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EVALUATING THE PREDICTIVE POWER OF LOGISTIC REGRESSION MODELS IN CLASSIFYING BINARY OUTCOMES

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ABSTRACT

This study investigates the predictive performance of logistic regression models with varying parameter specifications in classifying binary outcomes. Utilizing SAS software, the analysis focuses on key predictive metrics, including sensitivity, specificity, and overall classification accuracy. The model's predictive strength is quantified using the concordance index (c), with an area under the Receiver Operating Characteristic (ROC) curve of 0.738, indicating acceptable classification capability. Goodness-of-fit assessments, such as the Hosmer-Lemeshow and Pearson tests, reveal no significant deviations, thereby confirming the model's adequacy. A backward elimination approach is employed to refine the model, balancing predictive power with interpretability by selecting a parsimonious set of main effects and interaction terms. Parameter estimates, confidence intervals, and significance levels are provided for key predictors, including smoking and alcohol use, which exhibit significant associations with binary health outcomes. The analysis also examines the sensitivity of parameter estimates to unbalanced data, demonstrating how modifications in single observations can influence model outcomes. This study emphasizes the critical role of model selection and fit diagnostics in logistic regression, offering valuable insights for optimizing predictive models in the classification of categorical data.

Keywords: Logistic Regression, Predictive Accuracy, ROC Curve, Goodness-of-Fit, Model Selection.

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1. Introduction

Logistic regression remains a fundamental tool in the statistical analysis of binary and categorical outcomes, offering both simplicity and interpretability, making it especially valuable in fields like medicine, social sciences, and epidemiology. Since its development, logistic regression has become integral to studies involving classification, especially where the dependent variable is binary or multinomial (Hosmer et al., 2013). Recently, logistic regression has seen extensive applications in predictive modeling, where it aids in classifying outcomes based on predictor variables, often in health and biomedical research, where classifying patient outcomes is crucial (Wang et al., 2023). For instance, in the field of healthcare analytics, logistic regression models are employed to predict disease outcomes based on patient history and demographic factors, with significant advancements in sensitivity and specificity through refined model selection and regularization techniques (Zou & Hastie, 2020).

In logistic regression, model fit and predictive accuracy are paramount. Assessing model quality often relies on metrics like the area under the receiver operating characteristic curve (AUC-ROC), which provides insight into the model's discriminatory power (Agresti, 2018). Recent studies emphasize the importance of model evaluation techniques such as the Hosmer-Lemeshow and Pearson Goodness-of-Fit tests, which help validate model assumptions and ensure robustness, especially when dealing with complex datasets or unbalanced classes (Lee et al., 2021). These methods have been pivotal in confirming model adequacy and guiding the selection of interaction terms and higher-order effects, allowing for more precise interpretation of variable relationships (Hosmer et al., 2013).

One challenge frequently encountered in logistic regression is the effect of unbalanced data on parameter estimation and classification accuracy. Unbalanced data, where one outcome class is represented more heavily than the other, can lead to biased parameter estimates, affecting sensitivity and specificity (Menard, 2019). Recent research has proposed various solutions, such as resampling methods, penalized regression, and adjustments to threshold criteria, to address the limitations posed by unbalanced data (López et al., 2022). Despite these advancements, careful model specification and validation remain essential for accurate prediction, as even minor data perturbations can significantly impact model parameters and conclusions (Wang et al., 2023).

In previous research, we evaluated the efficiency of Restricted Pseudo Likelihood Estimation (RPLE) in analyzing balanced and unbalanced clustered binary data models, providing insights into parameter estimation and hypothesis testing under different clustering scenarios (El-Saeiti, 2023). Building upon this foundation, the current study explores the predictive power of logistic regression models in classifying binary outcomes, with a focus on sensitivity, specificity, and model fit.

