventory flow conservation equations through the planning horizon. Constraints (23) are the capacity constraints; the overall consumption must remain lower than or equal to the available capacity. If we produce a product p at period t, then constraints (24) impose that the quantity produced must not exceed a maximum production level $M_{pt} \cdot M_{pt}$ can be defined as below:

$$Min \left\{ \sum_t R_p Q_p, \frac{B_t - f_{pt}}{v_{pt}} \right\} \tag{29}$$

it is the minimum value between the total demand for product p during periods $[t, T]$ of the horizon and the maximum possible production quantity according to the plant capacity. Constraints (25) and (26) define upper bounds on, respectively, the demand shortage and the safety stock deficit for product p at period t. Constraints (27) and (28) characterize the variable's domains.

5. Solution method

Before going through algorithm steps in order to reduce the computational time required finding the optimal values of Q and s, we mention the following three theorems which are based on pseudo-convexity properties.

Theorem 1. *For a fixed Q, the expected cost rate is pseudo-convex in s (See the proof in* [35]*).*

Theorem 2. *Whenever $Q > [\frac{(g_1-c)\lambda_1(\lambda_1+\lambda_2)}{h\mu}]$, $C(s, Q)$ is an increasing function of s and hence $s^* = 0$ (See the proof in* [35]*).*

Theorem 3. *For a fixed s, the long-run expected cost rate is pseudo-convex in Q (See the proof in* [35]*).*

Finding optimum s, Q algorithm steps are:

Step 1. Find $Q_{\min} = \frac{(g_1-c)\lambda_1(\lambda_1+\lambda_2)}{h\mu}$.

Step 2. Find s_0 and Q_0 so that

$$C(s_0, Q_0) \le C(s, Q) \quad 0 < s < Q,\ s < Q < Q_{\min}.$$

To find optimum s for each $Q < Q_{\min}$ solve the equation below:

$$C(s+1, Q) - C(s, Q) = 0.$$

Step 3. find Q_1 so that

$$C(0, Q_1) \le -C(0, Q), \quad Q_{\min} \le Q,\ Q_1 \le \infty.$$

To find optimum $Q_{\min} \le Q$ that minimizes total costs solve $C(0, Q+1) - C(0, Q) = 0$.

Step 4. if $C(0, Q_1) < C(s_0, Q_0)$ then $Q^* = Q_1$, $s^* = 0$ and end.

Step 5. if $C(0, Q_1) \ge C(s_0, Q_0)$ then $s^* = s_0$, $Q^* = Q_0$.

Step 6. stop.

Considering a capacitated inventory system, after computing s, Q for all products we should check the following condition: (W = *total warehouse capacity*)

$$\sum_p \bar{I}_p \le W. \tag{30}$$

If the condition is not satisfied, to calculate new s, Q we go through the following steps:

Step 1. For each product p determine below values

$$C(s_p^*, Q_p^* - 1) - C(s_p^*, Q_p^*),$$
$$C(s_p^* - 1, Q_p^*) - C(s_p^*, Q_p^*).$$

Step 2. Find the minimum among the results of the previous step, if the minimum was from the first equation then $Q_p^* := Q_p^* - 1$, otherwise $s_p^* := s_p^* - 1$.

Step 3. If by new values $\sum_p \bar{I}_p \le W$ stop, else go to step 1.

Suggested algorithm by Isotupa [35] using MAPLE 9 had the total computational time of 14.14 s to determine the optimal cost for one product and the computational time for the extended algorithm in this article using MATLAB 7.1 and implementing numerical calculation techniques for 27 products in a capacitated warehouse was 35.7 s, the efficient computational time makes this model an applicable algorithm for real world problems.

