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On the Problem of Solution of Non-Linear (Exponential) Diophantine Equation $\boldsymbol{\beta^x + (\beta + 18)^y = z^2}$

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Abstract Diophantine equations have great importance in research and thus among researchers. Algebraic equations with integer coefficients having integer solutions are Diophantine equations. For tackling the Diophantine equations, there is no universal method available. So, researchers are keenly interested in developing new methods for solving these equations. While handling any such equation, three issue arises, that is whether the problem is solvable or not; if solvable, possible number of solutions and lastly to find the complete solutions. Fermat's equation and Pell's equation are most popularly known as Diophantine equations. Diophantine equations are most often used in the field of algebra, coordinate geometry, group theory, linear algebra, trigonometry, cryptography and apart from them, one can even define the number of rational points on circle. In the present manuscript, the authors demonstrated the problem of existence of a solution of a non-linear (exponential) Diophantine equation $\beta^x + (\beta + 18)^y = z^2$, where x, y, z are non-negative integers and $\beta, (\beta + 18)$ are primes such that β has the form $6n + 1$ of a natural number n. After it, authors also discussed some corollaries as special cases of the equation $\beta^x + (\beta + 18)^y = z^2$ in detail. Results of the present manuscript depict that the equation of the study is not satisfied by the non-negative integer values of the unknowns x, y and z. The present methodology of this paper suggests a new way of solving the Diophantine equation especially for academicians, researchers and people interested in the same field.

Keywords Solution, Prime Number, Existence, Diophantine Equation, Rational Points

Mathematics Subject Classification 11D61, 11D45, 11D72.

1. Introduction

Diophantine equations play a plausible role in providing the answers to the problems of balancing the equations of chemical reactions, word problems of algebra and showing the irrationality of a number. The decomposition method, method of inequalities, mathematical induction method, parametric method, method of infinite descent and modular arithmetic method are well known methods for solving particular Diophantine equations and are very well documented in the literature [1].

Acu [2] considered the Diophantine equation $2^x + 5^y = z^2$ and showed that this equation has only two solutions. Kumar et al. [3] have considered the equations $61^x + 67^y = z^2$ and $67^x + 73^y = z^2$ in their study and determined that these equations are not solvable for non-negative integer values of x, y and z. Kumar et al. [4] have tackled two equations $31^x + 41^y = z^2$ and $61^x + 71^y = z^2$ and proved that these equations are not satisfied by the non-negative integer values of x, y and z. Existence of irrational numbers is easily proved by the help of Diophantine equations [5, 7].

Rabago [6] studied the Diophantine equation $8^x + p^y = z^2$, where x, y, z belongs to the set of positive integers, and determined its three solutions for a particular value of $p = 17$. These three solutions are given by $\{x = 1, y = 1, z = 5\}$, $\{x = 2, y = 1, z = 9\}$ and $\{x = 3, y = 1, z = 23\}$. Sroysang [8-9] studied the two Diophantine equations $8^x + 19^y = z^2$ and $8^x + 13^y = z^2$. He showed that $\{x = 1, y = 0, z = 3\}$ is the only solution to these equations. Sroysang [10] used Catalan conjectures and examined the equation $31^x + 32^y = z^2$ for its solution. Aggarwal et al. [11] studied the Diophantine equation $181^x + 199^y = z^2$.

Aggarwal et al. [12] applied modular arithmetic method to the equation $223^x + 241^y = z^2$ and showed that there are no non-negative integer values of x, y and z that satisfy the equation $223^x + 241^y = z^2$. Gupta and Kumar [13] studied exponential Diophantine equation $n^x + (n + 3m)^y = z^{2k}$. Kumar et al. [14] proved that there are no non-negative integer values of x, y and z that satisfy the equation $601^p + 619^q = r^2$. Mishra et al. [15] examined the equation $211^\alpha + 229^\beta = \gamma^2$ and determined that the equation $211^\alpha + 229^\beta = \gamma^2$ is not satisfied by the non-negative integer values of x, y and z.

Modular arithmetic method was used by Bhatnagar and Aggarwal [16] for examining the nature of solution of equation $421^p + 439^q = r^2$. The Diophantine equation ${M_5}^p + {M_7}^q = r^2$ was examined for the existence of its solution by Goel et al. [17]. Kumar et al. [18] used modular arithmetic method and studied the problem of existence of the non-negative integer's solution of the equation $(2^{2m+1} - 1) + (6^{r+1} + 1)^n = \omega^2$. Kumar et al. [19] observed the equation $[(7^{2m}) + (6r + 1)^n = z^2]$ and showed that there are no non-negative integer values of unknowns that satisfied the equation $[(7^{2m}) + (6r + 1)^n = z^2]$. The equation $379^x + 397^y = z^2$ was studied by Aggarwal and Sharma [20] using modular arithmetic method.

