International Journal of Mathematics (IJMM)
Volume 1, Issue 1, January-December 2020, pp. 1-11. Article ID: IJMM_01_01_001
Available online at https://iaeme.com/Home/issue/IJMM?Volume=1&Issue=1
Journal ID: 3365-6114

δ –SHOCK MAINTENANCE MODEL FOR AN IMPROVING SYSTEM UNDER PARTIAL SUM PROCESS

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ABSTRACT

In this paper, a shock model for the maintenance problem of a repairable system is studied. Assume that shocks will arrive according to a Poisson process. If the inter arrival time of two successive shocks is less than a threshold, then the system will fail. For an improving system, we assume that the successive threshold values are geometrically decreasing after repair, and the consecutive repair times after failure form a decreasing partial sum process. A replacement policy N is adopted by which we shall replace the system by an identical new one at the time following the $N-th$failure. Then for an improving system, an optimal policy N^* for minimizing the long run average cost per unit time is determined explicitly.

Keywords: Maintenance Model, Repairable System, Shock Arrival Process, Geometrically Decreasing Threshold Values, Partial Sum Process

Cite this Article: Sutha. M and Sridhar. A, δ-Shock Maintenance Model for An Improving System Under Partial Sum Process, International Journal of Mathematics (IJMM), 1(1), 2020, pp. 1–11.
https://iaeme.com/Home/issue/IJMM?Volume=1&Issue=1

1. INTRODUCTION

Although the study of maintenance problem is an important topic in reliability, most important models just pay attention on the internal cause of a system failure, but do not on an external cause of the system failure. In practice, a system failure may be caused by some external cause, such as a shock. For example, a computer system may fail due to the invasion of some virus or an attack from a raider. As the virus or raider may arrive randomly, it is a stochastic shock. It might be a discrete stochastic shock.

If the virus is benign as it will create a virus program to display some rubbish messages, it might be a continuous stochastic shock if the virus is malignant as it will create a virus program to wipe out data or files in the computer, then the computer system will be paralyzed. In engineering, a precision instrument and meter system may fail due to the effect of operation of other equipment. This might be an example of discrete stochastic shock. On the other hand, a precision instrument and meter system may be installed in an environment with a high temperature and humidity such as in naval vessels. In this case, the system may reduce its lifetime as it operates in an adverse environment. This might be an example of continuous stochastic shock. Therefore we should consider the maintenance problem of a system subject to shocks. A shock is called a deadly shock if the system will fail after suffering such a shock. In 1975, Barlow and Proschan [1] studied this problem by considering the following shock model, whenever a shock arrive, it will cause a random amount of damage to the system. A shock is a deadly shock if the accumulated amount of damage to the system by the time of the shock arriving exceeds a specified threshold, and then the system fails .

Later on, Shantikumar and Sumitha [2,3] studied a more general shock model. Lam and Zhang [4] studied a maintenance model for a deteriorating system subject to shocks.

The shock model has been successfully applied to many different subjects, such as physics, communication, electronic engineering and medicine. As a result, more and more researchers are interested in doing research on this topic.

A δ −shock model is different from the above shock models. It pays attention on the "frequency" of shocks rather than the accumulated amount of damage of shocks. Now, a shock is a deadly shock if the time that elapses from the preceding shock to this shock is smaller than a specified thresholdδ, and then the system fails. In this paper, we shall study a δ- shock maintenance model for an improving system. To do this, at first, we should introduce the definition of stochastic order and the concept of partial sum process.

Definition 1.1

Given two random variables X and Yif $P(X > t) \geq P(Y > t)$ for all real t,

Then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by $X \geq_{st} Y$ or $Y \leq_{st} X$ (See e.g. Ross [5] for reference.)

Definition 1.2.

Given a stochastic process $\{Z_n, n = 1,2 \dots.\}$ if for all n, $Z_n \leq_{st} (\geq_{st}) Z_{n+1}$, then $\{Z_n, n = 1,2 \dots.\}$ is called a stochastically increasing (decreasing) process.

As a simple stochastically monotone process, Babu [1] introduced the following partial sum process.

Definition 1.3.