The proposed model in this article is a dynamic model combined of two separate models. Outputs of suggested algorithm to find s, Q are inputs of the second model which is coded using LINGO 8 optimization software

6. An illustrative case study: PAKSHOO Chemicals Company

PAKSHOO is a chemical manufacturing company which produces many different detergent products. Amidst the varied range of products we have chosen 27 products for our case study. Planning horizon in a year consists of 12 production periods. To avoid complicated calculations and a large number of iterations in computational algorithm and as all products are transported by truck, we changed the demand to truck scale, the capacity of current trucks is 640 boxes and all costs are scaled to one million. Input parameters for PAKSHOO Chemical Company are given in Table 1. Using the mean inventory level formula, the proposed algorithm can suggest the optimal warehouse capacity needed for current supply chain. According to numerical example we need a capacity of 680,000 boxes, and then by receiving the real capacity announced by the user which is 350,000 in this example, safety stock level and reorder quantity will be known. After changing these values to real scale we use them as inputs of MCLSS model in LINGO.

Table 1
Input parameters for PAKSHOO chemical company.

Product	β_p	λ_{1p}	λ_{2p}	v_{pt}	f_{pt}	K_p	C_p	g_{1p}	g_{2p}	h_p
1	76.8	5	11	53	30	3	43.776	89.549	81.408	0.288
2	107.52	41	90	53	30	3	61.286	125.368	113.971	0.288
3	69.12	3	7	53	30	3	39.398	80.594	73.267	0.288
4	69.12	7	16	53	30	3	39.398	80.594	73.267	0.288
5	46.08	5	11	43	30	2.3	26.266	53.729	48.845	0.256
6	107.52	10	23	53	30	3	61.286	125.368	113.971	0.288
7	107.52	10	23	53	30	3	61.286	125.368	113.971	0.288
8	107.52	10	23	53	30	3	61.286	125.368	113.971	0.288
9	76.8	5	11	53	30	3	43.776	89.549	81.408	0.288
10	76.8	5	11	53	30	3	43.776	89.549	81.408	0.288
11	107.52	41	90	53	30	3	61.286	125.368	113.971	0.288
12	107.52	25	56	53	30	3	61.286	125.368	113.971	0.288
13	69.12	8	19	53	30	3	39.398	80.594	73.267	0.288
14	96.768	7	15	53	30	3	55.158	112.831	102.574	0.288
15	23.04	4	8	32	30	1.8	13.133	26.865	24.422	0.224
16	69.12	22	49	53	30	3	39.398	80.594	73.267	0.288
17	46.08	6	13	43	30	2.3	26.266	53.729	48.845	0.256
18	128	8	19	64	30	2	72.96	149.248	135.68	0.384
19	46.08	3	6	43	30	2.3	26.266	53.729	48.845	0.256
20	23.04	14	31	32	30	1.8	13.133	26.865	24.422	0.224
21	46.08	17	38	43	30	2.3	26.266	53.729	48.845	0.256
22	107.52	10	23	53	30	3	61.286	125.368	113.971	0.288
23	107.52	7	13	53	30	3	61.286	125.368	113.971	0.288
24	76.8	8	14	53	30	3	43.776	89.549	81.408	0.288
25	76.8	18	32	53	30	3	43.776	89.549	81.408	0.288
26	69.12	4	9	53	30	3	39.398	80.594	73.267	0.288
27	46.08	3	6	43	30	2.3	26.266	53.729	48.845	0.256

As it is a mixed integer programming model, integer coefficients can deliver more realistic and sustainable outputs.

In the studied supply chain production capacity is measured by the available production time. In current situation according to economical policies, market demands and human resources costs, one of the production lines is working continuously (every 24 h and 7 days a week).

Also the second production line is producing just 2 days a week which capacity is counted as below:

LINE 1. 30 days per month*24 h a day *60 min an hour *60 s a minute=2592,000 s.

LINE 2. weeks a month, 2 days a week, 24 h a day, 60 min an hour, 60 s a minute = 691,200 s.

Total capacity : 691,200 + 2592,000 = 3283,200.

