Raman Scattering in High Temperature Superconductors

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ABSTRACT

The detailed study of Raman Scattering in high temperature Superconductors (HTS) via using the quantum dynamical approach of double time temperature dependent phonon-Green's functions. The theory of first-order Raman scattering, second-order Raman scattering, third-order Raman scattering and electron-phonon contribution Raman scattering is investigated for high temperature superconductors. Raman tensor, intensity of Raman lines and differential cross-sections of various orders of scattering have been investigated. It has been developed using almost complete Hamiltonian which consists of the contributions due to the unperturbed phonons and electrons, anharmonic phonon fields, localized phonon fields and that of electron-phonon interactions. The temperature and defect dependencies are discussed in detail along with the nature of continues and line spectra. The phonon Raman scattering spectral density function has been obtained and analyzed for high temperature superconductors.

Keywords: High Temperature Superconductors, Raman Scattering, Electron-Phonon Interaction, Green's Function Technique, Hamiltonian

I. INTRODUCTION

Raman scattering is an excellent technique to characterize and to investigate basic physical properties of high temperature superconductors. However, many properties have not yet been understood in the field of high temperature superconductors The Raman scattering has played an important role to investigate the properties of the gap function. A wide range of experimental techniques can be employed to investigate this type of properties [1, 2]. The multi-phonon interactions triggered for the calculation of thermal, optical and elastic properties of crystals. Mostly purposed theories are based on the harmonic approximation [3]. We should not expect the harmonic theory to be correct, even at the lowest temperatures. The influence of anharmonicities does not vanish even at the absolute temperature [4]. In addition to anharmonicity the presence of impurity dramatically modifies the phonon spectrum of the crystal and, hence, substantial changes occur in energy dependent properties of the crystal [5]. The interactions such as: phonon interactions in the anharmonic crystal fields, phonon interactions in the localized fields of crystal impurity, anharmonic phonon interactions with localized phonons and electron-phonon interactions. These interactions give rise to anharmonic mode, impurity modes and impurity-anharmonicity interference modes. The involvement of anharmonic and impurity effects can cause considerable alternation in the phonon Raman intensity. In this work, we shall amply the powerful Green's function technique to investigate the Raman intensity in high temperature superconductors. This thermodynamic Green's function technique has become an invaluable tool in the study of complicated systems of interacting particles [6-8]. In case of Raman line both the Raman shifts and line widths, and also the scattered intensities vary with temperature.

RAMAN SCATTERING

Raman Tensor

The intensity of linearly polarized Raman scattering per unit solid angle per unit energy is given by [9, 10]

$$I(\in_R) = \frac{\in_i^4}{2\pi c^3} \sum_{\alpha\beta,\gamma\lambda} n_\alpha n_\beta i_{\alpha\gamma,\beta\lambda}(\in_R) E_\gamma^- E_\beta^+ \tag{1}$$

In above equation $\in_i$ is the energy of incident radiation, $\in_R$ stand for the Raman shift with $\in_R < 0$ corresponds to the Stokes line, while $\in_R > 0$ gives the anti-Stokes (Raman) line, c is the speed of light, n is a unit vector of the one linearly polarized component of the scattered light and is perpendicular to the scattering direction, and E_γ^- and $E_\beta^+ (\equiv E_\gamma^+)^*$ are the positive and negative energy components of the electric vector of the incident light. The various Raman tensors can be obtained as follows

$$i_{\alpha\gamma,\beta\lambda}(\in_R) = i_{\alpha\gamma,\beta\lambda}^{(1)}(\in_R) + i_{\alpha\gamma,\beta\lambda}^{(2)}(\in_R) + i_{\alpha\gamma,\beta\lambda}^{(3)}(\in_R) \tag{3}$$

$$i_{\alpha\gamma,\beta\lambda}^{(1)}(\in_R) = \frac{1}{2\pi} \sum_{k_1 j_1, k_1' j_1'} \int_{-\infty}^{+\infty} dt \exp(-i\in_R t) P_{\alpha\gamma,\beta\lambda}^{(1)} \binom{k_1\, k_1'}{j_1\, j_1'} \langle A_{k_1 j_1}(t) A_{k_1' j_1'}(0) \rangle \tag{4a}$$