Let $\{X_n, n = 1,2,3 \dots..\}$be a sequence of independent non-negative random variables and let $F(x)$ be the distribution function of X_1.Then $\{X_n, n = 1,2,3 \dots..\}$ is called a partial sum process, if the distribution function of X_{m+1} is $F(\beta_n x)$, $m = 1,2,3$ ...where $\beta_n > 0$ are constants with $\beta_n = \beta_0 + \beta_1 + \beta_2 + \cdots \beta_{n-1}$ and $\beta_0 = \beta > 0$

According to Definition 1.3.We have

(i) $E\ (X_1) = \mu$ Then for $i = 1,2,3, \dots \dots \dots$

(ii) $E\ (X_{i+1}) = \frac{\mu}{2^{i-1}\beta}$ (1)

(iii) For real β_i $(i = 1,2,3 \dots \dots ..)$ $\beta_i = 2^{i-1}\beta$.

Then the distribution function of X_{i+1} is $F\left(2^{i-1}\beta x\right)$ for $i = 1,2,3, \dots \dots \dots$

The density function of X_{i+1} is $f_{i+1}(x) = \beta_i\, f(\beta_i\, x)$

(iv) The partial sum process $\{X_n\ , n = 1,2,3 \dots ..\}$ with parameter $\beta > 0$ is stochastically decreasing and hence it is a monotone process.

In this paper, we shall study a new δ −shock model for the maintenance problem for an improving system. It is a new model, because the threshold of a deadly shock is not a constant but monotone. Moreover the successive repair times after failure form a partial sum process.

By making different assumptions the model could be applied to improving system. The model is introduced in section 2. Assume further that a replacement policy N is adopted, by which a system is replaced by an identical new one at the time following the $N - th$ failure.

In section 3, the long run average cost per unit time is evaluated. Then in section 4, for an improving system an optimal replacement policy N^* is determined analytically.

2. THE δ –SHOCK MODEL

We introduce the δ –shock model for the maintenance problem of a repairable system by making the following assumptions.

Assumption 1. At the beginning a new system is installed. Whenever the system fails, it will be repaired , the system will be replaced by an identical new one sometime later.

Assumption 2. The repair cost rate is C, the reward rate when the system is operating is r. The replacement cost is R. The replacement cost comprises two parts, one part is the basic replacement cost R, the other part is proportional to the replacement time W, at rate C_p.

Assumption 3. A replacement policy N is adopted by applying a replacement policy N, the system will be replaced by an identical new one at the time following the $N - th$ failure. The replacement time is a random variable W with$E(W) = \tau$.

Assumption 4. The system is subject to a sequence of shocks. The shock will arrive according to a poisson process with rate θ. If the system has been repaired for n times $(n = 0,1,2 \dots)$ The threshold of a deadly shock will be $\alpha^n\delta$ where α $(0 < \alpha \leq 1)$ is the rate and δ is the threshold of a deadly shock for a new system. This means that whenever the time to the first shock is less than or the inter arrival time of two successive shocks after the n^{th} repair is less than $\alpha^n\delta$, the system will fail. During the repair time, the system is closed; this means that any shock arriving when the system is under repair is ineffective.

Assumptions 5. Let X_1 be the repair time after the 1^{st} failure and let $F(x)$be the distribution function of X_1. Then the distribution function of X_{i+1} is $F(2^{i-1}\beta x)$ where $\beta > 0$ are constants, for $i = 1,2,3 \dots$ That is the successive repair times $\{X_{i+1}, i = 1,2 \dots .\}$ after failure constitute a decreasing partial sum process and also assume that $E(X_1) = \mu > 0$ and $E\ (X_{i+1}) = \frac{\mu}{2^{i-1}\beta}$.

Assumption 6. The poisson process and partial sum process are independent.

REMARKS

Usually, whenever a system fails, it needs to wait for repair, In our model, we study the maintenance problem for a system with one repair facility. In this case, the repair facility will repair the system when it fails until is is recovered from failure. Therefore the repair facility will be free if the system is operating. Thus, once the system fails, it could be repaired without delay.

In real life, there do have some improving systems. For example, some systems could be improved, this might be due to the fact that the operator can accumulate the operating experience so that the damage caused by a shock will be lightened, this might be due to the repair facility becoming more familiar with the system. So that the successive repair times might be decreasing. Then, for an improving system, the older the system, the more solid the system is. Thus, the threshold of a deadly shock should be decreasing geometrically, while the successive repair times of the system will constitute a decreasing partial sum process.