Outputs of inventory-queue model and production planning model are given in Tables 2 and 3, respectively.

Production quantities of each period (month) are outputs of this model. At the end, the total supply chain's profit will be calculated using the following equation.

$$Profit\ Function = \sum_p \sum_t (R_p Q_p - S^-_{pt} - r_{pt})\beta_p - \sum_p \sum_t (c_p x_{pt} + k_p y_{pt} + g_{1p} r_{pt} + h_p S^+_{pt} + g_{2p} S^-_{pt}). \quad (31)$$

Total profit of the studied SC by this method is 428.269.653.590.

Computational results show the inventory capacity restrictions. Therefore we have analyzed the effects of capacity changes on total profit. As the real warehouse space is 10,000 m^2 it is obvious that we cannot analyze changes made by 1 box capacity increase, so to be more applicable we will discuss capacity changed for every 2500 m^2. Table 4 and Fig. 2 show the results.

The trend in Fig. 2 shows the possibility of profit increase by increasing the warehouse capacity, one of the main reasons of having such ascending trend is the high production capacity comparing with inventory capacity. According to Eq. (30) it can be seen that economic warehouse capacity occurs in equal condition.

Table 2
Outputs of inventory-queue model.

Product	s_p	Q_p	c_p	R_p	I_p
1	0.745	17	0.711*1000	0.929	9.549
2	4.548	143	8.1132*1000	0.905	74.938
3	0.409	12	0.4011*1000	0.823	6.789
4	0.983	23	0.92*1000	0.986	12.703
5	0.756	15	0.4289*1000	1.047	8.515
6	1.472	38	2.0453*1000	0.858	20.569
7	1.472	38	2.0453*1000	0.858	20.569
8	1.472	38	2.0453*1000	0.858	20.569
9	0.745	18	0.711*1000	0.878	10.049
10	0.745	18	0.711*1000	0.878	10.049
11	4.548	143	8.1132*1000	0.905	74.938
12	3.037	90	5.0175*1000	0.889	47.545
13	1.107	27	1.0797*1000	0.986	14.78
14	1.081	25	1.2286*1000	0.87	13.809
15	0.591	10	0.163*1000	1.167	5.846
16	2.547	64	2.8356*1000	1.093	34.178
17	0.904	18	0.509*1000	1.037	10.117
18	1.089	30	1.9915*1000	0.891	16.318
19	0.416	10	0.2419*1000	0.886	5.781
20	2.064	32	0.6076*1000	1.363	17.659
21	2.241	46	1.4704*1000	1.173	24.914
22	1.472	38	2.0453*1000	0.858	20.569
23	1.115	24	1.2399*1000	0.825	13.362
24	1.139	22	0.9766*1000	0.988	12.358
25	2.219	47	2.2169*1000	1.05	25.583
26	0.576	14	0.5208*1000	0.916	7.918
27	0.416	10	0.2419*1000	0.886	5.781

In this case we have 680,000 boxes as optimal value for warehouse capacity. We should know that capacity increase to more than 680,000 boxes will have no effect on SC profit and the total profit function might begin to decrease at this point. Increasing the warehouse capacity to different arbitrary values from the practical point of view is not applicable. Moreover, PAKSHOO has some limitations in warehouse space selection and 680,000 boxes as warehouse capacity are not applicable and we ignore it in our sensitivity analysis. Therefore, the two feasible capacities are 700,000 and 750,000 boxes more than 680,000 which the warehouse capacity of 700,000 boxes is optimal.

Table 3
Outputs of production planning model.