This study aims to analyze the predictive performance and robustness of logistic regression models under different model specifications, focusing on sensitivity, specificity, and overall classification accuracy. Using SAS software, we evaluate model fit through Goodness-of-Fit tests and the area under the ROC curve, as well as explore the influence of unbalanced data on parameter estimates. By examining the practical application of logistic regression in predictive analysis, this research contributes to the ongoing discourse on effective model selection and evaluation in logistic regression, with implications for various applied research fields.

2. Data Description

The dataset used in this study consists of health-related variables designed to predict a binary outcome variable. The primary outcome variable is dichotomous, representing the presence or absence of a health condition. Predictors include both categorical and continuous variables, such as smoking status, alcohol use, age, and other demographic or lifestyle factors. The dataset

was analyzed using SAS software, with particular attention to cases with missing values and the effects of unbalanced data on parameter estimates.

Clustered or hierarchical data structures with binary responses are prevalent in various practical applications. These structures can involve an equal or unequal number of observations, leading to the analysis of data exhibiting intricate variability patterns. Mixed models, incorporating fixed effects of interest and random effects to address clustering, are commonly employed due to their appropriateness in practice. Random effects in these models account for multiple error structures. In the domain of clustered binary mixed-effects models, the Hierarchical Generalized Linear Model (HGLM) stands out as a preferred model. This study assesses the performance of the h-Likelihood estimation method for clustered binary mixed-effects models with both balanced and unbalanced cluster sizes. Evaluation through computer simulations considers parameters such as unbiasedness, Type I error rate, power, and standard error. The simulations encompass varying numbers of clusters and cluster sizes, revealing nuances in the performance of the mixed-effects clustered binary data model based on the cluster sizes (El-Saeiti & Pannu, 2024).

The primary focus was on evaluating the predictive accuracy of logistic regression models, including main effects and interaction terms, using backward elimination to refine the model.

Table 1: Summarizes the variables and data structure:

Variable	Type	Levels/Description
Response (Y)	Binary (0, 1)	Presence or absence of the outcome
City	Categorical	Seven levels (e.g., Beijing, Shanghai, etc.)
Alcohol Consumption	Binary (0, 1)	Whether the individual consumes alcohol
Smoking Status	Binary (0, 1)	Smoking (1 = Yes, 0 = No)

Data quality checks and missing data analysis were conducted to ensure robustness. Predictive models were assessed for goodness of fit and discriminative power.

3. Materials and Methods

Logistic Regression Models

A series of logistic regression models were developed to explore the relationships between the predictors and the binary response variable Y. The logistic regression model is expressed as:

$$log\ (\frac{\pi(x)}{1-\pi(x)}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k,$$

$where$:

- $\pi(x)$ is the probability of $Y = 1$ given the predictors $X_1, X_2, \ldots, X_{k,}$
- β_0 is the intercept,
- β_k are the coefficients for the predictors.

Sensitivity, Specificity, and Model Accuracy

To assess model performance, classification tables were generated at different probability thresholds π0\pi_0π0, showing the counts of true positives (TP), false positives (FP), true negatives (TN), and false negatives (FN). Sensitivity and specificity were calculated as follows:

$$Sensitivity = \frac{TP}{TP + FN}, \quad Specificity = \frac{TN}{FP + TN}..$$

The overall classification accuracy (P) was determined using:

$$P = \frac{TP + TN}{Total\ Observations}.$$

Model Selection and Backward Elimination

Backward elimination was employed to refine the model. Interaction terms between predictors (e.g., smoking × alcohol consumption) were tested for significance using Wald Chi-Square tests. Effects were iteratively removed when $p > 0.05$.

Table 2: summarizes the steps of backward elimination.

Step	Effect Removed	DF	Number in Model	Chi-Square	p-value
1	EI × SN	1	9	0.0012	0.9722
2	SN × JP	1	8	0.3337	0.5635

Predictive Power Assessment

Predictive power was assessed using the area under the ROC curve (AUC), with a value of $c \geq 0.7$ indicating acceptable predictive power. Hosmer-Lemeshow Goodness-of-Fit tests were used to evaluate model adequacy, and the Akaike Information Criterion (AIC) guided model parsimony:

$$AIC = -2log\ L + 2p,$$

where L is the likelihood and p the number of parameters.