Aggarwal and Kumar [21] have considered the equation $[19]^{2m} + [2^{2r+1} - 1] = \rho^2$ in their study and proved that the unknowns appearing in equation $[19]^{2m} + [2^{2r+1} - 1] = \rho^2$ have no non-negative integer values. Aggarwal and Kumar [22] examined the equation $(19^{2m}) + (12\gamma + 1)^n = \rho^2$ using famous modular arithmetic method. Aggarwal [23] has considered the equation $(2^{2m+1} - 1) + (13)^n = z^2$ in his study and proved that the unknowns that appear in this equation have no non-negative integer values. Aggarwal and Kumar [24] tackled the equation $(19^{2m}) + (6^{\gamma+1} + 1)^n = \rho^2$ and told that there are no non-negative integer values of unknowns that satisfied the equation $(19^{2m}) + (6^{\gamma+1} + 1)^n = \rho^2$.

Aggarwal and other scholars [25-27] have considered the equations $193^x + 211^y = z^2, 313^x + 331^y = z^2$ and $331^x + 349^y = z^2$. They determined that there are no non-negative integer values of x, y and z that satisfy all these three equations. Aggarwal and Kumar [28] proved that the equation ${M_3}^p + {M_5}^q = r^2$, where M_3, M_5 are Mersenne primes, has no non-negative integer values of p, q and r that satisfy it. Aggarwal and Kumar [29] used modular arithmetic method and studied the equation $(19^{2m}) + (6\gamma + 1)^n = \rho^2$. Kumar and Aggarwal [30] applied modular arithmetic method to the equation $439^p + 457^q = r^2$ and proved that the equation $439^p + 457^q = r^2$ has no non-negative integer values of p, q and r that satisfy it.

The aim of the present paper is to demonstrate the problem of existence of the solution of exponential non-linear Diophantine equation $\beta^x + (\beta + 18)^y = z^2$, where x, y, z are non-negative integers and $\beta, (\beta + 18)$ are primes such that β has the form $6n + 1$ with natural number n.

2. Notation

I_0 : The set of non-negative integers
$\equiv$: Congruent to
N : The set of natural numbers
$<$: Less than
I^+ : The set of positive integers
$\Rightarrow$: Implies
O : The set of odd numbers
$\in$: Belongs to
$\forall$: For all
E : The set of even numbers

3. Preliminaries

LEMMA*: 1* *If β is a prime number of the form $6n + 1$ with $n \in N$ then the Diophantine equation $\beta^x + 1 = z^2, \forall\ x, z \in I_0$, has no solution in I_0.*

PROOF: According to our hypothesis, β is a prime number of the form $6n + 1$with natural number n so the prime $\beta \in O$.

$$\Rightarrow \beta^x \in O, \forall\ x \in I_0$$

$$\Rightarrow \beta^x + 1 = z^2 \in E, \forall\ x \in I_0$$

$$\Rightarrow z \in E$$

$$\Rightarrow z^2 \equiv 0(mod 3) \text{ or } z^2 \equiv 1(mod 3) \quad (1)$$

Now β is a prime number of the form $6n + 1$ with $n \in N$

$$\Rightarrow \beta \equiv 1(mod 6)$$

$$\Rightarrow \beta \equiv 1(mod 3)$$

$$\Rightarrow \beta^x \equiv 1(mod 3), \forall\ x \in I_0$$

$$\Rightarrow \beta^x + 1 \equiv 2(mod 3), \forall\ x \in I_0$$

$$\Rightarrow z^2 \equiv 2(mod 3) \quad (2)$$

Equation (2) contradicts equation (1). So, there is no solution of non-linear Diophantine equation $\beta^x + 1 = z^2$ *in* I_0.

LEMMA*: 2* *If β is a prime number of the form $6n + 1$ with $n \in N$ then the equation $(\beta + 18)^y + 1 = z^2, \forall\ y, z \in I_0$, has no solution in I_0.*

PROOF: According to our hypothesis, β is a prime number of the form $6n + 1$with natural number n so the prime $\beta \in O$.

$$\Rightarrow (\beta + 18)^y \in O, \forall\ y \in I_0$$

$$\Rightarrow (\beta + 18)^y + 1 = z^2 \in E, \forall\ y \in I_0$$

$$\Rightarrow z \in E$$

$$\Rightarrow z^2 \equiv 0(mod3) \text{ or } z^2 \equiv 1(mod3) \quad (3)$$

Now, β is a prime number of the form $6n + 1$ with $n \in N$

$$\Rightarrow \beta \equiv 1(mod6)$$

$$\Rightarrow \beta \equiv 1(mod3)$$

$$\Rightarrow \beta + 18 \equiv 1(mod3)$$

$$\Rightarrow (\beta + 18)^y \equiv 1(mod3), \forall\ y \in I_0$$

$$\Rightarrow (\beta + 18)^y + 1 \equiv 2(mod3), \forall\ y \in I_0$$

$$\Rightarrow z^2 \equiv 2(mod3) \quad (4)$$

Equation (4) contradicts equation (3). So, there is no solution of non-linear Diophantine equation $(\beta + 18)^y + 1 = z^2$ *in* I_0.

