International Journal of Mathematics (IJMM)
Volume 3, Issue 1, January-June 2024, pp. 1-7. Article ID: IJMM_03_01_001
Available online at https://iaeme.com/Home/issue/IJMM?Volume=3&Issue=1
ISSN Online: 2291-9598, Journal ID: 4011-0808
Impact Factor (2024): 18.34 (Based on Google Scholar Citation)

EFFECT OF VARIABLE VISCOSITY ON LINEAR CONVECTION IN ROTATING FERROMAGNETIC LIQUIDS UNDER GRAVITY CONDITION

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ABSTRACT

Linear stability analyses of Ferro convection in rotating Ferromagnetic liquids with variable viscosity have been studied numerically using the generalized Lorenz model. The linear stability is studied to discuss influence of variable viscosity in terrestrial gravity condition. Linear stability is studied effects of various parameters which include Taylor number, Ta, variable viscosity, V, internal Rayleigh number, R_I, buoyancy-magnetization parameter, M_1, non - buoyancy magnetization parameter, M_3 have been studied. We notice that the variable viscosity and heat source suppress the convection. However, due to the presence of variable viscosity, V, there is a change in the onset of Ferro convection.

Keywords: Rayleigh-Bénard Convection, Rotation, Variable Viscosity, Generalized Lorenz Model.

Cite this Article: Roopa G and A.Pranesha Setty, Effect of Variable Viscosity on Linear Convection in Rotating Ferromagnetic Liquids Under Gravity Condition, International Journal of Mathematics (IJMM), 2(1), 2023, pp. 1–7.
https://iaeme.com/Home/issue/IJMM?Volume=3&Issue=1

INTRODUCTION

Rayleigh-Bénard convection of ferromagnetic liquid in a rotating enclosure is a highly explored phenomenon in geophysics and astrophysics. It has a lot of industrial applications such as chemical engineering, geophysics, biomechanics etc. Ferro convection brings new applications in motion cooling, loud speakers, line transmission, and other equipment that already have a magnetic field. Ferro fluid is a form of functional fluid whose flow and energy transport processes can be regulated by modifying an external magnetic field, making it useful in sectors like electronic packaging, mechanical engineering, aerospace, bioengineering, and thermal engineering, among others.

By combining suspended single domain particles, Neuringer and Rosensweig [1] had discussed the flows of ferromagnetic liquids with a rotating magnetic field. Onset of convection, Niiler and Bisshop [2] had analyzed the variation of the Coriolis force in a shallow level for FIFI & RIRI boundaries. Rosensweig et al. [3] had studied the consistency of the liquids using the presence magnetic intensity assumed by dimensional stability. Finlayson [4] was researching convective instability in a ferromagnetic liquid in detail. Results on ferroconvection were studied by Gupta and Gupta [5]. The results of Ferro fluid layer heated from the bottom were discussed by Qin and Kaloni [6] on the theory of both linear and non-linear of combined buoyancy force. Venkatasubramanian and Kaloni [7] made a detailed study about rotation on boundaries of stress-free, rigid-paramagnetic and rigid-ferromagnetic to analyze consequences of instability on thermo-convective in the Ferro fluid layer horizontally. Natural convection in a revolving layer of a magnetic Ferro fluid was studied by Auernhammer and Brand [8]. Ganguly et al. [9] discussed the heat dissipation in ferromagnetic fluids and notice that the heat dissipation expansion due to thermal convection.

Siddheshwar and Abraham [10] used analytical methods to analyze Ferro convection in a micro polar magnetic fluid layer, concluding that micro polar ferromagnetic liquids are more reliable than Newtonian ferromagnetic liquids. Shivakumara [11] had studied velocity and temperature boundary at the quiescent Ferro fluid layer on the onset of ferro convection. The Bénard Marangoni ferroconvection with internal heat generation by applying a uniform vertical magnetic field was explored by Nanjundappa et al. [12]. Stability of linear and non-linear theory by using the rotating nanofluids-saturated porous medium with local thermal non-equilibrium effects were studied by Vanishree and Siddheshwar [13]. Each equilibrium's asymptotic stability was investigated by Kaloni and Mahajan [14]. For all feasible boundary combinations, Sekhar et al. [15], Siddheshwar et al. [16], Siddheshwar et al. [17] examined the effects of temperature-dependent viscosity on Bénard Marangoni magneto convection. Bénard-Marangoni Ferro convection in rotating layer of Ferro fluid boundary was researched by Shivakumara et al. [18].