$$i_{\alpha\gamma,\beta\lambda}^{(2)}(\in_R) = \frac{1}{2\pi} \sum_{\substack{k_1 j_1, k_1' j_1' \\ k_2 j_2, k_2' j_2'}} \int_{-\infty}^{+\infty} dt \exp(-i\in_R t) P_{\alpha\gamma,\beta\lambda}^{(2)} \binom{k_1\, k_2\, k_1'\, k_2'}{j_1\, j_2\, j_1'\, j_2'} \langle A_{k_1 j_1}(t) A_{k_2 j_2}(t) A_{k_1' j_1'}(0) A_{k_2' j_2'}(0) \rangle \tag{4b}$$

and

$$i_{\alpha\gamma,\beta\lambda}^{(3)}(\in_R) = \frac{1}{2\pi} \sum_{\substack{k_1 j_1, k_2 j_2, k_3 j_3 \\ k_1' j_1', k_2' j_2', k_3' j_3'}} \int_{-\infty}^{+\infty} dt \exp(-i\in_R t) P_{\alpha\gamma,\beta\lambda}^{(3)} \binom{k_1\, k_2\, k_3\, k_1'\, k_2'\, k_3'}{j_1\, j_2\, j_3\, j_1'\, j_2'\, j_3'}$$

$$\times \langle A_{k_1 j_1}(t) A_{k_2 j_2}(t) A_{k_3 j_3}(t) A_{k_1' j_1'}(0) A_{k_2' j_2'}(0) A_{k_3' j_3'}(0) \rangle \tag{4c}$$

In obtaining Eqn.(3) and (4) we have ignored the contributions from equal time correlation functions [11]. The correlation functions $\langle A_{k_1 j_1}(t) .. A_{k_1' j_1'}(0)..\rangle$ appearing in Eqn.(4) contain the entire physics of Raman scattering. These correlation functions can be evaluated from [12-17]

$$F_{kj\,k'j'}^{(i)}(t,t') = \int_{-\infty}^{+\infty} J_{kj\,k'j'}^{(i)}(\in) \exp[-i\in(t-t')] d\in \tag{5}$$

where $F_{kjk'j'}^{(i)}(t,t') = \langle A_{k_1 j_1}(t) A_{k_2 j_2}...(t) A_{k_i j_i}(t) A_{k_1' j_1'}(0) A_{k_2' j_2'}...(0) A_{k_i' j_i'}(0)\rangle$ $\tag{6}$

and $J_{kjk'j'}^{(i)}(\in)$ are the known as spectral density functions, given by

$$J_{kjk'j'}^{(i)}(\in) = -\{2/\exp(\beta\hbar\in -1)\} \operatorname{Im} G_{kj}^{(i)}(\in) \tag{7}$$

Differential Cross Section for Raman Scattering

The differential scattering cross section per unit solid angle per unit energy interval for Raman scattering is given by [10, 18]

$$\frac{d^2\sigma}{d\Omega\, d\in_R} = (\in/c)^4 \sum_{\alpha\beta,\gamma\lambda} n_\alpha n_\beta i_{\alpha\gamma,\beta\lambda}(\in_R) m_\gamma m_\beta \tag{8}$$

where Ω denotes solid angle, $d\in$ is solid angle located in the direction $\in$ and n and m are the unit polarization vectors of the radiation. Obviously, the differential cross section depends on the Raman intensity tensor $i_{\alpha\gamma,\beta\lambda}(\in_R)$, which will describe the one-, two-, and three-phonon differential cross section for the Raman scattering.