MAIN THEOREM: If $\beta, (\beta + 18)$ are prime numbers such that β has the form $6n + 1$ with natural number n then the Diophantine equation $\beta^x + (\beta + 18)^y = z^2$, where $x, y, z \in I_0$, has no solution in I_0.

PROOF: The proof of the above theorem is divided in four cases:

1. If $x = 0$ then the equation $\beta^x + (\beta + 18)^y = z^2$ becomes $1 + (\beta + 18)^y = z^2$. This equation has no solution in I_0 using lemma 2.
2. If $y = 0$ then the equation $\beta^x + (\beta + 18)^y = z^2$ becomes $\beta^x + 1 = z^2$. This equation has no solution in I_0 using lemma 1.
3. If $x, y \in I^+$, then $\beta^x, (\beta + 18)^y \in O$

$$\Rightarrow \beta^x + (\beta + 18)^y = z^2 \in E, \forall \quad x, y \in I^+$$

$$\Rightarrow z \in E$$

$$\Rightarrow z^2 \equiv 0(mod3) \textit{ or } z^2 \equiv 1(mod3) \quad (5)$$

Now, β is a prime number of the form $6n + 1$ with $n \in N$

$$\Rightarrow \beta \equiv 1(mod6)$$

$$\Rightarrow \beta \equiv 1(mod3)$$

$$\Rightarrow \beta^x \equiv 1(mod3) \textit{ and } \beta + 18 \equiv 1(mod3)$$

$$\Rightarrow \beta^x \equiv 1(mod3) \textit{ and } (\beta + 18)^y \equiv 1(mod3)$$

$$\Rightarrow \beta^x + (\beta + 18)^y \equiv 2(mod3)$$

$$\Rightarrow z^2 \equiv 2(mod3) \quad (6)$$

Equation (6) contradicts equation (5). So, the equation $\beta^x + (\beta + 18)^y = z^2$, where $x, y \in I^+$and $z \in I_0$, has no solution in I_0.

4. If x, y both are equal to zero then Diophantine equation $\beta^x + (\beta + 18)^y = z^2$ becomes $z^2 = 2$, which is impossible due to the nature of z. Hence Diophantine equation $\beta^x + (\beta + 18)^y = z^2$, where x, y are both are equal to zero and $z \in I_0$, has no solution in I_0.

COROLLARY: 1 If β is a prime number of the form $6n + 1$ with natural number n then the Diophantine equation $\beta^x + 1 = z^{2k}, \forall\ x, z \in I_0, k \in I^+$, has no solution in I_0.

PROOF: Let $z^k = w \in I_0$, then the Diophantine equation $\beta^x + 1 = z^{2k}$ becomes $\beta^x + 1 = w^2$, which has no solution in I_0 using lemma 1.

COROLLARY: 2 If β is a prime number of the form $6n + 1$ with natural number n then the Diophantine equation $\beta^x + 1 = z^{2(k+1)}, \forall\ x, z, k \in I_0$, has no solution in I_0.

PROOF: Let $z^{k+1} = w \in I_0$, then the Diophantine equation $\beta^x + 1 = z^{2(k+1)}$ becomes $\beta^x + 1 = w^2$, which has no solution in I_0 using lemma 1.

COROLLARY: 3 If β is a prime number of the form $6n + 1$ with $n \in N$ then the equation $(\beta + 18)^y + 1 = z^{2k}, \forall\ y, z \in I_0, k \in I^+$, has no solution in I_0.

PROOF: Let $z^k = w \in I_0$, then the Diophantine equation $(\beta + 18)^y + 1 = z^{2k}$ becomes $(\beta + 18)^y + 1 = w^2$, which has no solution in I_0 using lemma 2.

COROLLARY: 4 If β is a prime number of the form $6n + 1$ with natural number n then the Diophantine equation $(\beta + 18)^y + 1 = z^{2(k+1)}, \forall\ y, z, k \in I_0$, has no solution in I_0.

PROOF: Let $z^{k+1} = w \in I_0$, then the Diophantine equation $(\beta + 18)^y + 1 = z^{2(k+1)}$ becomes $(\beta + 18)^y + 1 = w^2$, which has no solution in I_0 using lemma 2.