3. LONG RUN AVERAGE COST

In our model, we say that a cycle is completed if a replacement is completed. Sice a cycle is actually a time interval between the installation of the system and the first replacement or a time interval between two consecutive replacements, the successive cycles will form a renewal process. Thus, the successive cycles together with the costs incurred in each cycle will consitute a renewal reward process. By applying the standard result in renewal process, the long run average cost per unit time is given by

$$C(N) = \frac{Expected\ cost\ incurred\ in\ a\ cycle}{Expected\ length\ of\ a\ cycle} \quad (2)$$

[See Ross[5] for reference]

Now let X_n be the operating time of the system following the $(n-1)-th$ repair in a cycle, denote the distribution function of X_n by F_n; and let Y_n be the repair time after the $n-th$ failure in the cycle. Suppose a replacement policy N is adopted, let the average cost be $C(N)$. It follows from (1) with the help of (1)

$$C(N) = \frac{E\left(C\sum_{n=1}^{N-1} Y_n - r\sum_{n=1}^{N} X_n + R + C_p W\right)}{E(\sum_{n=1}^{N} X_n + \sum_{n=1}^{N-1} Y_n + W)}$$

$$= \frac{C\sum_{n=1}^{N-1} E(Y_n) - r\sum_{n=1}^{N} E(X_n) + R + C_p E(W)}{\sum_{n=1}^{N} E(X_n) + \sum_{n=1}^{N-1} E(Y_n) + E(W)}$$

$$= \frac{C(\mu + \sum_{n=2}^{N-1} \mu_n) - r\sum_{n=1}^{N} \lambda_n + R + C_p\tau}{\sum_{n=1}^{N} \lambda_n + (\mu + \sum_{n=2}^{N-1} \mu_n) + \tau}$$

$$= \frac{C\left(\mu + \sum_{n=2}^{N-1} \frac{\mu}{2^{n-1}\beta}\right) - r\sum_{n=1}^{N} \lambda_n + R + C_p\tau}{\sum_{n=1}^{N} \lambda_n + \left(\mu + \sum_{n=2}^{N-1} \frac{\mu}{2^{n-1}\beta}\right) + \tau} \quad (3)$$

$$C(N) = \frac{(C+r)\mu\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+R+\left(C_p+r\right)\tau}{\sum_{n=1}^{N}\lambda_n+\left(\mu+\sum_{n=2}^{N-1}\frac{\mu}{2^{n-1}\beta}\right)+\tau} - r \quad (4)$$

$$C(N) = A(N) - r$$

Where $\lambda_n = E(X_n)$ is the operating time following the $(n-1)^{th}$ repair.

Now, we need to evaluate the values of λ_n, $n = 1,2,3 \dots \dots$..

Lemma 3.1. λ_n is non-decreasing in n.

Proof. Let U_{n1} be the arrival time of the first shock following the $(n-1)^{th}$ repair.

In general, let U_{nk} be the interarrival time between the $(k-1)^{th}$ and k^{th} shocks following the $(n-1)^{th}$ repair until the first deadly shock occured .Let $E(U_{11}) = \lambda$. Assume that $\{U_{ni}, i = 1,2 \dots \dots\}$ are independent and identically distributed (i.i.d) sequences for all n.

Let M_n, $n = 1,2,3 \dots$.... Be the number of shocks following the $(n-1)-th$ repair until the first deadly shock occurred.

Then $$M_n = min\{m/U_{n1} \geq \alpha^{n-1}\delta, \dots \dots \dots U_{n\,m-1} \geq \alpha^{n-1}\delta, U_{n\,m} < \alpha^{n-1}\delta\} \quad (5)$$

Let M_n be a random variable with exponential distribution $Exp(\theta)$ with mean $\frac{1}{\theta}$. Then, M_n will have a geometric distribution $G(P_n)$ with parameter

$$P_n = P(U_n < \alpha^{n-1}\delta) = \int_0^{\alpha^{n-1}\delta} \theta e^{-\theta x}\,dx$$

$$= 1 - \exp\left(-\theta\alpha^{n-1}\delta\right)$$

and $q_n = 1 - p_n$ thus $X_n = \sum_{i=1}^{M_n} U_{ni}$ (6)

Now, suppose that $M_n = m$, then $X_n = X_{nm} + U_{nm}$ (7)

with $X_{nm} = \sum_{i=1}^{m-1} U_{ni}$ and $U_{n1} \geq \alpha^{n-1}\delta, \dots \dots \dots . U_{n\,m-1} \geq \alpha^{n-1}\delta$,

but $U_{n\,m} < \alpha^{n-1}\delta$ (8)

Consequently, $X_{nm} = \sum_{i=1}^{m-1}(U_{ni} - \alpha^{n-1}\delta) + (m-1)\alpha^{n-1}\delta$.