HAMILTONIAN

In the present formulation we take the following Hamiltonian

$$H = H_p + H_e + H_{ep} + H_A + H_D \tag{9}$$

where $\quad H_p = \dfrac{1}{4}\sum_{kj} \in\binom{k}{j}\left\{A_{kj}^*A_{kj} + B_{kj}^*B_{kj}\right\}$ (unperturbed phonon p Hamiltonian) $\tag{10a}$

$$H_e = \sum_q \in_q b_q^* b_q, \quad \text{(electron e Hamiltonian)} \tag{10b}$$

$$H_{ep} = \sum_{kj,q,Q} g_{kj} b_Q^* b_q B_{kj}, \quad \text{(electron-phonon ep Hamiltonian)} \tag{10c}$$

$$H_A = \sum_{S \geq 3}\sum_{k_1 j_1,..,k_S j_S} V_S\begin{pmatrix} k_1\, k_2 & k_s \\ j_1\, j_2 & ..... & j_s \end{pmatrix} A_{k_1 j_1} A_{k_2 j_2 ......} A_{k_S j_S}, \text{ (anharmonic A Hamiltonian)} \tag{10d}$$

and $\quad H_D = \sum_{k_1 j_1, k_2 j_2}\left[D\begin{pmatrix} k_1\, k_2 \\ j_1\, j_2 \end{pmatrix} A_{k_1 j_1} A_{k_2 j_2} - C\begin{pmatrix} k_1\, k_2 \\ j_1\, j_2 \end{pmatrix} B_{k_1 j_1} B_{k_2 j_2} \right], \quad \text{(defect D Hamiltonian)} \tag{10e}$

In above expressions $A_{kj} = a_{kj} + a_{-kj}^* = +A_{-kj}^*$ (phonon field operator) and $B_{kj} = a_{kj} - a_{-kj}^* = -B_{-kj}^*$ (phonon momentum operator), $b_q(b_q^*)$ and $a_{kj}(a_{kj}^*)$ are electron and phonon annihilation (creation) operators with wave vectors q and kj respectively (j is the branch index). g_{kj}, $V_s\begin{pmatrix} k_1\, k_2 & k_s \\ j_1\, j_2 & ..... & j_s \end{pmatrix}$, $C\begin{pmatrix} k_1\, k_2 \\ j_1\, j_2 \end{pmatrix}$ and $D\begin{pmatrix} k_1\, k_2 \\ j_1\, j_2 \end{pmatrix}$ are electron-phonon coupling coefficient, anharmonic coupling coefficients, mass difference and force constant charge parameters, respectively. Also, $Q = kj + q$.

Phonon Green's function

Let us consider the double-time thermodynamic electron retarded Green's functions

$$G_{kk'}(t,t') = <<A_k(t); A_{k'}^*(t')>> = -i\theta(t - t')<[A_k, A_{k'}^*]> \tag{11}$$

The Green's functions can be evaluated with the help of equation of motion technique of quantum dynamics via Hamiltonian (10) by adopting the technique of successive approximations and some tedious algebra. The shift $\Delta(k,\in)$ and line width $\Gamma(k,\in)$ can be separated in four terms, namely; (i) electron-phonon contribution, (ii) anharmonic contribution (iii) defect contribution and (iv) anharmonic and defect contribution. Thus

$$\Delta(k,\in) = \Delta^{EP}(k,\in) + \Delta^A(k,\in) + \Delta^D(k,\in) + \Delta^{AD}(k,\in) \tag{12}$$

and $\quad \Gamma(k,\in) = \Gamma^A(k,\in) + \Gamma^D(k,\in) + \Gamma^{AD}(k,\in) + \Gamma^{ep}(k,\in) \tag{13}$

with

$$\Gamma^A(k,\in) = \Gamma^{3A}(k,\in) + \Gamma^{4A}(k,\in) \tag{14}$$

$$\Gamma^{3A}(k,\in) = 18\pi\xi(\in)\sum_{k_1,k_2}|V_3(k_1,k_2,k)|^2 \eta_1 [S_{+\alpha}\,\tilde{\in}_{+\alpha}\,\delta(\in^2 - \tilde{\in}_{+\alpha}^2) + S_{-\alpha}\,\tilde{\in}_{-\alpha}\,\delta(\in^2 - \tilde{\in}_{-\alpha}^2)] \tag{15a}$$

