The Development of Contraction Mappings in Metric Space Referencing

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ABSTRACT

The research into metric space has advanced greatly during the last few years. We have versions of metric spaces and b-metric spaces since few researchers have sought to change metric spaces. While others explored metric fixed-point theory to develop an infinite number of different contraction mappings. The goal of the research piece is to analyse and improve different contraction mappings with respect to metric spaces.

Subject Classification: 30L15, 30L99

KEYWORDS *: Contraction mappings, weakly contractive map, multivalued mappings, rational contraction mapping, Ciric's Contraction Mapping.*

INTRODUCTION

In 1905, Maurice Fréchet, a French mathematician, began exploring metric spaces. Metric space continues to exist in mathematics as an abstract set with a distance function known as a metric, only in the division topology. Let's analyze the different contraction mappings in metric space.

SUMMARY OF PRIOR RESEARCH

Definition: metric space (M.S.)

Assume metric or distance function d: $\wp \times \wp \rightarrow R^+$ for non-empty set $\wp$. If

(1) $d(v_1, w_1) = 0 \Leftrightarrow v_1 = w_1$ (2) $d(v_1, w_1) = d(w_1, v_1)$ (3) $d(v_1, w_1) \leq [d(v_1, u_1) + d(u_1, w_1)]$

for all $v_1, w_1, u_1 \in \wp$ Suppose d is a metric on $\wp$, we state, $(\wp, d)$ is metric space. [1]

Backthin had a thought about b-metric space in 1989, and Czerwik later wrote that thought down in terms of b-metric space in 1993. Czerwik thus established b-metric space as the first concept.

Definition: b-metric space (b-M.S.)

Assume $\wp$ is a set and take $s \geq 1$, $s \in R$. A function d: $\wp \times \wp \rightarrow R^+$ is considered a b-metric if $\forall\, v_1, w_1, u_1 \in \wp \Rightarrow$

(1) $d(v_1, w_1) = 0 \Leftrightarrow v_1 = w_1$ (2) $d(v_1, w_1) = d(w_1, v_1)$

(3) $d(v_1, w_1) \leq s\,[d(v_1, u_1) + d(u_1, w_1)]$

A couple $(\wp, d)$ is denoted as a b-metric space. [14]

Example: -

The space L_p $(0<p<1)$ pertaining to all real functions $v(t), t \epsilon [0,1]$ so that:

$\int_0^1 |v(t)|^p\, dt < \infty$ is a b metric space if we proceed

$d(v,w) = (\int_0^1 |v(t) - w(t)|^p)^{1/p}$ for each

$v,w \epsilon L_p$, where $a = 2^{(1/p)}$

Augustin Louis Cauchy, a French mathematician, developed the Cauchy sequence idea. According to his theories, a series is said to be Cauchy if its terms arbitrarily approach one another as it moves forward. [8]

Definition: Cauchy Sequences & convergence in b-metric Spaces: -

Consider the sequence $(v_n)_{n \in N}$ in b-metric space is called:

: Cauchy whenever ε>0 one can found, n(ε)ϵ N:∀n, m ≥ n(ε), we have d $(v_n, w_m) < \varepsilon$

: Convergent whenever ∃ v ϵ ℘ to each ε>0, ∃ n(ε)ϵN: for each n≥n(ε)

We have $d(\mathfrak{v}_n, \mathfrak{v}) < \varepsilon$ i.e. $\lim_{n\to\infty} \mathfrak{v}_n = \mathfrak{v}$ [8]

Definition: Complete b-Metric Space: -

(1) Assume that the space (℘,d) is b-metric. Subsequently a subset ב⊂℘ is then referred to as compact if and only if there is a Subsequence approaching an element in ב." for every sequence of ב's elements.

: Closed whenever every sequence $(v_n)_{n\epsilon N}$ in Y converges to v⇒ v ϵ ב

: The b-metric space is complete whenever each Cauchy sequence converges. [8]

Definition: b-Convergent Sequence: -

Let (℘, d) is a b-metric space. Then the sequence $\{v_n\}$ in ℘ is called:

: b- Convergent if and only if there exists v∈℘ such that

$$d(\mathfrak{v}_n, \mathfrak{v}) \to 0 \text{ as } n \to \infty \text{ symbolically } \lim_{n\to\infty} \mathfrak{v}_n = \mathfrak{v}$$

: b- Cauchy ⇔ $d(v_n, v_m) \to 0$ as n, m → ∞ [6].

Theorem: Every sequence in a metric space that converges is a Cauchy.

