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EFFECTS OF THERMAL RADIATION AND PRESSURE WORK ON MAGNETO MARANGONI FLUID FLOW OVER A CIRCULAR SURFACE

Elsayed M. A. Elbeshbeshy[1], Hamada G. Asker[2], Mahmoud M. Kamal[3]. Ashraf Kotb[3], Hany Saad[3], Ahmed Sedki[1]

[1]Ain Shams University, Faculty of Science, Department of Mathematics, Egypt.

[2]Ain Shams University, Faculty of Engineering, Department of Math and Engineering Physics, Egypt.

[3]Ain Shams University, Faculty of Engineering, Department of Mechanical Power Engineering, Egypt.

ABSTRACT

A computational work has been performed on a thermally radiative and magneto Marangoni boundary layer on a heated circular surface upon considering the pressure work and the surface tension. As developed from the governing partial differential equations with proper boundary conditions, the current work numerical model comprises a set of ordinary differential equations which pronounce multiple physical parameters. The numerical analysis of the effective variables revealed that increasing the buoyancy parameter increases the velocity magnitudes while opposite effects were found for the magnetic parameter and the ratio of the surface tension. The temperature magnitudes increased upon increasing the magnetic effect. It was indicated that the Marangoni ratio number, Prandtl number and the buoyancy parameter enhance Nusselt number.

Keywords: Marangoni Fluid Flow; Thermal Radiation; Magnetic Field; Pressure Work; Circular Surface

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1. Introduction

The Marangoni consequence occurs as a mass transfer process which is triggered by the difference in the surface tension between two fluid surfaces. This phenomenon has numerous industrial uses, including the flow coat technology, the microfluidics, the foams, and the film drainage in emulsions. While intensive research works were devoted to the Marangoni phenomenon, the results on the Marangoni flow over a stretching surface showed that for small values of Prandtl number (P_r), Nusselt number (Nu) is proportional $(P_r)^{2/3}$ [1]. Under such conditions of low Prandtl number values, the non-linear temperature gradient becomes linear [2]. On the other hand, for large values of (P_r), (Nu) is proportional to $(P_r)^{1/6}$. While the Marangoni convection flow was numerically treated in the work of Pop et al. [3], an analytical approach was performed on the Marangoni convection heat transfer over a liquid-vapor surface [4]. As the thermo-solute surface tension increases, there is a decrease in the velocity and an increase in both the temperature and the concentration [5]. Increasing Dufour number in combination with the decrease in Soret number leads to an increase in the temperature and a decrease in the concentration. In addition, Nusselt number is enhanced via increasing Prandtl number [6], while increasing Dufour number in combination with the nanofluid effect enhances the heat transfer rates [7].

Inasmuch as the thermal radiation is important in various engineering applications such as the space vehicles and the nuclear power plants, many researches focused on the characteristics of thermal radiation for fluids which flow over different surfaces. Increasing the combined effect of the magnetic field and the nanoparticle volume fraction decreases the thickness of the nano-momentum boundary layer and increases the thickness of the nano-thermal boundary layer. In addition, the local Nusselt number decreases with the magnetic parameter [8]. The existence of the porosity and Chandrasekhar parameter postpone the non-

Darcy-Benard Marangoni convection [9], while decreasing the values of the horizontal wavenumber and the internal Rayleigh number can delay the Darcy-Benard-Magneto Marangoni convection [10]. Different nanoparticles present an efficient tool for enhancing the Marangoni radiative effect on the permeable surfaces with inclined magnetic fields [11]. Upon increasing the radiation and the heat source effects, the velocity decreases for both the H_2O and $C_2H_6O_2$ based nanofluids [12]. Increasing the mixed convection decreases the skin friction coefficient but a slightly increase the temperature. Conversely, the existence of a porous medium increases the skin friction coefficient [13].