$$\Gamma^{4A}(k,\in) = 48\pi\xi(\in)\sum_{k_1,k_2,k_3}|V_4(k_1,k_2,k_3,k)|^2 \eta_2 [S_{+\beta}\,\tilde{\in}_{+\beta}\,\delta(\in^2 - \tilde{\in}_{+\beta}^2) + 3S_{-\beta}\,\tilde{\in}_{-\beta}\,\delta(\in^2 - \tilde{\in}_{-\beta}^2)] \tag{15b}$$

$$\Gamma^D(k,\in) = \Gamma_m(k,\in) + \Gamma_f(k,\in) + \Gamma_{mf}(k,\in) \tag{16}$$

$$\Gamma_m(k,\in) = 8\pi\xi(\in)\sum_{k_1}|C(k_1,k)|^2 \in_{k_1}^3 \in_k^{-2}\delta(\in^2 - \tilde{\in}_{k_1}^2) \tag{16a}$$

$$\Gamma_f(k,\in) = 8\pi\xi(\in)\sum_{k_1}\left|D(k_1,k)\right|^2 \in_{k_1}\delta(\in^2 - \tilde{\in}^2_{k_1}) \tag{16b}$$

$$\Gamma_{mf}(k,\in) = 128\,\pi\xi(\in)\sum_{k_1}\left|D(k_1,k)\right|^2\left|C(k_1,k)\right|^2 \in_k^{-2}\in_{k_1}\ \delta(\in^2 - \tilde{\in}^2_{k_1}) \tag{16c}$$

$$\Gamma^{AD}(k,\in) = \Gamma^{3Am}(k,\in) + \Gamma^{4Am}(k,\in) \tag{17}$$

$$\Gamma^{3Am}(k,\in) = 288\,\pi\xi(\in)\sum_{k_1,k_2}\left|V_3(k_1,k_2,k)\right|^2\left|C(k_1,k)\right|^2\eta_1[S_{+\alpha}\ \tilde{\in}_{+\alpha}\ \delta(\in^2 - \tilde{\in}^2_{+\alpha}) + S_{-\alpha}\ \tilde{\in}_{-\alpha}\ \delta(\in^2 - \tilde{\in}^2_{-\alpha})]\in_k^{-2} \tag{18a}$$

$$\Gamma^{4Am}(k,\in) = 768\,\pi\xi(\in)\sum_{k_1,k_2,k_3}\left|V_4(k_1,k_2,k_3,k_1)\right|^2\left|C(k_1,-k_1)\right|^2\eta_2[S_{+\beta}\ \tilde{\in}_{+\beta}\ \delta(\in^2 - \tilde{\in}^2_{+\beta}) + 3S_{-\beta}\ \tilde{\in}_{-\beta}\ \delta(\in^2 - \tilde{\in}^2_{-\beta})]\in_k^{-2} \tag{18b}$$

$$\Gamma^{EP}(k,\in) = 8\pi\sum_q g_k^2 \in_k^{-2}\in_Q^2 \delta(\in - 2\,\tilde{\in}_Q)N_Q \tag{19}$$

Where

$$\tilde{\in}_{\pm\alpha} = \tilde{\in}_{k_1} \pm \tilde{\in}_{k_2}\,;\ \ \tilde{\in}_{\pm\beta} = \tilde{\in}_{k_1} \pm \tilde{\in}_{k_2} \pm \tilde{\in}_{k_2}\quad S_{\pm\alpha} = n_{k_2} \pm n_{k_1}\,;\ S_{\pm\beta} = 1 \pm n_{k_1}n_{k_2} + n_{k_2}n_{k_3} \pm n_{k_3}n_{k_1}$$

$$\eta_1 = \frac{\in_{k_1}\in_{k_2}}{\tilde{\in}_{k_1}\tilde{\in}_{k_2}}\,;\ \eta_2 = \frac{\in_{k_1}\in_{k_2}\in_{k_3}}{\tilde{\in}_{k_1}\tilde{\in}_{k_2}\tilde{\in}_{k_3}}\ \text{and}\ n_{k_i} = \coth(\beta \in_{k_i}/2)\quad N_Q = <b_Q^*b_Q> = \{\exp(\beta\,\tilde{\in}_Q)+1\}^{-1}$$

In the above equations the superscripts $3A$, $4A$, m, f, mf, and Am stand for cubic-and quartric-anharmonicities, mass change, force constant change, mass and force constant change and mass difference anharmonicity interactions.