Remark: Not every Cauchy sequence in a metric space necessarily converges.

Example: Cauchy sequence

Assume sequence $v_1 = 1.41$, $v_2 = 1.414$, $v_3 = 1.4142$, $v_4 = 1.41421$……in usual metric space Q, converging to √2 ⇒ it is Cauchy but √2 ∉ Q.

Let $\wp = (0,1]$ and $d(\mathfrak{v}, \mathfrak{w}) = |\mathfrak{v} - \mathfrak{w}|$ is metric and $\{\mathfrak{v}_n\} = \{\frac{1}{n} : n \in N\}$ ⇒ is a Cauchy ∀ ε > 0 we have $d(\mathfrak{v}_m, \mathfrak{v}_n) = \left|\frac{1}{m} - \frac{1}{n}\right| < \epsilon$

for all m,n>1/ε on other hand $v_n \to 0$ but 0 does not belong to ℘ hence sequence is not convergent.

However, if ℘=[0,1] the sequence is Cauchy and is convergent. [4,5]

Definition: - Fixed point

Assume a nonempty set ℘ and ℵ:℘→℘. If ℵ $(v_1) = v_1$ ⇒ It's a fixed point. [7]

Table1: Fixed point of functions

Function	Fixed point	Contraction mapping	Metric space
$e^{\mathfrak{v}}$	No fixed point	Yes	(0,1)
log 𝔳	No fixed point	Yes	R+
$\mathfrak{v}^2$	0 and 1	Yes, only in metric [0,1/2]	[0,1/2]
Sinh𝔳	0	No	----
Tanh𝔳	0	Yes	R+
𝔳 +1	No	Yes	R+
$\frac{1}{\mathfrak{v}+1}$	0.618033	No	R
$m\mathfrak{v} + b$	b/[1-m]	Yes	R

Definition: Fixed point of multivalued mapping

Nadler (1969): Assume $v_0 \in \wp$ is a fixed point of the multivalued mapping f if, $v_0 \in \aleph v_0$ [19]

The term "contractive mapping" was initially introduced by Edelstein for the metric space.

Definition: Contraction mapping (C.M.)

Assume ℵ: ℘→℘ in metric space (℘,d) is C.M. if $v_1 \neq w_1$ entails $d(\aleph(v_1), \aleph(w_1)) < d(v_1, w_1)$ [18]

Example 1: Contraction Mapping (C.M.)

Define ℘=(0,1] alongwith,d(v,w)=|v-w|

Let ℵ: ℘→℘ | ℵ (v)=1/2v is contraction mapping.

In the research Ciric [12], [13] investigated C.M., also known as Ciric's type C.M. In one research article "A generalization of Banach's contraction principle", author defined quasi C.M.

Definition: Ciric's type C.M.

Consider $\aleph: \wp \to \wp$ in b-metric space named as Ciric's type contraction $\Leftrightarrow \forall$ v, w$\in \wp$ $\exists$ h<1 so as

$$d(\aleph(\mathfrak{v}), \aleph(\mathfrak{w})) \leq h.\max\{d(\mathfrak{v}, \mathfrak{w}), d(\mathfrak{v}, \aleph\mathfrak{v}), \frac{d(\mathfrak{v},\aleph\mathfrak{w})+d(\mathfrak{w},\aleph\mathfrak{v})}{2}\}$$

Example2: Ciric's C.M.

Define $\wp=[1,\infty)$ with d(v,w) =|v-w|. Assume $\aleph(v)=1/v$ is Ciric's C.M. where h=0.8

$$\text{Proof: } LHS = d(\aleph(\mathfrak{v}), \aleph(\mathfrak{w})) = \left|\frac{1}{\mathfrak{v}} - \frac{1}{\mathfrak{w}}\right| = \left|\frac{\mathfrak{v}-\mathfrak{w}}{\mathfrak{v}.\mathfrak{w}}\right| < |\mathfrak{v} - \mathfrak{w}|$$

RHS= 0.8 |v-w| Hence LHS≤RHS

Let's point out that Ciric [14] invented and researched quasi-C.M. According to the well-known Ciric finding having unique fixed point (U.F.P) [12], [13]

Example: 4:

Let $\aleph, \varrho: \wp \to \wp$ as $\aleph(v)=v$ and $\varrho(v) = v^2$ then

Clearly 1 is the coincidence point.

Definition: [17]

Let $\psi_1 : [0,\infty)\to[0,\infty)$ that fulfills the given requirements:

Assures continuity of ψ_1

Non-decreasing is the function ψ_1

$\psi_1(\daleth) = 0 \Leftrightarrow \daleth = 0$

Alber and Guerre-Delabriere [16] first described the idea of the weak contraction in 1997.