COROLLARY: 5 If $\beta, (\beta + 18)$ are prime numbers such that β has the form $6n + 1$ with natural number n then the Diophantine equation $\beta^x + (\beta + 18)^y = z^{2k}$, where $x, y, z \in I_0, k \in I^+$, has no solution in I_0.

PROOF: Let $z^k = w \in I_0$, then the Diophantine equation $\beta^x + (\beta + 18)^y = z^{2k}$ becomes $\beta^x + (\beta + 18)^y = w^2$, which has no solution in I_0 using our main theorem.

COROLLARY: 6 If $\beta, (\beta + 18)$ are prime numbers such that β has the form $6n + 1$ with natural number n then the Diophantine equation $\beta^x + (\beta + 18)^y = z^{2(k+1)}$, where $x, y, z, k \in I_0$, has no solution in I_0.

PROOF: Let $z^{k+1} = w \in I_0$, then the Diophantine equation $\beta^x + (\beta + 18)^y = z^{2(k+1)}$ becomes $\beta^x + (\beta + 18)^y = w^2$, which has no solution in I_0 using our main theorem.

REMARK: The existence of solutions (non-negative integer) of Diophantine equations $\beta^x + (\beta + 18)^y = z^2$, where x, y, z are non-negative integers, $\beta < 1000$ and $\beta, (\beta + 18)$ are primes such that β has the form $6n + 1$ with natural number n are presented in Table 1.

Table 1. Existence of non-negative integer solutions of Diophantine equations $\beta^x + (\beta + 18)^y = z^2$ for $\beta < 1000$

S.N.	n	$\beta = 6n + 1 = \text{prime}$	$(\beta + 18) = \text{prime}$	$\beta^x + (\beta + 18)^y = z^2, x, y, z$ are non − negative integers	Existence of non-negative integer solution
1	2	13	31	$13^x + 31^y = z^2$	No solution
2	3	19	37	$19^x + 37^y = z^2$	No solution
3	7	43	61	$43^x + 61^y = z^2$	No solution
4	10	61	79	$61^x + 79^y = z^2$	No solution
5	13	79	97	$79^x + 97^y = z^2$	No solution
6	18	109	127	$109^x + 127^y = z^2$	No solution
7	23	139	157	$139^x + 157^y = z^2$	No solution
8	27	163	181	$163^x + 181^y = z^2$	No solution
9	30	181	199	$181^x + 199^y = z^2$	No solution
10	32	193	211	$193^x + 211^y = z^2$	No solution
11	35	211	229	$211^x + 229^y = z^2$	No solution
12	37	223	241	$223^x + 241^y = z^2$	No solution
13	52	313	331	$313^x + 331^y = z^2$	No solution
14	55	331	349	$331^x + 349^y = z^2$	No solution
15	58	349	367	$349^x + 367^y = z^2$	No solution
16	63	379	397	$379^x + 397^y = z^2$	No solution
17	70	421	439	$421^x + 439^y = z^2$	No solution
18	73	439	457	$439^x + 457^y = z^2$	No solution
19	87	523	541	$523^x + 541^y = z^2$	No solution
20	100	601	619	$601^x + 619^y = z^2$	No solution
21	102	613	631	$613^x + 631^y = z^2$	No solution
22	107	643	661	$643^x + 661^y = z^2$	No solution
23	112	673	691	$673^x + 691^y = z^2$	No solution
24	115	691	709	$691^x + 709^y = z^2$	No solution
25	118	709	727	$709^x + 727^y = z^2$	No solution
26	122	733	751	$733^x + 751^y = z^2$	No solution
27	123	739	757	$739^x + 757^y = z^2$	No solution
28	125	751	769	$751^x + 769^y = z^2$	No solution
29	128	769	787	$769^x + 787^y = z^2$	No solution
30	135	811	829	$811^x + 829^y = z^2$	No solution
31	143	859	877	$859^x + 877^y = z^2$	No solution
32	153	919	937	$919^x + 937^y = z^2$	No solution
33	165	991	1009	$991^x + 1009^y = z^2$	No solution

4. Conclusions

In the present manuscript, authors successfully demonstrate the problem of existence of solution to Diophantine equation (non-linear) $\beta^x+(\beta+18)^y=z^2$, where x,y,z are non-negative integers and $\beta,(\beta+18)$ are prime numbers such that β has the form $6n+1$ with natural number n. Results indicate that Diophantine equation $\beta^x+(\beta+18)^y=z^2$, where x,y,z are non-negative integers and $\beta,(\beta+18)$ are prime numbers such that β has the form $6n+1$ with natural number n, has no solution in I_0.

Data Availability

Authors confirm that the data-sets that are used to support the finding of this paper are available from the author upon request.

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