The pressure work has significant effects on the devices which work with accelerated rotational velocity or exist in high gravitational fields on large scales. Therefore, the effects of the pressure stress work on the forced/free convection flow along horizontal cylinders, vertical flat plates, truncated cones, stretching surfaces and radial surfaces have extensively been studied. It was found that the pressure work decreases the velocity and the temperature profiles [14]. The skin friction coefficient is enlarged by increasing the chemical reaction parameter, the nonlinearity, the Brownian motion and the surface permeability. Upon increasing the volume of nanoparticles, the thermal radiation, the chemical reaction, both the thermophoresis effect and the rate of mass transfer increase [15]. The flow in a shallow cylinder is affected by the Marangoni convection [16]. While the surface tension increases, the permeability effects decrease the fluid velocity [17]. Under such circumstances, the buoyancy parameter raises the flow velocity, while the porosity value controls the magnitudes of the heat transmission properties [18]. Furthermore, the Marangoni convection has a great impact on the hybrid nanofluid [19]. In the light of such previous findings, there is a great potential for combining the thermal radiative heat transfer with the magneto Marangoni convection within a fluid which flows over a vertically heated circular plate while the consideration is given to the pressure work. The effects of the surface heating and the surface tension are analyzed. The surface tension has important applications such as the applications which require the stability of flow over films and the circumstances that are experienced in the manufacturing of integrated circuits.

2. Mathematical Model

A two-dimensional, laminar, stable and incompressible Marangoni boundary layer is currently examined, where the flow across a heated circular plate is under pressure and stress.

The thermal gradient-driven Marangoni convective and radiative flow above a planar interface is considered. The r-axis is considered in the direction of motion which is perpendicular to the z-axis (Fig. 1). While the plane ($z = 0$) represents the surface, the flow is confined in the region wherein $z > 0$. A uniform magnetic field of strength B_0 is applied in the z-direction (which is perpendicular to the flow). The Marangoni phenomenon occurs when the mass transmission develops due to the variation of the surface tension at the interface of two surfaces. Such variation in the surface tension (which may result from a gradient in the temperature or in the concentration) causes the fluid to get transported from the lower interfacial tension to the higher one.

With the above-mentioned assumptions, the basic equations of flow are given as follows:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \frac{\mu}{\rho}(\frac{\partial^2 u}{\partial z^2}) + g\beta_{\mathrm{T}}(T - T_\infty) - \frac{\sigma^* B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{T\beta u}{\rho c_p}\frac{\partial P}{\partial r} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial z} \tag{3}$$

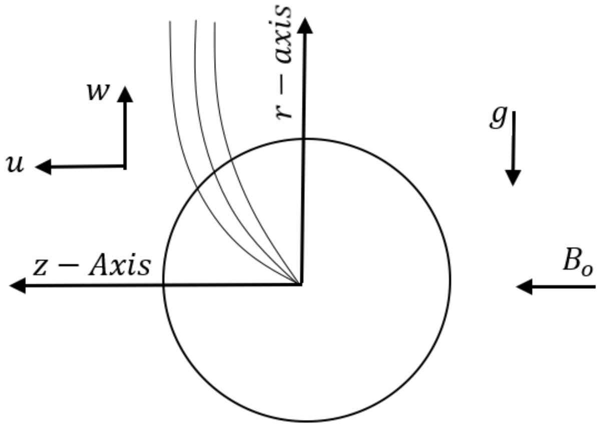

Fig. 1: Diagram illustrating the issue

The boundary conditions are:

$$at\ z = 0:\ \mu \frac{\partial u}{\partial z} = \frac{\partial \sigma}{\partial r},\quad w = 0,\ \ T = T_{w,}\quad z \to \infty:\ u = 0, w = 0, T = T_{\infty} \tag{4}$$

Where,

$$r\frac{\partial P}{\partial r} = \rho g \tag{5}$$

By using Rosseland diffusion approximation

$$q_r = -\frac{4\sigma_1}{3k^*}\frac{\partial T^4}{\partial z} \tag{6}$$

Taylor expansion is used to expand T^4 about T_{∞} while ignoring the higher-order terms yields:

$$T^4 = 4T_{\infty}^3 T - 3T_{\infty}^4 \tag{7}$$

Therefore, equation (3) becomes

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \left[\alpha + \frac{16\sigma^* T_{\infty}^3}{3\rho c_p k^*}\right]\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{T\beta\rho g}{r\rho c_p}\, u \tag{8}$$