Definition: Weakly contractive map (W.C.M.)

A self-map $\aleph$ is considered a W.C.M. if $\exists\ \varphi_1 : [0,+\infty)\to[0,+\infty)$ such that non-decreasing is the function φ_1 and assuring its continuity, and $\varphi_1(\daleth) = 0 \Leftrightarrow \daleth = 0$ satisfying $d(\aleph(v_1), \aleph(w_1)) \leq d(v_1, w_1) - \varphi_1(d(v_1, w_1)), \forall\ v_1, w_1 \in \aleph$ [17]

Remark 1

Consider the set $\aleph=[1,\infty)$, $\varphi_1 : [0, \infty)\to[0,\infty)$, with $\varphi_1(\daleth) = \daleth^2$, $\aleph(v_1) = v_1$, $d(v_1, w_1) = |v_1 - w_1|$ φ_1 fails to be a W.C.P.

Remark 2:

Whereas for the set $\aleph=[1,\infty)$, $\varphi_1:[0,\infty)\to[0,\infty)$, $\varphi_1(\daleth) = \daleth/2$, $\aleph(v) = 1/v$, $d(v_1, w_1) = |v_1 - w_1|$

Undoubtedly φ_1 is a W.C.M.

Remark 3:

The set $\wp = [2, \infty)$, *with* $\varphi(\daleth) = \frac{\daleth}{6}, \psi(\daleth) = \frac{\daleth}{3}, \aleph(\mathfrak{v}) = \mathfrak{v}$

Where $d(\mathfrak{v}, \mathfrak{w}) = |\mathfrak{v} - \mathfrak{w}|$ *it is not* (ψ, φ) –W.C.

Remark 4:

The set $\wp = [3, \infty), \psi, \varphi: [0, \infty) \to [0, \infty)$ *with* $\varphi(t) = \frac{t}{6}, \psi(\daleth) = \frac{t}{3}, f(\mathfrak{v}_1) = \frac{1}{\mathfrak{v}_1}$, $d(\mathfrak{v}_1, \mathfrak{w}_1) = |\mathfrak{v}_1 - \mathfrak{w}_1|$ It is (ψ, φ) – weak contraction map.

Definition: - Altering distance function (A.D.F.)

Consider $\psi:[0,\infty)\to[0,\infty)$ It is termed as an A.D.F. if:

i) Assures continuity of ψ (ii)It is nondecreasing

ii) $\psi(\daleth)=0$ exactly when $\daleth=0$ [16]

Remark 5:

$\psi(t) = t^2$ and te^t are A.D.F.

Definition: - Infinite altering distance function (I.A.D.F.)

An A.D.F. is an I.A.D.F. if is defined on R [16]

Kirk et al. [24] worked on cyclic contraction mappings.

Definition: - Generalized cyclic contractive mapping (G.C.C.M.) [16], [6]

Let $\beth$ = AUB, Assume, $\aleph:\beth\to\beth$, $A\subseteq\wp$, $B\subseteq\wp$: $A\neq\varnothing$, $B\neq\varnothing$, and $(\wp,d,s)$ be a b-metric space. A G.C.C.M. is $\aleph$ if:

i) $\beth$ = AUB represents Y cyclically with regard to $\aleph$, that is, $\aleph(A) \subset B$ and $\aleph(B)\subset A$

ii) There exist $\psi_1 \in \Psi$, $\varphi_1 \in \Phi_u$, L≥0 (constant) such that $\psi(s^4 d(\aleph v, \aleph w)) \leq \varphi(\psi(M(v, w))) + LN(v,w)$, $\forall$ (v,w) ϵ A×B or (v,w) ϵ B×A wherever

$$M(\mathfrak{v}, \mathfrak{w}) = \max\{d(\mathfrak{v}, \mathfrak{w}), d(\mathfrak{v}, \aleph\mathfrak{v}), d(\mathfrak{w}, \aleph\mathfrak{w}), \frac{d(\mathfrak{v},\aleph\mathfrak{w})+d(\mathfrak{w},\aleph\mathfrak{v})}{2s},$$

$$\frac{d(\aleph^2\mathfrak{v}, \mathfrak{v}) + d(\aleph^2\mathfrak{v}, \aleph\mathfrak{w})}{2s}, d(\aleph^2\mathfrak{v}, \aleph\mathfrak{v}), d(\aleph^2\mathfrak{v}, \mathfrak{w}), d(\aleph^2\mathfrak{v}, \aleph\mathfrak{w})\}$$

$$N(\mathfrak{v}, \mathfrak{w}) = \min\{d(\mathfrak{v}, \aleph\mathfrak{v}), d(\mathfrak{w}, \aleph\mathfrak{v}), d(\aleph^2\mathfrak{v}, \aleph^2\mathfrak{w})\},$$

In 1998, Jungck and Rhoades [20] developed the idea of weakly compatible maps and demonstrated that weakly compatible maps are compatible maps, but the reverse is not necessarily accurate.

