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DETERMINANTS OF PRODUCTIVITY INDEX IN INFINITE-ACTING RESERVOIRS: A CRITICAL EXAMINATION OF MILLER-DYES-HUTCHINSON AND VOGEL MODELS

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ABSTRACT

This work presents a critical analysis of the Productivity Index (PI) of infinite-acting reservoirs. A PI model was developed from the combination of Miller-Dyes-Hutchinson and Vogel`s models by incorporating key reservoir factors such as permeability, net pay thickness, porosity, total compressibility, viscosity, formation volume factor, wellbore radius, and skin factor. The Plackett-Burman design was employed to investigate the impact of these factors on PI, and the results revealed that permeability, net pay thickness, skin factor, and viscosity are the most significant factors affecting PI. The findings of this research provide valuable insights into the behaviour of infinite-acting reservoirs, indicating that permeability and net pay thickness have a positive impact on productivity index, while skin factor and viscosity have a negative impact. The PI is time-dependent. Thicker net pay zones increase productivity index, Lower skin factor increases petroleum index, and production index decreases over time due to the logarithmic term. Conclusively, analysing early-time data makes the understanding of well behaviour easier, the understanding of reservoir properties is crucial in optimizing *production strategies, and optimizing well completion design minimizes skin factor. By applying transient analysis, operators can gain valuable insights into the behaviour of infinite-acting wells and optimize production strategies.*

Keywords: Infinite-acting reservoir, Productivity Index, reservoir factors, well performance optimization.

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1. Introduction

The study of infinite-acting reservoirs and their productivity index (PI) is crucial for optimizing hydrocarbon recovery. The productivity index (PI) is a crucial parameter in petroleum engineering that quantifies the ability of a well to produce hydrocarbons from a reservoir. Accurate estimation of PI is essential for optimizing well performance, predicting reservoir behavior, and making informed decisions regarding field development and production strategies. Infinite-acting reservoirs, characterized by their vast size and lack of significant boundary effects during the life of the well, pose unique challenges in PI estimation. Arnold (2019)

Two widely used models for estimating PI in oil reservoirs are the Miller-Dyes-Hutchinson (MDH) model and the Vogel model.

Miller, dyes and Hutchinson (1950) presented a method of evaluating the effective permeability and the pressure of the reservoir when shut-in, the model is based on the conditions of stabilized well of a homogeneous reservoir formation, unsteady state flow of the reservoir fluid, and increase in bottom hole pressure.

The Vogel model provides a straightforward approach to estimating productivity index based on reservoir properties, while the MDH model offers a more comprehensive framework that accounts for the impact of various factors on well performance. Despite their widespread adoption, these models have limitations and assumptions that can affect their accuracy in different reservoir settings.

This literature review aims to provide an overview of the behaviour of infinite-acting reservoirs, the productivity index, and the factors affecting them.

Infinite-acting reservoirs are characterized by their ability to exhibit transient flow behaviour for an extended period (Matthews and Russell 1967). Infinite-acting reservoirs do not exhibit boundary effects, making it challenging to determine reservoir properties. (Mohammad and Sohrab (2019) noted that infinite-acting reservoirs require careful analysis to understand their behaviour

The productivity index (PI) is a measure of a well's ability to produce fluids (Ibragimov, A., Khalmanova, D., Valko, P.P., and Walton, J.R. 2005). PI is defined as the ratio of flow rate to pressure drawdown. (Kubota, L. 2015).

Several factors affect the behaviour of infinite-acting reservoirs, including permeability, net pay thickness and porosity. Ahmed and McKinney (2011) noted that reservoir properties such as permeability and porosity play a significant role in determining reservoir behaviour.

The productivity index is affected by various factors, including skin factor, viscosity, and wellbore radius. Cinco-Ley and Samaniego (14) found that skin factor significantly impacts PI. Dake (1978) noted that viscosity affects PI, while Brons and Marting (1961) highlighted the importance of wellbore radius.

Firas and Kabbawi (2024) presented new correlations for calculating the Productivity Index (PI) of horizontal wells in a closed, rectangular reservoir. The correlations are based on an infinite-conductivity model and account for various well and reservoir configurations, enabling calculation of PI, average pressure drop, and starting time of pseudo-steady state (PSS). The results show that dimensionless well length significantly impacts productivity index.

Cao, Xiao, & Wang (2015) investigated the impact of nonlinear flow mechanisms on the productivity index (PI) of multiple-fractured horizontal wells (MFHW) in steady-state conditions. A mathematical model was proposed to analyse the effects of non-Darcy flow and pressure-dependent properties on Productivity index. The results show that PI is influenced by fracture properties, reservoir geometry, and flow mechanisms, and is negatively impacted by non-Darcy flow and pressure sensitivity.