<div align="center">PHONONIC RAMAN SCATTERING</div>

The Raman scattering tensor has been developed adopting the method of double-time temperature dependent Green's function. The correlation functions appearing in Eq.(4) are the direct consequence of double-time thermodynamic Green's function. To investigate the first-, second-, and third-order Raman scattering, let us consider the evaluation of one-, two-, and three-phonon Green's functions respectively. For this purpose we have used the imaginary part of phonon line width. The phononic FOR-, SOR-, and TOR- Scattering has been calculated by C. P. Painuli et al.[11], but the phonon-electron contribution to the phononic Raman scattering is added in the present work.

First-order Phononic Raman Scattering

The first order Raman scattering can be obtained in this form

$$i^{(1)}_{\alpha\gamma,\beta\lambda}(\in_R) = \left(2/\pi^2\right)\sum_{k_1j_1,k_1'j_1'}\int_{-\infty}^{+\infty}dt\exp(-i\in t)P^{(1)}_{\alpha\gamma,\beta\lambda}\binom{k_1k_1'}{j_1j_1'}\int_{-\infty}^{+\infty}d\in\eta_{kj\,k'j'}\in^2\binom{k_1}{j_1}\left(\frac{\exp(i\in t)}{[\exp(\beta\in)-1]}\right)$$

$$\times\left[\frac{\Gamma_{kj}(\in)}{\{[\in^2 - \overline{\in}^2\binom{k}{j}]^2 + 4\in^2\binom{k}{j}\Gamma^2_{kj}(\in)\}}\right] \tag{20}$$

The integrand is

$$\frac{2\in^2\binom{k}{j}\Gamma_{kj}(\in)}{\pi\{[\in^2 - \overline{\in}^2\binom{k}{j}]^2 + 4\in^2\binom{k}{j}\Gamma^2_{kj}(\in)\}} \tag{21}$$

The Raman tensor can be written in the form of diagonal and non-diagonal and these are given by

$$i_{\alpha\gamma,\beta\lambda}^{1,d}(\in_R)_{\text{FOR}} = \left(\frac{8}{\pi}\right)\sum_{\substack{kj\\kj=k'j'}}P_{\alpha\gamma,\beta\lambda}^{(1)}\binom{k\ -k}{j\ \ j}\sum_{kj}\left[C\binom{k\ k}{j\ j}C\binom{-k-k}{j\ \ j}\in^2\binom{k_1}{j_1}+D\binom{k\ -k}{j\ \ j}D\binom{-k\ k}{j\ \ j}\in^2\binom{k}{j}\right.$$

$$\left.+16C\binom{k\ k}{j\ j}C\binom{-k\ k}{j\ j}D\binom{k\ k}{j\ j}D\binom{-k\ k}{j\ j}\right]n\left[\tilde{\in}\binom{k}{j}\right]\frac{\in\binom{k_1}{j_1}}{\tilde{\in}\binom{k}{j}\left[\tilde{\in}^2\binom{k}{j}-\overline{\in}^2\binom{k}{j}\right]^2} \tag{22a}$$

$$i_{\alpha\gamma,\beta\lambda}^{1,nd}(\in_R)_{\text{FOR}} = \left(\frac{32}{\pi}\right)\sum_{\substack{kj\\kj\neq k'j'}}P_{\alpha\gamma,\beta\lambda}^{(1)}\binom{k\ k'}{j\ j'}C\binom{k-k'}{j\ j'}\in^2\binom{k_1}{j_1}\sum_{kj}\left[C\binom{k\ k}{j\ j}C\binom{-k-k}{j\ \ j}\in^2\binom{k_1}{j_1}+D\binom{k-k}{j\ j}D\binom{-k\ k}{j\ \ j}\in^2\binom{k}{j}\right.$$