Using Boussinesq approximation, the surface tension is considered as follows:

$$\sigma = \sigma_0[1 - \gamma(T - T_{\infty})] \tag{9}$$

Where σ_0 is the interface surface tension ratio and $\gamma = -\frac{1}{\sigma_0}\frac{\partial \sigma}{\partial T}$,

The stream function $\Psi\,(r, z) = -\frac{\upsilon r^2}{L} f(\eta)$ is chosen such that

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z}\ ,\quad w = \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{10}$$

are satisfying the continuity equation. For similarity solution, the following variables are introduced:

$$\eta = \frac{z}{L} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w - T_\infty = T_0 r^2 \tag{11}$$

Where T_0 is the reference temperature constant, while the controlling equations can be written as follows:

$$f''' - f'^2 + 2ff'' - Mf' + \xi\theta = 0 \tag{12}$$

$$(1+N)\theta'' + 2Prf\theta' - 2Prf'\theta - Pr\epsilon\left(\theta + \frac{1}{\theta_w - 1}\right)f' = 0 \tag{13}$$

With the following boundary conditions:

$$z = 0: f'' = -2S, \quad \theta = 1 \; z \to \infty: f = 0, \quad \theta = 0 \tag{14}$$

Where

$$\zeta = \frac{g\beta(T_w - T_\infty)L^4}{rv^2}, M = \frac{L^2\sigma^* B_0^2}{\rho_\infty}, Pr = v/\alpha,\ \theta_w = \frac{T_w}{T_\infty}, N = \frac{16\sigma^* T_\infty^3}{3kk^*}, \epsilon = \frac{\beta v g}{c_p}, S = \frac{\sigma_0 \gamma T_0}{\mu v} \tag{15}$$

The practical physical parameters are defined as follows:

$$Nu = \frac{rq_w}{k_f(T_w - T_\infty)} \tag{16}$$

Where q_W represents the surface heat flux which is thus given by

$$q_w = -k_f\left(\frac{\partial T}{\partial z} + q_r\right)_{z=0} = -\frac{(T_w - T_\infty)}{L} k_f (1+N)\,\theta'\,|_{\eta=0} \tag{17}$$

Finally, the following equation is used:

$$\frac{L}{r}Nu = -(1+N)\theta'(0) \tag{18}$$

3. Solution Methodology

Equations (12) and (13) are transformed into a system of linear equations. In this case, $y_1 = f, y_2 = f', y_3 = f'',\ y_4 = \theta, y_4 = \theta'$

$$y_1' = y_2 \tag{19}$$

$$y_2' = y_3 \tag{20}$$

$$y_3' = y_2^2 - 2y_1y_3 + \lambda y_2 - \xi y_4 \tag{21}$$

$$y_4' = y_5 \tag{22}$$

$$y_5' = \frac{1}{1+N}(2\,Pr\,(y_1y_4 - y_2y_4)\ + Pr\epsilon(y_4 + \frac{1}{\theta_w - 1})y_2) \tag{23}$$

Where the boundary conditions are given as follows:

$$y_1(0) = 0,\ y_2(0) = s_1,\ y_3(0) = -2S, \qquad y_4(0) = 1, y_5(0) = s_2 \tag{24}$$

Where s_1 and s_2 are arbitrary parameters. The equations from (19) to (23) are solved numerically by using 4th/5th Runge-Kutta method and Mathematica software.

4. Results and Discussion

A satisfactory agreement is obtained upon comparing the present work results with the results of Zhang and Zheng [20] as shown in Table 1, which correspond to the values of $f'(0)$. Such agreement testifies to the accuracy of the current work analysis.