Mahajan and Arora [19] investigated the instability of convection in a thin layer of a nano fluid using rotation. The effect of magnetic and non-magnetic variables on Marangoni convection was investigated by Sekhar et al. [21]. In a Brinkman porous media, Nanjundappa et al. [22] Investigated the effects of cubic temperature profiles on ferro convection. For all feasible boundary combinations, Sekhar et al. [23] investigated a linear stability analysis of thermal convection in variable viscosity. Many studies have been conducted on the Rayleigh-Bénard convection of a ferromagnetic liquid in revolving enclosures. The influence of MFD viscosity on Benard-Marangoni Ferro convection on viscosity in a rotating Ferro fluid layer was investigated by Arunkumar et al. (2018).

On a rotating anisotropic Ferro fluid layer, Amit et al. (2019) investigated both linear and weakly non-linear stability studies. The finite-amplitude equation was solved using the Runge-Kutta- Gill numerical method. Anthony et al. (2020) used the Lorenz model to investigate the effects of various factors on heat transfer in a nonlinear investigation of the influence of rigid body rotation on Ferro convection.

The major aim of this analysis is to examine at the stability of both linear and non-linear Coriolis force systems with varying ferroconvection viscosity, as well as a numerical analysis of how rotation affects convection and heat transfer in ferromagnetic liquids. The normal mode approach is used to investigate linear stability. The Lorenz model was used to do the non-linear stability study. It has been discussed how many physical characteristics affect heat transport.

MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of an electrically non-conducting incompressible ferromagnetic fluid with a thickness of 'd' that is permeated by a uniform applied magnetic field H_0 acting in the vertical direction. The layer is rotating uniformly about its vertical axis with constant angular velocity $\vec{\Omega} = \Omega\,\hat{k}$. The lower and upper surfaces $T_0 + \Delta T$ and T_0 respectively, are maintained at constant temperature. The equations defining for the motion of the fluid are the continuity equation, momentum equation, transport equation and Maxwell's equations (Finlayson [4] and Gupta and Gupta [5]). In the continuity equation $\vec{q} = (u, v, w)$ is the velocity vector. The momentum equation is containing viscous force $\nabla.[\mu(H,T)(\nabla\vec{q} + \nabla.\vec{q}^{Tr})]$ in the rotating frame and $(\nabla\vec{q} + \nabla.\vec{q}^{Tr})$ is the rate of strain tensor. (See Fig.1).

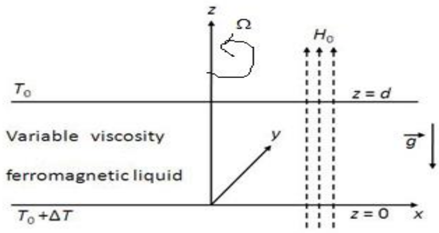


Figure 1: The physical representation of the proble

The basic governing equations for the present problem describe:

Continuity equation:

$$\nabla.\vec{q} = 0, \tag{1}$$

Conservation of linear momentum**:**

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\,\vec{q}\right] = -\nabla p + \rho\,\vec{g} + \mu_0[\vec{M}.\nabla]\,\vec{H} + \nabla.[\mu(H,T)(\nabla\vec{q} + \nabla.\vec{q}^{Tr})] + 2\,\rho_0\,(\vec{q}\times\Omega), \tag{2}$$

In equation (2), third term represents the ponder motive force. In the last term density 'ρ_0' is a constant. T is the temperature, $\vec{H}$ is the magnetic field, $\vec{M}$ is the magnetization, p is the pressure.

Energy balance equation:

$$\rho_0\left[\frac{\partial T}{\partial t} + (\vec{q}.\nabla)\,T\right] = \kappa\,\nabla^2 T + Q\,(T - T_0), \tag{3}$$

Where 'κ' is the thermal conductivity and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator.

Equation of the state**:**

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{4}$$

where $'\alpha'$ is the thermal expansion coefficient.