$$\left.+16C\binom{k\ k}{j\ j}C\binom{-k\ k}{j\ j}D\binom{k\ k}{j\ j}D\binom{-k\ k}{j\ j}\right]n\left[\tilde{\in}\binom{k}{j}\right]\frac{\in\binom{k_1}{j_1}}{\in\binom{k}{j}\tilde{\in}\binom{k}{j}\left[\tilde{\in}^2\binom{k}{j}-\overline{\in}^2\binom{k}{j}\right]^2} \tag{22b}$$

where

$$n\left[\tilde{\in}\binom{k}{j}\right]=\frac{1}{\exp[\,\beta\,\tilde{\in}\binom{k}{j}]-1}$$

After substitution of Eqn.(15) into Eqn.(8) with appropriate simplifications, one can very easily obtain the FOR scattering differential cross section in the form

$$\left(\frac{d^2\sigma}{d\Omega d\in_R}\right)=\left(\frac{d^2\sigma}{d\Omega d\in_R}\right)^{1,d}+\left(\frac{d^2\sigma}{d\Omega d\in_R}\right)^{1,nd} \tag{23}$$

with

$$\left(\frac{d^2\sigma}{d\Omega d\in_R}\right)^{1,d}=\left(\frac{\in}{c}\right)^4\sum_{\alpha\beta,\gamma\lambda}m_\gamma\,m_\beta\,n_\alpha\,n_\beta\,i_{\alpha\gamma,\beta\lambda}^{1,d}(\in_R)_{\text{FOR}} \tag{24a}$$

and

$$\left(\frac{d^2\sigma}{d\Omega d\in_R}\right)^{1,nd}=\left(\frac{\in}{c}\right)^4\sum_{\alpha\beta,\gamma\lambda}m_\gamma\,m_\beta\,n_\alpha\,n_\beta\,i_{\alpha\gamma,\beta\lambda}^{1,nd}(\in_R)_{\text{FOR}} \tag{24b}$$

Higher-order Phononic Raman Scattering
Second-order Raman Scattering

The second-order electronic Raman tensor is given by

$$i_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{SOR}}=i_{\alpha\gamma,\beta\lambda}^{1,d}(\in_R)_{\text{SOR}}+i_{\alpha\gamma,\beta\lambda}^{1,nd}(\in_R)_{\text{SOR}}+i_{\alpha\gamma,\beta\lambda}^{(2)}(\in_R)_{\text{SOR}} \tag{25}$$

where the first term is obtained from (15) for this processes in which two phonons contribute in inelastic scattering processes via $\Gamma_{kj}^{(3)}(\in)$ and is given by

$$i_{\alpha\gamma,\beta\lambda}^{1,d}(\in_R)_{\text{SOR}}=\left(\frac{18}{\pi}\right)\sum_{\substack{kj\\k_1j_1,k_2j_2}}P_{\alpha\gamma,\beta\lambda}^{(1)}\binom{k-k}{j\ \ j}\in^2\binom{k}{j}\left[\xi_{kk_1k_2}^{(1)}+16\chi_{kk_1}\xi_{kk_1k_2}^{(1)}\right]S_1^{(\pm)}(\alpha) \tag{26a}$$

$$i_{\alpha\gamma,\beta\lambda}^{1,nd}(\in_R)_{\text{SOR}}=\left(\frac{72}{\pi}\right)\sum_{\substack{kj,k'j'\\k_1j_1,k_2j_2}}P_{\alpha\gamma,\beta\lambda}^{(1)}\binom{k\ k'}{j\ j'}C\binom{k-k'}{j\ j'}\in\binom{k}{j}\left[\xi_{kk_1k_2}^{(1)}+16\chi_{kk_1}\xi_{kk_1k_2}^{(1)}\right]S_1^{(\pm)}(\alpha) \tag{26b}$$