Table 1: Comparison values $f'(0)$ for various values of M at $\xi = 0$

M	Zhang and Zheng [20]	Present Work
0.0	2.5199	2.5200
1.0	2.1572	2.1570
2.0	1.6240	1.6399

The effects of the M parameter on the fluid velocity boundary layers tends to produce a Lorentz force which causes a flow retardation effect. This causes the fluid velocity to decrease (as shown in Fig. 2a). On the other hand, enhancing the values of M enlarges the width of the thermal boundary layer, Fig. 2b. The influence of the buoyancy parameter is shown in Fig. 3. It is noted that the increase in the buoyancy parameter tends to increase the velocity. Physically, the buoyancy force is related to the inertial force. Figure 4 shows the temperature distribution for different values of the pressure work parameter (ϵ). The increase in the pressure work parameter causes an increase in the temperature. Figure 5 shows that as the value of Pr increases, the value of Θ decreases. Physically, increasing Pr decreases the thermal diffusion such that the thickness of the thermal boundary layer decreases and the heat transfer rate becomes smaller.

Figure 6 shows that the temperature magnitudes increase as the N parameter increases. Therefore, the radiation heat transfer can be used to control the thermal boundary layers quite effectively where the thermal boundary layer thickness increases with the increase in the N parameter. Figure 7 indicates that the increase in the θ_w parameter increases the temperature. In Fig. 8, as the S parameter increases, more induced flow is produced within the boundary layer. It is noted that the temperature is reduced by involving a higher Marangoni convection parameter (S) but the opposite behavior occurs for the velocity profile. Physically, the Marangoni convection is inversely proportional to the viscosity. It means that the larger value of the Marangoni parameter is correlated to low viscosity which decelerates the fluid velocity. Table 2 shows that the Marangoni number S, Prandtl number and buoyancy parameter enhance the heat transfer rate, but the opposite result is obtained for the magnetic parameter, the Pressure work parameter, the radiation parameter and the surface heating parameter.