Because exponential distribution is memoryless

$U_{ni} = \alpha^{n-1}\delta, i = 1,2 \dots \dots . m-1,$ are i.i.d random variables, each has the same exponential distribution $Exp(\theta)$, as U_n has. This implies that $X_{nm} - (m-1)\alpha^{n-1}\delta$ will have a gamma distribution $\Gamma(m-1,\theta)$. Thus the density g_{nm} of X_{nm} is given by

$$g_{nm}(x) = \begin{cases} \frac{\theta^{m-1}}{(m-2)!}(x-k)^{m-2}e^{-\theta(x-k)} & x > k, \\ 0 & elsewhere \end{cases}$$

(9)

Where $k = (m-1)\alpha^{n-1}\delta,$

As a result

$$E(X_{nm}) = \frac{m-1}{\theta} + (m-1)\alpha^{n-1}\delta \tag{10}$$

On the other hand, Let U_n be an exponentially distributed random variable with mean $\frac{1}{\theta}$ because $U_{nm} < \alpha^{n-1}\delta,$

we have $E(U_{nm}) = E(U_n/U_n < \alpha^{n-1}\delta)$

$$= \int_0^{\alpha^{n-1}\delta} u\theta e^{-\theta u} \,/\, (1 - \exp\,(-\theta\alpha^{n-1}\delta))\, du$$

$$= \frac{1}{\theta} - \frac{\alpha^{n-1}\delta \exp(-\theta\alpha^{n-1}\delta)}{1 - \exp(-\theta\alpha^{n-1}\delta)} \tag{11}$$

Then (7) with the help of (10) and (11) yields

$$\lambda_n = \sum_{m=1}^{\infty} E(X_n/M_n = m)P(M_n = m)$$

$$= \sum_{m=1}^{\infty} E(X_{nm} + U_{nm})q_n^{m-1}p_n$$

$$= \sum_{m=1}^{\infty} \left\{\frac{m-1}{\theta} + (m-1)\alpha^{n-1}\delta + \frac{1}{\theta} - \frac{\alpha^{n-1}\delta \exp(-\theta\alpha^{n-1}\delta)}{1-\exp(-\theta\alpha^{n-1}\delta)}\right\} q_n^{m-1}p_n$$

$$= \frac{1-p_n}{p_n}\left(\frac{1}{\theta} + \alpha^{n-1}\delta\right) + \frac{1}{\theta} - \frac{\alpha^{n-1}\delta \exp(-\theta\alpha^{n-1}\delta)}{1-\exp(-\theta\alpha^{n-1}\delta)}$$

$$\lambda_n = \frac{1}{\theta(1 - \exp(-\theta\alpha^{n-1}\delta))} \tag{12}$$

Since $0 < \alpha \leq 1$ from equation (12)it follows that λ_n is non-decreasing, consequently, from (1), the average cost is given by,

$$C(N) = \frac{C\mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] - \mathrm{r}\sum_{n=1}^{N} \frac{1}{\theta[1 - exp(-\theta\alpha^{n-1}\delta)]} + R + C_p\tau}{\sum_{n=1}^{N} \frac{1}{\theta(1 - \exp(-\theta\alpha^{n-1}\delta))} + \mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + \tau} \quad (13)$$

$$= \frac{(C + r)\mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + R + \left(C_p + \mathrm{r}\right)\tau}{\sum_{n=1}^{N} \lambda_n + \mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + \tau} - r$$

$$= A(N) - r$$

where

$$A(N) = \frac{(C + r)\mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + R + \left(C_p + \mathrm{r}\right)\tau}{\sum_{n=1}^{N} \lambda_n + \mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + \tau} \quad (14)$$

4 THE OPTIMAL POLICY N^*

In this section we determine an optimal replacement policy for minimizing $C(N)$.
From equation (14) we have,

$$A(N) = \frac{(C + r)\mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + R + \left(C_p + \mathrm{r}\right)\tau}{h(N)}$$

where

$$h(N) = \sum_{n=1}^{N} \lambda_n + \mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + \tau$$