Maxwell equations**:**

$$\nabla.\vec{B} = 0,\ (\nabla\times\vec{H}) = 0 \text{ or } \vec{H} = -\nabla\Phi \tag{5}$$

Magnetic induction of a state**:**

$$\vec{B} = \mu_0[\vec{M} + \vec{H}], \tag{6}$$

Magnetic equation of a state**:**

$$\vec{M} = M_0 + \chi_m(H - H_0) - k_l(T - T_0), \tag{7}$$

is linearized about the magnetic field H_0 and the temperature T_0. Where $k_l = -\left(\frac{\partial M}{\partial T}\right)_{H_0,T_0}$, the pyro magnetic coefficient and $\chi_m = \left(\frac{\partial M}{\partial H}\right)_{H_0,T_0}$ is the magnetic susceptibility.

$M_0 = M\,(H_{0,}T_0)$, is the constant mean value of magnetization.

Effective viscosity**:**

$$\mu = \mu_0[\,1 + \delta_H(H_b - H_0)^2 - \delta_T(T_b - T_0)^2\,] \tag{8}$$

We have neglected the inertia of the suspended particles and their motion, so that the equations of motion could be tractable. Therefore, in the present study, the effective viscosity is taken to be a quadratic function of magnetic field and temperature.

Quiescent state solution:

The basic state solutions are:

$\vec{q}_b = 0$, $p = p_b(z)$, $\rho = \rho_b(z)$, $\mu = \mu_b(z)$,

$$T = T_b(z), \vec{M} = (0,0,M_b(z)), \vec{H} = (0,0,H_b(z)) \quad (9)$$

Where $T_b(z) = T_1 - \Delta T z$, where T_1 is the constant temperature of the lower boundary = $(T_0 + \Delta T)$.

$\rho_b(z) = \rho_0[1 + \alpha\, \Delta T z]$, $\vec{H}_b(z) = \left[H_0 - \frac{k_l \Delta T z}{(1+\chi_m)}\right]$ and $\vec{M}_b(z) = \left[M_0 + \frac{k_l \Delta T z}{(1+\chi_m)}\right]$.

The effective viscosity reduces to the form:

$$\mu_b(z) = \mu_0\left[1 + \delta_H(H_b - H_0)^2 - \delta_T(T_b - T_0)^2\right]$$

$$= \mu_0\left[1 - \left(\delta_T(\Delta T)^2(1-z)^2 + \left(\frac{\delta_H k_l^2}{1+\chi_m}\right)(\Delta T)^2(1-z)^2\right)\right]$$

$g_1(z) = [1 - V(1-z)^2]$, where

$g_1(z) = \frac{\mu_b(z)}{\mu_0}$ and $f(z) = (1-z)^2$,

$V = \left(\delta_T - \frac{\delta_H k_l^2}{1+\chi_m}\right)(\Delta T)^2$ is the variable viscosity.

After perturbation, we will now have,

$\vec{q} = \vec{q}_b + q'$, $p = p_b + p'$, $\vec{M} = M_b + M'$, $T = T_b + T'$, $\rho = \rho_b + \rho'$, $\vec{H} = H_b + H'$

In the dimensionless formulation scales for length, velocity, time and temperature are taken as

$$x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad z^* = \frac{z}{d}, \quad t^* = \frac{t\chi}{d^2}, \quad w^* = \frac{w\chi}{d}, \quad T^* = \frac{T}{\Delta T},$$

$\Phi^* = \frac{\Phi(1+\chi_m)}{\kappa \Delta T\, d^2}$, $f^*(z) = \frac{f(z)}{\Delta T}$, where Φ is the magnetic scalar potential.