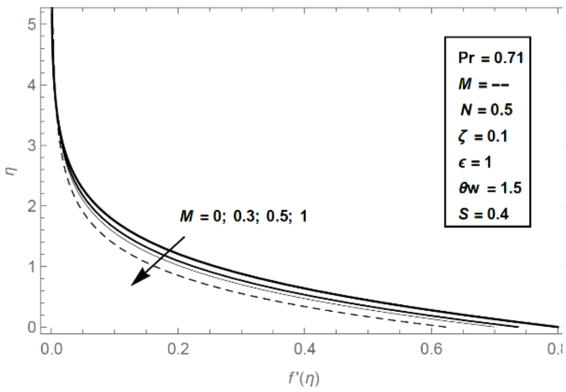

Fig. 2a. Velocity graph f' for M

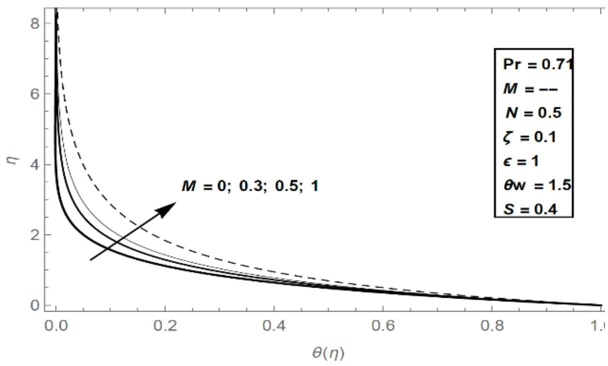

Fig. 2b. Temperature graph θ for M

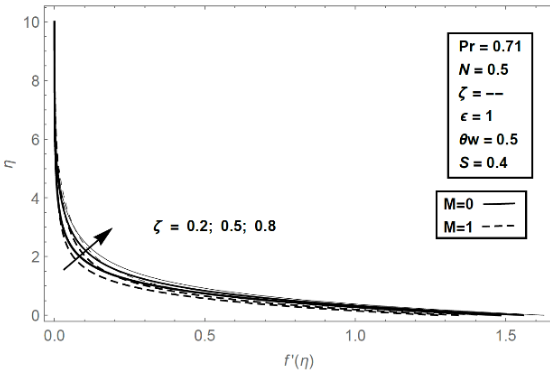

Fig. 3: Velocity graph f' for ζ

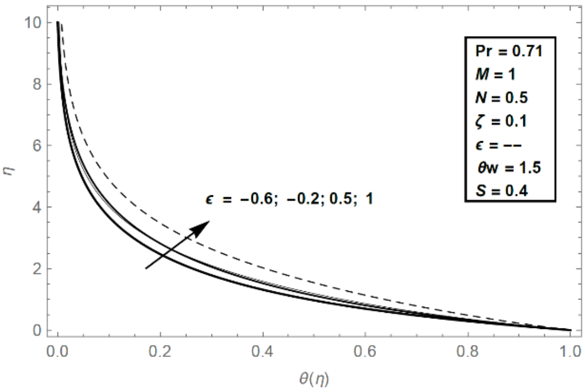

Fig. 4: Temperature graph θ for ε

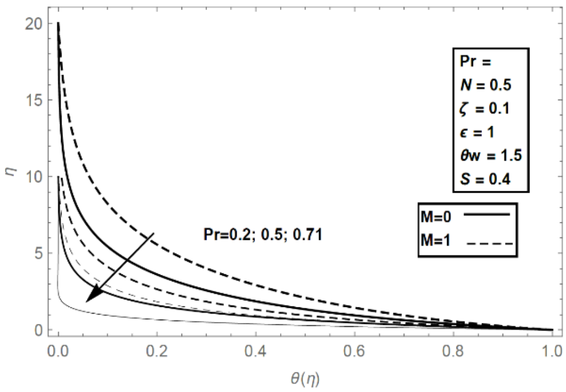

Fig. 5: Temperature graph θ for Pr

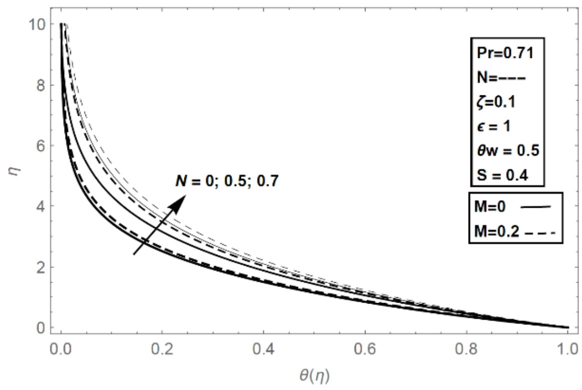

Fig. 6: Temperature graph θ for N

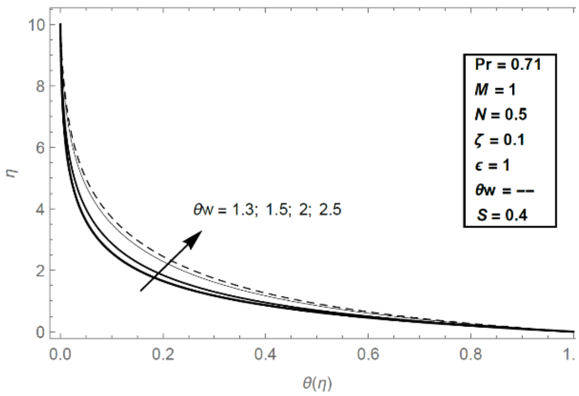

Fig. 7: Temperature graph θ for θ_w

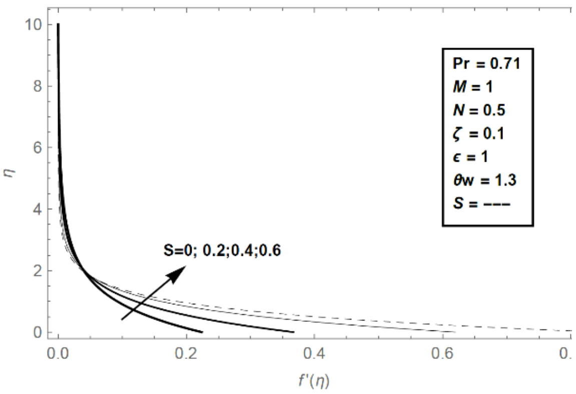

Fig.8a. Velocity graph f' for S

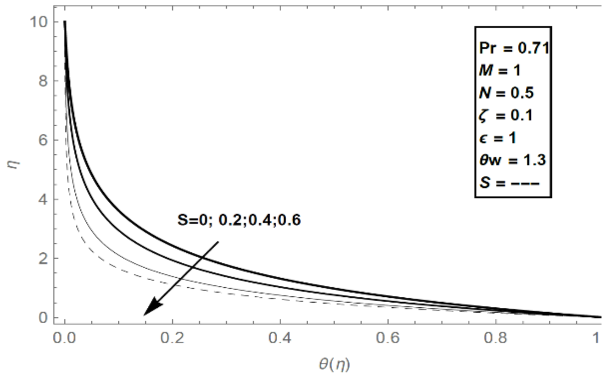

Fig.8b. Temperature graph θ for S

5. Conclusions

The magneto Marangoni fluid flow along an upright circular surface with non-linear pressure work and heat radiation was studied numerically. The effects of the magnetic field, the pressure work and the thermal radiation on the velocity and the temperature profiles are thoroughly examined and illustrated graphically. Additionally, Nusselt number was calculated for the values of different factors which were shown in a tabular form. The key elements of the current results are provided as follows:

- The velocity magnitudes increase for the higher values of buoyancy parameter. In contrast, the opposite effect occurs for both the magnetic parameter and the ratio of the surface tension.
- The temperature values increase for the individual increase in the magnetic field, the thermal radiation heat flux, the surface heating parameter and the pressure work parameter while the temperature values decrease due to the increase in Marangoni number and Prandtl number.

- Increasing Marangoni ratio number, Prandtl number and the buoyancy parameter enhance Nusselt number. In contrast, the opposite result is obtained for the magnetic field, the pressure work parameter, the radiation parameter and the surface heating parameter.