Definition: - Weakly Compatible

Consider ℵ,ϱ:℘→℘. in metric space (℘,d) If ℵu =ϱu ⇒ ℵϱu = ϱℵu for some u∈℘. Then ℵ and ϱ are identified as weakly compatible. [2]

Definition: - K-contraction

Assuming ℵ:℘→℘ in metric space (℘,d) is referred to as K-contraction if there can be found a ∈ (0,½): ∀ v,w ∈ ℘ the subsequent inequality is valid:

d(ℵv,ℵw)≤ a[d(v,ℵv)+d(w,ℵw)] [3]

In 1968, Kannan demonstrated that any K contraction on complete metric space (℘,d) has a U.F.P.

A fixed-point theorem for C-contraction mappings was developed by Chatterjee in 1972. According to this theorem, a mapping ℵ is a C-contraction if a ∃ point at α ∈ (0,½) where it fulfills condition below for every values of v and w:

d(ℵv,ℵw)≤ ∝[d(v,ℵw)+ d(w,ℵv)]

Definition: q-set-valued α-quasi-contraction

Assume b-metric space (℘,d) and ∝:℘×℘→(0,∞] is a mapping. The set-valued mapping

ℵ:℘→$P_{b,d}$ (℘) termed as q-set-valued α-quasi-contraction if

∝(v_1, w_1) H(ℵv_1, ℵw_1) ≤ qM (v_1, w_1), for all v_1, w_1 Є ℘,0≤q<1 and

M (v_1, w_1) =

max {d (v_1, w_1), d (v_1, ℵv_1), d (w_1, ℵw_1), d (v_1, ℵw_1), d(w_1, ℵv_1)} [16]

Definition: - Multivalued mappings (M.P.)

Assume ℘≠∅,ℶ≠∅ and ℵ:℘→$2^{ℶ}$ then ℵ is known as M.P. [19]

By applying a control function, Sessa et al. [23] established an altering distance function in the year 1984.

Definition: - Comparison Function (C.F.)

Suppose $φ_1$:[0,∞) →[0,∞) φ: is termed as a C.F. if

(i) Assurance of increasing and

(ii) $φ_1^n$ (ℸ)→0 as n→∞ for any ℸ ∈[0,∞). [10]

Definition: - b-comparison function (b-C.F.)

Let s ≥1 a real number. Suppose φ1:[0,∞) →[0,∞) is a b-C.F. if

(1) If $φ_1$ is monotonically increasing and

(2) ∃ n_0 ∈N,a ∈(0,1) and a convergent series of nonnegative terms $\sum_{n=1}^{\infty} V_n$ so that s^{n+1} $φ_1^{n+1}$ (ℸ)≤a s^n $φ_1^n$ (ℸ) + V_n For n ≥ n_0 and any ℸ∈[0,∞) [10]

Definition: - α-ψ contractive mappings

Assume ℵ:℘→℘ in metric space (℘,d). Then ℵ is α-ψ contractive mappings. Supposing there are two functions

∃ functions∝:℘×℘ →[0,∞) and ψ∈Ψ:

∝ (v,w)d(ℵv,ℵw)≤ψ(d(v,w))for all v,w Є ℘ [10]

Authors Arsalan Hojat Ansari, Vishal Gupta, Naveen Mani in the year as 2010 had newly defined the infinite altering distance function.

Definition: - Infinite altering distance function (I.A.D.F.) [6]

Suppose ψ:R→R termed as an I.A.D.F. if the following criteria are met:

i) ψ is non-decreasing as well as continuous

ii) ψ(ℸ) = 0 ⇔ ℸ=0

In the year 2012 Samet et al originated concept α admissible mapping as mentioned below.