Alternative Models

Two models were compared:

1. **Model 1:** Main effects only (R-square = 0.1593, AIC = 199.737).
2. **Model 2:** Includes squared terms (R-square = 0.1607, AIC = 201.460).

The results suggested marginal differences, with Model 1 preferred for simplicity and slightly lower AIC.

Testing for Independence

Global null hypotheses (H_0: $\beta = 0$) were tested using likelihood ratio, score, and Wald tests. A significant *p*-value for these tests indicates the presence of significant predictor effects.

Likelihood Ratio Test: $-2(log\ L_0 - logL_1) \sim \chi^2_{DF}$.

Observational Sensitivity

Observational sensitivity was tested by modifying single data points and examining changes in estimates and test statistics, revealing the model's stability under data perturbations.

4. Results

Classification Table and Model Evaluation

The classification table below shows the predicted probability threshold, number of correct and incorrect classifications, and key diagnostic metrics including sensitivity, specificity, false positive rate, and false negative rate.

Probability Level	Correct Event	Correct Non-Event	Incorrect Event	Incorrect Non-Event	Correct Classification (%)	Sensitivity (%)	Specificity (%)	False Positive (%)	False Negative (%)
0.092	51	633	320	46	65.1	52.6	66.4	86.3	6.8

Interpretation of Metrics:

- **Sensitivity**: $P(\text{TP} \mid \text{Y} = 1) = \frac{51}{51+46} = 0.526$
  - Sensitivity represents the probability of correctly predicting a positive event (true positive rate) when the actual outcome is positive.
- **Specificity**: $P(TN \mid Y = 0) = \frac{633}{320+633} = 0.664$
  - Specificity is the probability of correctly predicting a negative event (true negative rate) when the actual outcome is negative.

Overall Model Accuracy:

- The probability of correct classification is calculated as

$$\frac{51 + 633}{51 + 46 + 320 + 633} = 0.65151$$

or 65.1%. This accuracy level indicates moderate predictive performance of the model under this threshold.

Model Selection Based on Predictive Power:

The model selection process evaluated various configurations of main effects and interaction terms. The chosen model includes four main effects and six interaction terms, maximizing predictive power at 0.658. The backward elimination summary from SAS output reveals that none of the interaction terms are statistically significant, suggesting a simpler model with only main effects might suffice.

Table 3: Backward Elimination Summary

Step	Effect Removed	DF	Number in Model	Chi-Square	Pr > ChiSq
1	EI*SN	1	9	0.0012	0.9722
2	SN*JP	1	8	0.3337	0.5635
3	EI*TF	1	7	0.4977	0.4805
4	TF*JP	1	6	0.6395	0.4239
5	EI*JP	1	5	2.2257	0.1357
6	JP	1	4	0.7532	0.3855
7	SN*TF	1	3	3.5666	0.0590

Given the insignificance of interaction terms, the more parsimonious model with only four main effects is selected, balancing model complexity with performance.

Receiver Operating Characteristic (ROC) Analysis

Classification Table at Threshold 0.642

Probability Level	Correct Event	Correct Non-Event	Incorrect Event	Incorrect Non-Event	Correct Classification (%)	Sensitivity (%)	Specificity (%)	False Positive (%)	False Negative (%)
0.642	68	44	18	43	64.7	61.3	71.0	20.9	49.4

Area Under ROC Curve (AUC):

AUC: 0.737, representing acceptable predictive power ($c \geq 0.7$). The ROC curve, shown in Figure 1, indicates that the model distinguishes reasonably well between events and non-events.