Table 2: Variation of $-\theta'(0)$ with $M, \xi, \epsilon, Pr, N, \theta$ and S.

	M	ζ	ϵ	Pr	N	θ_w	S	$-\theta'(0)$
M	0.3	0.1	1	0.71	0.5	1.5	0.4	1.2972
	0.5							1.2335
	1							1.0930
ζ	1	0.2	1	0.71	0.5	0.5	1	0.6065
		0.5						0.6228
		0.8						0.6401
ϵ	1	0.1	-0.6	0.71	0.5	0.5	0.4	0.7859
			-0.2					0.6930
			0.5					0.6362
Pr	1	0.1	1	0.2	0.5	1.5	0.4	0.4544
				0.5				0.8582
				0.72				1.0929
N	0.2	0.1	1	0.71	0	0.5	0.4	0.6149
					0.5			0.4849
					0.7			0.4490
θ_w	1	0.1	1	0.71	0.5	1.4	0.4	1.1708
						1.5		1.0930
						2		0.9337
S	1	0.1	1	0.71	0.5	1.3	0.1	0.7561
							0.2	0.9713
							0.4	1.2980

5. Nomenclature

B_0	The constant magnetic field intensity	r, z	coordinates (m)
c_p	specific heat at constant pressure, (J kg^{-1} K^{-1})		**Greek symbols**
f	dimensionless velocity	α	Thermal diffusivity
g	gravity, (m s^{-2})	β	Thermal expansion coefficient
$k*$	mean absorption coefficient (m^{-1})	ψ	Stream function
M	magnetic parameter	η	Dimensionless coordinator
N	radiation parameter	θ	Dimensionless temperature
Nu	local Nusselt number	θ_w	Surface heating parameter
P	the variable hydrostatic pressure	μ	Dynamics viscosity of the fluid (N. s/m^2)
P_r	Prandtl number	ν	Kinematic viscosity, (m^2 s^{-1})
q_r	radiative heat flux, (kg m^{-2})	ϵ	Pressure work parameter
Re	local Reynolds number	ρ	Density of the fluid (kgm^{-3})

S	Surface tension ratio (Marangoni number)	σ	The surface tension at the interface (N/m)
T	Temperature, (K)	σ_1	Stefan-Boltzmann constant
T_w	Surface temperature, (K)	σ_0	Interface surface tension ratio
T_∞	Free temperature, (K)	ξ	Buoyancy parameter
u, w	Velocity components (m s^{-1})		

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