Where * denotes the perturbed quantity. Now we need to remove pressure p from x and y components of eq. (2) and the stream function is defined as

$$u = -\left(\frac{\partial \Psi}{\partial z}\right), \qquad w = \left(\frac{\partial \Psi}{\partial x}\right)$$

We obtain a dimensionless form of governing equation as

$$\frac{1}{Pr} d_t \nabla^2\Psi = -Ra(1+M_1)\frac{\partial T}{\partial x} - RaM_1\left(\frac{\partial^2\Phi}{\partial x \partial z}\right) + RaM_1 J\left(T, \frac{\partial \Phi}{\partial x}\right) + \frac{1}{Pr} J\left(\Psi, \nabla^2\Psi\right) + g_1(z)\nabla^4\Psi + 2D[g_1(z)]\nabla^2\Psi + D^2[g_1(z)]\left(\frac{\partial^2\Phi}{\partial z^2}\right) - D^2[g_1(z)]\left(\frac{\partial^2\Phi}{\partial x^2}\right) - \sqrt{Ta}\frac{\partial v}{\partial z}, \quad (10)$$

Where $\vec{q}' \times \Omega = v$

$$\frac{1}{Pr}\left[\frac{\partial V}{\partial t} + J(\Psi, v)\right] = g_1(z)\nabla^2 v + D[g_1(z)]\frac{\partial v}{\partial z} + \sqrt{Ta}\frac{\partial \Psi}{\partial z}, \quad (11)$$

$$\frac{\partial T}{\partial t} = J(\Psi, T) + \nabla^2 T + R_I T - g_2(z)\frac{\partial \Psi}{\partial x}, \quad (12)$$

$$M_3 \nabla_1^2\Phi + \left(\frac{\partial^2\Phi}{\partial z^2}\right) - \frac{\partial T}{\partial z} = 0. \quad (13)$$

The dimensionless parameters appearing in the above equations are

$$Pr = \frac{\mu_0}{\rho_0\,\chi}, \; Ra = \frac{\alpha\,\rho_0\, g d^3 \Delta T}{\mu_0\,\chi}, \; R_I = \frac{Q_1 d^2}{\kappa \Delta T}, \; M_1 = \frac{\mu_0\, k_1^2 \Delta T}{\alpha\,\rho_0\, g(1+\chi_m)d}, \; M_3 = \frac{1+\frac{M_0}{H_0}}{(1+\chi_m)} \;\&\; Ta = \left(\frac{2\,\rho\Omega d^2}{\mu_0\,\chi_m}\right)^2$$

Eqs. (10) – (13) are solved by using following boundary conditions appropriate for free –free isothermal boundaries. (Vanishree et al.[13]).

$$\Psi = \frac{\partial^2\Psi}{\partial z^2} = T = \frac{\partial v}{\partial z} = \frac{\partial \Phi}{\partial z} = 0, \text{ at } z = (0, 1) \quad (14)$$

STABILITY ANALYSIS OF LINEAR STUDY

Linearized steady state eqs.(10)–(12) are considered to obtain resulting equation

$$Ra\,(1+M_1)\,\frac{\partial T}{\partial x} - RaM_1\left(\frac{\partial^2 \Phi}{\partial x \partial z}\right) + g_1(z)\nabla^4\Psi + 2\,D\,[g_1(z)]\,\nabla^2\Psi - D^2\,[g_1(z)]\left(\frac{\partial^2 \Phi}{\partial x^2}\right) - \sqrt{Ta}\,\frac{\partial v}{\partial z} = 0, \quad (15)$$

$$g_1(z)\nabla^2 v + D\,[g_1(z)]\frac{\partial v}{\partial z} + \sqrt{Ta}\,\frac{\partial \Psi}{\partial z} = 0, \quad (16)$$

$$\nabla^2 T + R_I T - g_2(z)\,\frac{\partial \Psi}{\partial x} = 0, \quad (17)$$

$$M_3\,\nabla_1^2\Phi + \left(\frac{\partial^2 \Phi}{\partial z^2}\right) - \frac{\partial T}{\partial z} = 0. \quad (18)$$

We have the following time derivative equations.

$$-q^2 f(V_1)A(\tau) - \frac{R\,k^2}{q^2}\left[\frac{k^2M_3+k^2M_1M_3+\pi^2}{k^2M_3+\pi^2}\right]B(\tau) + \frac{\sqrt{Ta}\pi k}{q^2}E(\tau) = 0, \quad (19)$$

$$\frac{4\pi^2}{(4\pi^2-R_I)}A(\tau) + (R_I - q^2)\,B(\tau) = 0, \quad (20)$$

$$-f(V_2)E(\tau) + \frac{\sqrt{Ta}\pi}{q^2}A(\tau) = 0. \quad (21)$$

We have considered non-trivial solutions from eqs. (19) - (21) in the following form

$$\begin{vmatrix} -q^2 f(V_1) & \frac{R\,k^2}{q^2}\left[\frac{k^2M_3+k^2M_1M_3+\pi^2}{k^2M_3+\pi^2}\right] & \frac{\sqrt{Ta}\pi k}{q^2} \\ \frac{4\pi^2}{(4\pi^2-R_I)} & (R_I - q^2) & 0 \\ \frac{\sqrt{Ta}\pi}{q^2} & 0 & -f(V_2) \end{vmatrix} = 0 \quad (22)$$