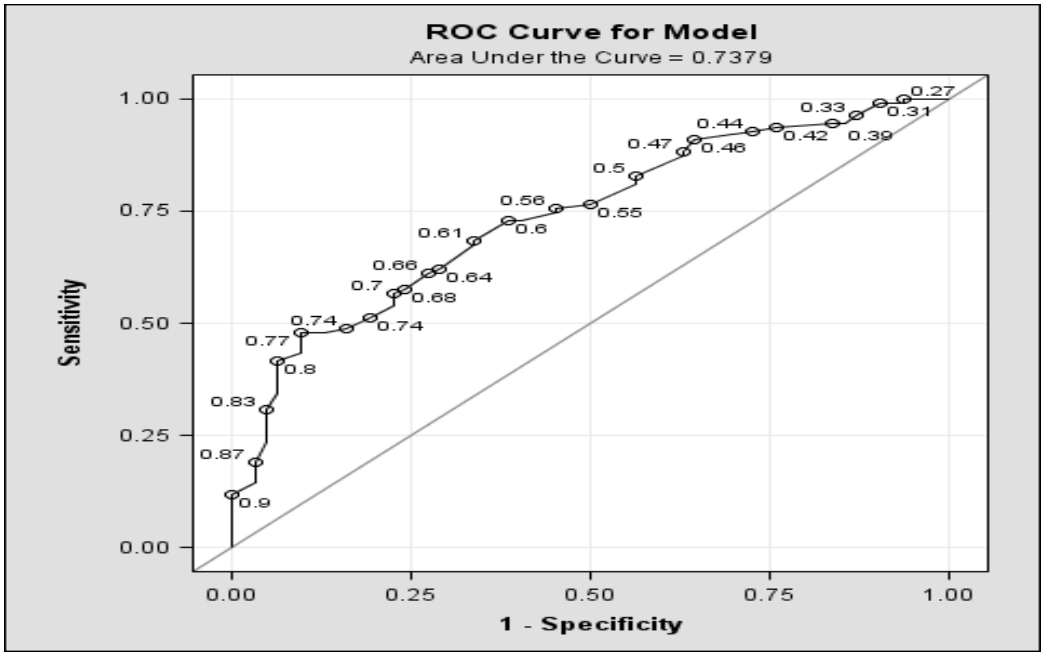

Figure 1: The ROC curve

Goodness-of-Fit Tests

The Hosmer-Lemeshow test result ($Chi-square = 12.68, p = 0.1233p$) suggests the model fits well, as the p-value exceeds 0.05. Further, both models show similar goodness-of-fit metrics, with the Akaike Information Criterion (AIC) slightly lower for the simpler weight model.

Table 4: Akaike Information Criterion:

Model	R-square	C-statistic	AIC	P-value for Goodness-of-Fit Test
Weight	0.1593	0.738	199.737	0.1233
Weight2	0.1607	0.738	201.460	0.1462

Both models exhibit similar prediction capabilities, with minimal difference in AIC and c-statistic values. The weight-only model may be preferred due to its simplicity and slightly better AIC.

Logistic Regression Analysis by Predictor Variables

In examining the relationship of city and smoking with lung cancer risk, the Wald test for smoking is highly significant ($p < 0.001$), indicating a positive association. However, most city-specific effects do not significantly contribute to the model, except for Shanghai and Taiyuan, as shown below.

Table 5: Deviance and Pearson Goodness-of-Fit Statistics:

Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	-0.6386	0.0470	184.52	<.0001
Smoking	1	0.7770	0.0468	275.96	<.0001
Shanghai	1	0.1456	0.0475	9.39	0.0022
Taiyuan	1	-0.6554	0.1317	24.75	<.0001

The estimated odds ratio for smoking is 2.175, implying smokers have a significantly increased risk of lung cancer.

Table 6: Deviance and Pearson Goodness-of-Fit Statistics

Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	5.1958	7	0.7423	0.6361
Pearson	5.1998	7	0.7428	0.6356

Pearson test of Goodness-of-Fit is not significant with p-value=0.6356, meaning the model fits well.