$$i_{\alpha\gamma,\beta\lambda}^{(2)}(\in_R)_{\text{SOR}}=\left(\frac{1}{2\pi}\right)\sum_{k_1j_1,k_2j_2}P_{\alpha\gamma,\beta\lambda}^{(2)}\binom{k_1\ k_2-k_1-k_2}{j_1\ j_2\ \ j_1\ \ j_2}\eta_1\left[\tilde{n}(\tilde{\in}_{+\alpha})S_1^{(+)}+\tilde{n}(\tilde{\in}_{-\alpha})S_1^{(-)}\right] \tag{26c}$$

with $\quad \xi_{kk_1k_2}^{(1)}=\sum_{\substack{kj,k_1j_1,\\k_2j_2}}V_3\binom{k_1\ k_2-k}{j_1\ j_2\ \ j}V_3\binom{-k_1-k_2-k}{j_1\ \ j_2\ \ j}\eta_1\,;\quad S_1^{(\pm)}(\alpha)=\frac{\tilde{n}(\tilde{\in}_{+\alpha})S_{+\alpha}}{\tilde{\in}_{+\alpha}\left[\tilde{\in}_{+\alpha}^2-\overline{\in}^2\binom{k}{j}\right]^{-2}}+\frac{\tilde{n}(\tilde{\in}_{-\alpha})S_{-\alpha}}{\tilde{\in}_{-\alpha}\left[\tilde{\in}_{-\alpha}^2-\overline{\in}^2\binom{k}{j}\right]^{-2}}$

$$\chi_{kk_1} = C\binom{k_1 - k}{j_1 \; j} C\binom{-k_1 - k}{j_1 \; j} \in^{-2}\binom{k}{j} \; ; \; \tilde{n}(\in_{\pm\alpha}) = \frac{1}{[\exp(\beta \in_{\pm\alpha}) - 1]} \; ; \tilde{\in}_{\pm\alpha} = \tilde{\in}\binom{k_1}{j_1} \pm \tilde{\in}\binom{k_2}{j_2} \text{ and } S_{\pm\alpha} = n\binom{k_2}{j_2} \pm n\binom{k_1}{j_1}$$

The SOR cross section can be obtained in the form

$$\left(\frac{d^2\sigma}{d\Omega\, d \in_R}\right)_{\text{SOR}} = \left(\frac{\in}{c}\right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta \left[i^{1,d}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{SOR}} + i^{1,nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{SOR}} + i^{(2)}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{SOR}} \right] \qquad (27)$$

Third-order Raman Scattering

After certain simplifications the third-order Raman scattering tensor takes the form

$$i_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} = i^{1,d}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} + i^{1,nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} + i^{(3)}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} \qquad (28)$$

where in the first term the first-order polarizability is involved and the second term appears due to the fluctuations of the electronic polarizability in its third-order derivative; these terms are given by

$$i^{1,d}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} = \left(\frac{48}{\pi}\right) \sum_{\substack{kj=k'j' \\ k_1 j_1, \dots, k_3 j_3}} P^{(1)}_{\alpha\gamma,\beta\lambda}\binom{k-k}{j \; j} \in^2\binom{k}{j}\left[\xi^{(2)}_{kk_1 k_2 k_3} + 16\chi_{kk_1}\xi^{(2)}_{kk_1 k_2 k_3} \right] S_2(\beta) \qquad (29)$$

$$i^{1,nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} = \left(\frac{192}{\pi}\right) \sum_{kj \neq k'j'} \sum_{k_1 j_1, \dots, k_3 j_3} P^{(1)}_{\alpha\gamma,\beta\lambda}\binom{k \; k'}{j \; j'} C\binom{k-k'}{j \; j'} \in\binom{k}{j}\left[\xi^{(2)}_{kk_1 k_2 k_3} + 16\chi_{kk_1}\xi^{(2)}_{kk_1 k_2 k_3} \right] \left[S_2(\beta) \right] \qquad (30)$$