In order to obtain the optimal policy N^*, we study the difference between

$A(N + 1) - A(N)$

$$A(N + 1) - A(N) = \left[\left(\frac{(C + r)\mu\left[1 + \sum_{n=2}^{N} \frac{1}{2^{n-1}\beta}\right] + R + \left(C_p + \mathrm{r}\right)\tau}{h(N + 1)}\right) - \left(\frac{(C + r)\mu\left[1 + \sum_{n=2}^{N-1} \frac{1}{2^{n-1}\beta}\right] + R + \left(C_p + \mathrm{r}\right)\tau}{h(N)}\right)\right]$$

$$= \frac{\left((C+r)\mu\left[1+\sum_{n=2}^{N}\frac{1}{2^{n-1}\beta}\right]+R+\left(C_p+\mathrm{r}\right)\tau\right)\left(\sum_{n=1}^{N}\lambda_n+\mu\left[1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right]+\tau\right)-\left((C+r)\mu\left[1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right]+R+\left(C_p+\mathrm{r}\right)\tau\right)\left(\sum_{n=1}^{N+1}\lambda_n+\mu\left[1+\sum_{n=2}^{N}\frac{1}{2^{n-1}\beta}\right]+\tau\right)}{h(N)h(N+1)}$$

$$= \frac{\left(\begin{array}{c}(C+r)\mu\tau\frac{1}{2^{N-1}\beta}-(C+r)\mu\lambda_{N+1}+(C+r)\mu\frac{1}{2^{N-1}\beta}\sum_{n=1}^{N}\lambda_n-(C+r)\mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\lambda_{N+1}\\ -R\lambda_{N+1}-R\mu\frac{1}{2^{N-1}\beta}-\left(C_p+\mathrm{r}\right)\tau\lambda_{N+1}-\left(C_p+\mathrm{r}\right)\tau\mu\frac{1}{2^{N-1}\beta}\end{array}\right)}{h(N)h(N+1)}$$

$$= \frac{\left(\begin{array}{c}(C+r)\mu\left[\frac{1}{2^{N-1}\beta}\sum_{n=1}^{N}\lambda_n-\lambda_{N+1}\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}-\lambda_{N+1}+\frac{\tau}{2^{N-1}\beta}\right]-\\ R\left[\lambda_{N+1}+\frac{\mu}{2^{N-1}\beta}\right]-\left(C_p+\mathrm{r}\right)\tau\left[\lambda_{N+1}+\frac{\mu}{2^{N-1}\beta}\right]\end{array}\right)}{h(N)h(N+1)}$$

$$= \frac{\left(\begin{array}{c}\frac{(C+r)\mu}{2^{N-1}\beta}\left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}-2^{N-1}\beta\lambda_{N+1}\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}+\tau\right]-\\ \frac{1}{2^{N-1}\beta}\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]\end{array}\right)}{h(N)h(N+1)}$$

$$A(N+1)-A(N)= \frac{\left(\begin{array}{c}(C+r)\mu\left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+\tau\right]-\\ \left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]\end{array}\right)}{2^{N-1}\beta h(N)h(N+1)}$$

Then define the following auxiliary function

$$B(N)=\frac{(C+r)\mu\left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+\tau\right]}{\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]}$$

As the denominator of $A(N+1)-A(N)$ is always positive. It is clear that the sign of $A(N+1)-A(N)$ is the same as the sign of its numerator. Consequently, we have the following lemma.

Lemma 4.1. $A(N+1) \gtreqless A(N) \Leftrightarrow B(N) \gtreqless 1$

Lemma(3.1) shows that the monotonicity of $A(N)$ can be determined by the value of $B(N)$.

Next we shall prove $B(N)$ is decreasing in N.

Lemma 4.2. To prove $B(N)$ is decreasing in N.