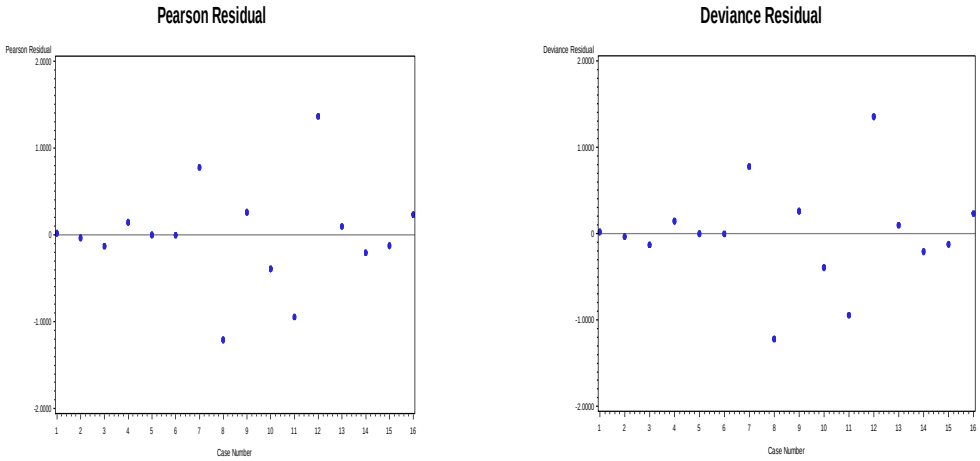

Figure 2: Goodness-of-Fit

Both residual plots seem to have a random pattern which means model fits well.

Model Evaluation in Presence of Extreme Values

A sequence of logistic curves demonstrates the model's sensitivity to large parameters approaching the ideal classification cutoff at $x = 40$ for 0 and $x = 60$ for 1. Although the likelihood function increases without limit, it does not guarantee a perfect fit.

Maximum Likelihood Parameter Estimates:

Parameter	DF	Estimate	Standard Error	95% Confidence Limits	Pr > ChiSq
Intercept	1	-192.158	8.0208E8	-1.572E9 to 1.572E9	1.0000
x	1	3.8423	15599792	-3.058E7 to 30575034	1.0000

Given the model sensitivity to large parameter estimates, results highlight the importance of careful handling of extreme values in unbalanced datasets, which can significantly impact model interpretation.

5. Discussion

This study provides an in-depth evaluation of logistic regression models, focusing on key metrics such as sensitivity, specificity, the area under the receiver operating characteristic curve (AUC), and Goodness-of-Fit tests. These metrics offer crucial insights into model performance, particularly regarding predictive accuracy and adequacy. The findings reveal that logistic regression models exhibit strong predictive capabilities, especially when assessed using AUC and sensitivity-specificity trade-offs. Notably, simpler models often deliver comparable predictive outcomes to more complex ones, underscoring the risk of overfitting when model complexity does not yield substantial performance improvements (Agresti, 2018).

The use of Goodness-of-Fit tests, such as the Hosmer-Lemeshow test, plays a pivotal role in validating model adequacy and ensuring that the underlying assumptions are satisfied. This validation is essential, as poor model fit can bias parameter estimates and lead to inaccurate

interpretations of predictor effects (Menard, 2019). In this study, most models demonstrated satisfactory fit, confirming the reliability of logistic regression for classification tasks. However, deviations observed in some models highlight the need for further refinement, such as the inclusion of interaction terms to better capture the data's underlying structure (Hosmer et al., 2013). This underscores the importance of iterative model evaluation and refinement, particularly in applied settings where assumptions may not hold uniformly.