Product	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$	$t = 11$	$t = 12$
1	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
2	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4
3	19.8	0.0	9.9	19.8	0.0	19.8	0.0	19.8	0.0	9.9	19.8	0.0
4	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7	22.7
5	15.7	15.7	15.7	15.7	15.7	15.7	15.7	15.7	15.7	15.7	15.7	15.7
6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6
7	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6
8	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6
9	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
10	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8	15.8
11	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4	129.4
12	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0
13	26.6	26.6	26.6	26.6	26.6	26.6	26.6	26.6	26.6	26.6	26.6	26.6
14	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7
15	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7
16	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
17	18.7	18.7	18.7	18.7	18.7	18.7	18.7	18.7	18.7	18.7	18.7	18.7
18	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7
19	17.7	0.0	17.7	0.0	17.7	0.0	17.7	0.0	17.7	0.0	17.7	0.0
20	43.6	43.6	43.6	43.6	43.6	43.6	43.6	43.6	43.6	43.6	43.6	43.6
21	53.9	53.9	53.9	53.9	53.9	53.9	53.9	53.9	53.9	53.9	53.9	53.9
22	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6	32.6
23	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8
24	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7
25	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3
26	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8
27	8.9	17.7	0.0	17.7	0.0	17.7	0.0	17.7	0.0	17.7	0.0	8.9

Table 4
Supply chain total profit by different warehouse capacities.

Space	Warehouse capacity (box)	Revenue	Cost	Total profit
22,500	790,000	100,480,401,101	57,376,100,000	43,104,301,101
20,000	700,000	100,480,401,101	57,376,100,000	43,104,301,101
17,500	620,000	100,404,890,235	57,333,000,000	43,071,890,235
15,000	530,000	100,282,014,843	57,262,800,000	43,019,214,843
12,500	440,000	100,105,280,471	57,162,000,000	42,943,280,471
10,000	350,000	99,835,465,359	57,008,500,000	42,826,965,359

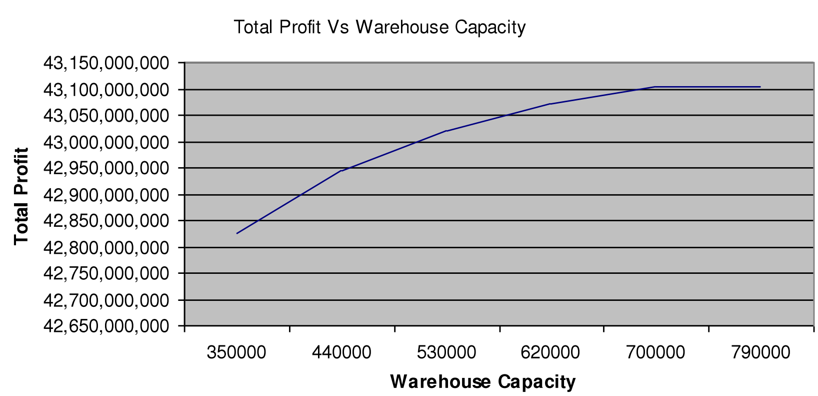

Fig. 2. Total profit vs. warehouse capacity.

7. Conclusions

The studied supply chain process is a real time process in chemical detergent industry. We decided to plan its processes under real world's uncertain demand situations which will lead to make decisions of the inventory level, reorder quantity and production amount in every period. The proposed model is a dynamic model made with the combination of two separate models, one is an inventory control system and the other is a production planning model. To make the model more applicable we designed it under uncertain demand situations and despite common approaches such as robust optimization, stochastic programming, games theory, linear programming and parametric programming, we had to distribute all products from the central warehouse in the real situations by considering the demand uncertainty effects on production processes. It is the first time that anyone has used queueing techniques to construct this production-inventory model. Moreover, solving this problem can help to strategic decision making about supply chain decoupling point.

In this article we have extended the Isotupa [35] model to multi-item supply chain with capacitated warehouse and by implementing numerical calculation methods which decrease the computational time for efficient application in real world problems. As we combined inventory control system with production planning model for a MTS system, it is suggested that other researchers construct a similar model for MTO system or any other production system. In the second part of the model we have used LINGO to solve the formulated problem; a large-scale solution can also be a topic of future studies. It can help to link both models more efficiently.

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