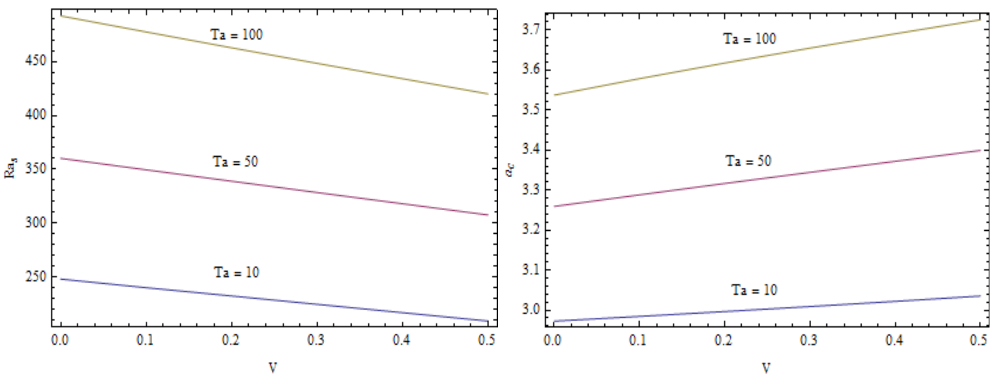
The stationary Rayleigh number Ra_s is obtained in the form

$$Ra_s = \frac{q^6(k^2M_3+\pi^2)}{k^2(k^2M_3+k^2M_1M_3+\pi^2)}\left\{-q^2 f(V_1)\left(\frac{4\pi^2-R_I}{4\pi^2}\right) - \left(\frac{q^2 f(V_1)(4\pi^2-R_I)}{4\pi^2}\right) + \left(\frac{Ta\,\pi^2\,k\,(q^2-R_I)(4\pi^2-R_I)}{q^4 f(V_2)4\pi^2}\right)\right\} \quad 23)$$

Where

$$f(V_1) = 1 + \left(\frac{-1}{3} + \frac{1}{2\pi^2}\right)V + \left(\frac{2\pi^2 V}{q^4}\right) - \left(\frac{2k^2V}{q^4}\right) - \frac{V}{q^2}, \quad f(V_2) = \frac{1}{k} + \left(\frac{-1}{3k} + \frac{1}{2\pi^2 k}\right)V + \frac{kV}{q^2}.$$

The following are the graphs plotted for linear stability analysis.



(a) (b)

Fig 2: Plots of Ra_s and a_c versus V for different with fixed values of $M_1 = 5, M_3 = 1, R_I = 0.1$.

The magnetic numbers are field dependent with $M_1 \sim 10^{-4}$ - 10^{2} and $M_3 \geq 1$ for typical magnetic field strengths. In the stationary convection case 'r' is greater than one. Therefore in the present paper, the values of the parameters, $R_1 = 10$, $M_3 = 1.1$, are chosen for numerical calculations.

CONCLUSION

In the presence of varying viscosity, the effect of Taylor number (Ta) on the stationary Rayleigh number and the corresponding wave number is investigated. In the presence of changing viscosity, increasing Ta lowers the stationary Rayleigh number, implying that the function of Taylor number is to destabilize the system, whereas the wave number has the opposite effect. This indicates that the process is being stabilized by the system.

Increasing the variable viscosity V and Taylor number Ta, the Nusselt number (Nu) decreases. It indicates that increasing these parameters lowers the heat transfer rate. It can be observed for other parameters such as RI, M_1, M_3, the system stabilizes and the amount of the heat transfer reduces. Therefore, we try to control the convection process and also synchronize the transfer of heat with the help of Ferro magnetic liquid and the magnetic field strength.

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