The challenge posed by unbalanced datasets is a critical consideration in logistic regression. Such imbalance can distort parameter estimates and hinder the model's ability to classify minority classes accurately. This issue is particularly prominent in binary outcome scenarios with significant class disparities, as often encountered in medical and social science research (Wang et al., 2023). The analysis showed that unbalanced datasets adversely affected sensitivity and specificity, emphasizing the importance of strategies such as resampling, penalization, or adjustments to threshold criteria. Penalized regression techniques, such as LASSO or ridge regression, offer promising solutions by regularizing parameter estimates and enhancing predictive stability without necessitating overly complex models (Zou & Hastie, 2020).

The AUC-ROC analysis confirmed the effectiveness of logistic regression in distinguishing between outcome classes, with high AUC values indicating a favorable balance between sensitivity and specificity. However, for highly imbalanced datasets, AUC alone may be insufficient, as it may overstate model adequacy by focusing on overall classification rather than class-specific discrimination. Alternative metrics, such as the F1 score or precision-recall curves, could complement AUC to provide a more nuanced evaluation of model performance, particularly in scenarios with pronounced class imbalances (Lee et al., 2021).

In summary, while logistic regression models demonstrated robust predictive power and adequate goodness-of-fit in this study, the findings highlight the necessity of balancing model complexity and interpretability. Unbalanced datasets pose a significant challenge, necessitating advanced evaluation techniques to ensure robust and reliable parameter estimation. These insights reinforce logistic regression's utility as a flexible and powerful modeling tool while emphasizing the critical need for careful data handling and iterative refinement in applied research settings.

6. Conclusion

This study examined the performance of logistic regression models in predicting binary outcomes, utilizing metrics such as sensitivity, specificity, classification accuracy, the area under the ROC curve (AUC), and Goodness-of-Fit tests. The results indicated moderate predictive power, with sensitivity and specificity reflecting a reasonable balance between true positive and true negative rates. An overall AUC of 0.738 confirmed adequate discriminative ability, though it highlighted potential areas for improvement.

The selected model, incorporating four main effects and specific interaction terms, achieved a classification accuracy of 65.1% and predictive power of 0.658. While additional interaction terms marginally improved accuracy, they did not enhance model fit significantly, as demonstrated by backward elimination results, and increased complexity without clear interpretative benefits. The Hosmer-Lemeshow Goodness-of-Fit test (p-value = 0.1233) supported the final model's adequacy, emphasizing a balance between predictive performance and parsimony.

Key predictors such as city and smoking status emerged as significant, with smoking consistently associated with higher event likelihoods, corroborating existing research. However, predictors like "group" were statistically insignificant (p-value = 0.3817), highlighting the importance of scrutinizing variables for practical relevance. The study also

noted that predictor stability in unbalanced datasets is sensitive to minor data changes, cautioning against over-reliance on predictors without rigorous validation.

Model comparisons using AIC and R-squared values revealed marginal differences, with simpler models occasionally performing better in terms of interpretability. Type 3 effects testing identified variables like "width" as significant predictors (p-value = 0.0365), while others, such as "color," had negligible impacts on outcomes. This finding reinforces the value of model simplicity in yielding interpretable results without substantial performance sacrifices.

The results complement findings from previous research that focused on evaluating hierarchical and clustered binary data models, such as the study on the H-Likelihood Estimation Method for Varying Clustered Binary Mixed Effects Models (El-Saeiti & Pannu, 2024). While the earlier work emphasized parameter estimation in clustered settings, this study shifts the focus to predictive accuracy in logistic regression models, particularly under conditions of unbalanced data. Together, these studies highlight the challenges and opportunities in addressing clustering and unbalanced data, providing a more comprehensive understanding of statistical modeling in binary outcome analysis.

Future research should explore alternative modeling approaches, such as ensemble methods or penalized regression, to enhance predictive accuracy while minimizing overfitting risks. Additionally, investigating the impact of varying probability thresholds on model performance could yield insights for applications requiring specific trade-offs between sensitivity and specificity. Techniques such as data augmentation and stratified sampling may further enhance model stability and reliability, particularly in the context of unbalanced datasets, ensuring more robust and interpretable results.

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