Definition: -α admissible mapping

If for, ℵ: ℘→℘,∃ ∝:℘×℘→[0,∞) f is α-admissible mapping if ∀ v,w Є ℘, ∝ (v,w) ≥ 1⇒

∝ (ℵv,ℵw)≥1 [10]

Remark 11

Let ℘=(0,∞) and ℵ: ℘→℘, ∃ ∝:℘×℘→[0,∞) by

ℵv_1 = √(v_1) ∀ v_1 ∈ ℘. Then f is α admissible mapping where

$$\propto (v_1, w_1) = \begin{cases} e^{v_1 - w_1}, & v_1 \geq w_1 \\ 0, & v_1 < w_1 \end{cases}$$ [22]

Definition: - C-comparison function (C-C.F.)

The function $\varphi_1: [0,\infty) \to [0,\infty)$ is a **C-C.F.**

(i) where it is increasing and

(ii) $\exists\, n_0 \epsilon N, a\epsilon(0,1)$ and a convergent series of nonnegative terms $\sum_{n=1}^{\infty} V_n$ so that $\varphi_1^{n+1}(\daleth) \le a\varphi_1^{n}(\daleth) + V_n$

For $n \ge n_0$ and any $\daleth \epsilon[0,\infty)$ [8]

Definition: - Ordered - ψ -Suzuki type rational contractive mapping

For partially ordered b-metric space ($\wp$, d, $\preccurlyeq$). $\aleph$: $\wp \to \wp$ is an ordered - ψ -Suzuki type rational contractive mapping if 1/2s $d(v_1, \aleph v_1) \le d(v_1, w) \Rightarrow sd(\aleph v_1, \aleph w) \le \psi(M(v_1,w))$ where

$$M(v_1, w_1) = max\left\{ d(v_1, w_1), \frac{d(v_1, \aleph v_1)d(w_1, \aleph w_1)}{1 + s[d(v_1, w_1) + d(v_1, \aleph w_1)]}, \frac{d(v_1, \aleph w_1)d(v_1, w_1)}{1 + s[d(v_1, \aleph v_1) + d(w_1, \aleph w_1)]} \right\}$$

$\forall\, (v_1, w_1) \epsilon X$ with $v_1 \preccurlyeq w_1$ [25]

Later, Karapınar [21] changed admissible mappings by including additional requirements to create a triangular admissible mapping.

Authors Piyachat Borisut, Poom Kumam, Vishal Gupta and Naveen Mani [9] introduced A class of generalized (ψ, α, β) − weak contraction.

Definition: [9]

Suppose $\aleph, \varrho, S$: $\wp \to \wp$ are generalized (ψ, α, β) − weak contraction if $\forall$ v,w$\in \wp$

$\psi(d(\aleph v, \varrho w)) \le \propto (d(Sv, Sw)\, \beta(d(Sv, Sw)$ for all $v \ge w$ Where $\alpha \epsilon F$, $\psi \epsilon \Psi$, $\beta:[0,\infty) \to [0,\infty)$ is a continuous function with condition $0 < \beta(\daleth) < \psi(\daleth)$

Example: [9]

Let $\wp$=NU{0} Define $d(v_1, w_1) = \begin{cases} v_1 + w_1, & if\ v_1 \ne w_1 \\ 0, & if\ v_1 = w_1 \end{cases}$

Definition: - [6]

A mapping F: $[0,\infty)^2 \to R$ is C-class function if it is continuous and

(i) $F(v_1, w_1) \le v_1$

(ii) $F(v_1, w_1) = v_1 \Rightarrow$ either $v_1 = 0$ or $w_1 = 0$ for all $v_1, w_1 \epsilon [0,\infty)$ Moreover F (0, 0) = 0.

Authors [15] defined F-contractive type mappings in b-metric spaces as mentioned below.

Definition: [15]

Assume ϱ: $\wp \to \wp$ in b-metric space (X, d, s), is F-contractive type mapping if $\exists\, \tau > 0 \mid d(v,\varrho v)d(w,\varrho w) \neq 0$ implies

$\tau + F(sd(\varrho v,\varrho w)) \le 1/3\ [Fd(v,w) + Fd(v,\varrho v) + Fd(w,\varrho w)]$ and $d(v,\varrho v)d(w,\varrho w) = 0$ implies

$\tau + F\, sd(\varrho v,\varrho w) \le 1/3\ [Fd(v,w) + Fd(v,\varrho w) + Fd(w,\varrho v)]$ for all v,w$\in \wp$

CONCLUSION

Various scholars studied and worked in field of metric space. In the scholarly article we tried to rescript the various contraction mappings and the basic concepts in the reference of metric space. There are other further contraction mappings in the body of literature. The goal of this essay was to summaries the limited literature that was available.

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