Proof.

$$
\begin{aligned}
&B(N+1)-B(N)\\
&\qquad=\left(\frac{(C+r)\mu\left[\sum_{n=1}^{N+1}\lambda_n-2^N\beta\lambda_{N+2}\left(1+\sum_{n=2}^{N}\frac{1}{2^{n-1}\beta}\right)+\tau\right]}{\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+2}2^N\beta+\mu\right]}\right)\\
&\qquad-\left(\frac{(C+r)\mu\left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+\tau\right]}{\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]}\right)
\end{aligned}
$$

$$
=\frac{(C+r)\mu\left(\begin{array}{c}\left[\sum_{n=1}^{N+1}\lambda_n-2^N\beta\lambda_{N+2}\left(1+\sum_{n=2}^{N}\frac{1}{2^{n-1}\beta}\right)+\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]-\\ \left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+\tau\right]\left[\lambda_{N+2}2^N\beta+\mu\right]\end{array}\right)}{\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+2}2^N\beta+\mu\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]}
$$

Let

$$
\xi(N)=\frac{(C+r)\mu}{\left[R+\left(C_p+\mathrm{r}\right)\tau\right]\left[\lambda_{N+2}2^N\beta+\mu\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]}
$$

$$
\begin{aligned}
&B(N+1)-B(N)\\
&=\ \xi(N)\left(\begin{array}{c}\left[\sum_{n=1}^{N+1}\lambda_n-2^N\beta\lambda_{N+2}\left(1+\sum_{n=2}^{N}\frac{1}{2^{n-1}\beta}\right)+\tau\right]\left[\lambda_{N+1}2^{N-1}\beta+\mu\right]-\\ \left[\sum_{n=1}^{N}\lambda_n-2^{N-1}\beta\lambda_{N+1}\left(1+\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)+\tau\right]\left[\lambda_{N+2}2^N\beta+\mu\right]\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
=\xi(N)\Bigg(&\lambda_{N+1}2^{N-1}\beta\left(\sum_{n=1}^{N}\lambda_n+\lambda_{N+1}+\mu+\tau+\mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)\\
&-\ 2^N\beta\lambda_{N+2}\left(\sum_{n=1}^{N}\lambda_n+\lambda_{N+1}+\mu+\tau+\mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)\\
&+\mu(\lambda_{N+1}-2\lambda_{N+2})\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
=\xi(N)\Bigg(&\left(\sum_{n=1}^{N}\lambda_n+\lambda_{N+1}+\mu+\tau+\mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)(\lambda_{N+1}2^{N-1}\beta-2^N\beta\lambda_{N+2})\\
&+\ \mu(\lambda_{N+1}-2\lambda_{N+2})\Bigg)
\end{aligned}
$$

$$= \xi(N)\left((\lambda_{N+1} - 2\lambda_{N+2})\left(\mu + 2^{N-1}\beta\left(\sum_{n=1}^{N}\lambda_n + \lambda_{N+1} + \mu + \tau + \mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)\right)\right)$$

$$B(N+1) - B(N) = \frac{(C+r)\mu\left((\lambda_{N+1} - 2\lambda_{N+2})\left(\mu + 2^{N-1}\beta\left(\sum_{n=1}^{N}\lambda_n + \lambda_{N+1} + \mu + \tau + \mu\sum_{n=2}^{N-1}\frac{1}{2^{n-1}\beta}\right)\right)\right)}{\left[R + \left(C_p + \mathrm{r}\right)\tau\right][\lambda_{N+2}2^N\beta + \mu][\lambda_{N+1}2^{N-1}\beta + \mu]}$$

This implies that $B(N)$ is decreasing in N, because λ_n is non-decreasing in n and $\beta > 0$.

Therefore by using lemma 2.1 and lemma 2.3 we have the following theorem.

Theorem 4.1. The optimal replacement policy ${N_i}^* = \infty$ is the unique optimal for the improving system.

Proof. In fact, because $B(N)$ is decreasing in N, there exists an integer N_i, such that $N_i = min\{N \,/\, B(N) \le 1\}$

Therefore, Lemma 2.2 yields that $C(N)$ and $A(N)$ are both unimodel functions of N and both take maximum at N_i. This implies that the minimum of $C(N)$ will be given by

$$\min C(N) = min\{C(1), C(\infty)\}$$

$$= min\left\{\frac{R + C_p\tau - r\lambda}{\lambda_1 + \tau}, -r\right\}$$

$$= -r$$

Consequently, ${N_i}^* = \infty$ is the unique optimal replacement policy for the improving system.

5. CONCLUSION

As the optimal policy for the improving system is always ${N_i}^* = \infty$.

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Citation: Sutha. M and Sridhar. A, δ-Shock Maintenance Model for An Improving System Under Partial Sum Process, International Journal of Mathematics (IJMM), 1(1), 2020, pp. 1–11.

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