$$i^{(3)}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} = \left(\frac{1}{2\pi}\right) \sum_{\substack{k_1 j_1, \\ k_2 j_2, k_3 j_3}} P^{(3)}_{\alpha\gamma,\beta\lambda}\binom{k_1 \; k_2 \; k_3 \; -k_1 \; -k_2 \; -k_3}{j_1 \; j_2 \; j_3 \; j_1 \; j_2 \; j_3} \eta_2\left[\tilde{n}_1 S_1 + \tilde{n}_2 S_2 + \tilde{n}_3 S_3 + \tilde{n}_4 S_4 \right] \qquad (31)$$

The third-order Raman scattering cross section can also be obtained in the form

$$\left(\frac{d^2\sigma}{d\Omega\, d \in_R}\right)_{\text{TOR}} = \left(\frac{\in}{c}\right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta \left[i^{1,d}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} + i^{1,nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} + i^{(3)}_{\alpha\gamma,\beta\lambda}(\in_R)_{\text{TOR}} \right] \qquad (32)$$

Electron-Phonon Contribution Raman scattering

The electron-phonon Raman scattering can be obtained from Eq.(20) via using the electron-phonon contribution in density of states

$$i_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} = i^d_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} + i^{nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} \qquad (33)$$

where

$$i^d_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} = \left(\frac{2}{\pi}\right) \sum_{\substack{kj \\ kj=k'j'}} P^{(1)}_{\alpha\gamma,\beta\lambda}\binom{k-k}{j \; j} g^2\binom{k}{j} \frac{\tilde{n}(2\tilde{\in}_Q)N_Q}{\left[4\tilde{\in}_Q^2 - \overline{\in}^2\binom{k}{j}\right]^2} \qquad (34a)$$

$$i^{nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} = \left(\frac{8}{\pi}\right) \sum_{\substack{kj,k'j' \\ kj \neq k'j'}} P^{(1)}_{\alpha\gamma,\beta\lambda}\binom{k \; k'}{j \; j'} C\binom{k-k}{j \; j} g^2\binom{k}{j} \frac{\tilde{n}(2\tilde{\in}_Q)N_Q}{\in\binom{k}{j}\left[4\tilde{\in}_Q^2 - \overline{\in}^2\binom{k}{j}\right]^2} \qquad (34b)$$

The electron-phonon Raman scattering cross section can be written as

$$\left(\frac{d^2\sigma}{d\Omega\, d \in_R}\right)_{ep} = \left(\frac{\in}{c}\right)^4 \sum_{\alpha\beta,\gamma\lambda} m_\gamma m_\beta n_\alpha n_\beta \left[i^d_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} + i^{nd}_{\alpha\gamma,\beta\lambda}(\in_R)_{ep} \right] \qquad (35)$$

II. CONCLUSION

The phonon Raman scattering in High temperature superconductors can be successfully explained with the help of present theory. It is observed that the FOR scattering provides only one Stokes component and one anti-Stokes component and is induced due to processes in which fluctuations in the first order electronic polarizability are encountered. This scattering is governed by the defect terms only. The Eq.(26c) is purely anharmonic and mainly influenced by the temperature dependent distribution functions $\tilde{n}(\tilde{\in}_{\pm\alpha})$ and $S_1^{(\pm)}(\alpha)$. The SOR spectrum simultaneously depends on the one-phonon density of states via $i_{\alpha\gamma,\beta\lambda}^{1,d}(\in_R)_{SOR}$ and $i_{\alpha\gamma,\beta\lambda}^{1,nd}(\in_R)_{SOR}$ and two phonon density of states via $i_{\alpha\gamma,\beta\lambda}^{(2)}(\in_R)_{SOR}$ The intensity of TOR lines is also affected by the temperature. The electron-phonon Raman scattering can be obtained via using the electron-phonon contribution in density of states and separated into two classes. These classes are also temperature dependent via $\tilde{n}(2\tilde{\in}_Q)$ and N_Q.

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