Asadi and Zendehboudi (2019) enhanced the Distributed Volumetric Source (DVS) method for evaluating hydrocarbon production from complex well-reservoir systems, including horizontal wells and hydraulic fractures. The modified DVS method accurately captures production mechanisms and indices across transient and pseudo-steady state regimes. It is particularly effective in modelling horizontal well productivity, including the impact of well penetration ratio. The study demonstrated the potential of DVS to optimize Multi-Fractured Horizontal Well design, offering a technically and economically viable approach.

Diyashev and Economides (2006) developed a general approach to evaluating petroleum-well performance using a dimensionless productivity index (JD). JD is calculated from production data and well/reservoir information, allowing for comparison against optimized production benchmarks. This evaluation enables identification of underperforming wells and informs decisions on refracturing, treatment redesign, and well type selection.

Muskat (1937) worked on the fundamental principles of fluid flow in porous media. van Everdingen and Hurst (1949) developed solutions for transient flow in porous media. Vogel's equation is used for solution-gas drive reservoirs, estimating PI based on bottom-hole pressure and reservoir properties. Fetkovich's model approach combines transient and boundary-dominated flow equations to estimate PI.

2. Methodology

In an infinite-acting reservoir, the well's performance will be dominated by transient effects, and the productivity index (PI) will continue to change over time. Investigating the critical Productivity Index (PI) of an infinite-acting well requires considering the transient nature of the pressure response. Where Determining the critical PI in an infinite-acting well is challenging because the well never reaches steady-state conditions, making it difficult to define a critical PI based on stabilized pressure drawdown.

3. Transient Analysis of an Infinite-Acting Well

To investigate the Productivity Index (PI) of an infinite-acting well using transient analysis, starting with the equation of Miller-Dyes-Hutchinson: miller, C.C., Dyes, A.B., and Hutchinson 1950

$$P\omega_F = Pi - \frac{162.6\, q\mu B}{Kh}\left[log(t) + \log\left(\frac{k}{\varphi\,\mu\, Ct\;\; rw\text{^}2}\right) - 3.23 + 0.87S\right] \quad \ldots\ldots\ldots \quad 1$$

Where:

- $P\omega_F$ = flowing bottomhole pressure
- Pi = initial reservoir pressure
- q = flow rate
- μ = fluid viscosity

- B = formation volume factor
- k = permeability
- h = net pay thickness
- φ = porosity
- c_t = total compressibility
- r_w = wellbore radius
- s = skin factor
- t = time

and then expressing Vogel`s equation

$$PI = J = \frac{q}{Pi - Pwf} \quad \ldots\ldots\ldots\ldots\ldots \quad 2$$

Substituting the expression for P_{wf} gives

$$PI = \frac{q}{162.6\ q\mu B\left[log(t) + \log\left(\frac{k}{\varphi\ \mu\ Ct\ rw^{\wedge}2}\right) - 3.23 + 0.87S\right]} \quad \ldots\ldots\ldots\ldots \quad 3$$

Simplifying equation 3, gives

$$PI = \frac{Kh}{162.6\ q\mu B\left[log(t) + \log\left(\frac{k}{\varphi\ \mu\ Ct\ rw^{\wedge}2}\right) - 3.23 + 0.87S\right]} \quad \ldots\ldots\ldots\ldots \quad 4$$

The PI is time-dependent, and its value changes as the well produces over time

4. Influencing Parameters of Productivity Index

The Plackett-Burman design is a statistical technique used to screen for significant factors in a system. A table of these parameters with low, medium, and high values for each reservoir was designed: Using the Placket-Burman design, a set of runs is created to analyse the impact of each factor on the Productivity Index (PI)

Table 1: Parameters influencing the productivity index of oil reservoir in the Niger Delta

Parameter	**Low**	**Medium**	**High**
permeability (K), md	100	1000	2000
Net Pay Thickness(h), ft	3000	10000	18000
Porosity (φ)	0.1	0.2	0.3
Total compressibility (Ct) psi^{-1}	5×10^{-6}	12×10^{-6}	20×10^{-6}
Viscosity (μ), cp	0.5	2.0	5.0
Formation Volume factor(B), rb/stb	1.0	1.2	2.0
wellbore radius(rw), ft	0.125	0.185	0.25
Skin Factor (S),	-5	0	20
Flow Rate(q),	1300	2500	5000

Table 2: Plackett-Burmans runs for influencing parameters productivity index

Run	Q	K	h	φ	*Ct*	*μ*	B	rw	S	T
1	1300	2000	18000	0.3	5×10^{-6}	5.0	1.0	0.125	-5	10
2	1300	100	18000	0.3	20×10^{-6}	0.5	2.0	0.125	-5	10
3	5000	100	3000	0.3	20×10^{-6}	5.0	1.0	0.25	-5	10
4	5000	100	3000	0.1	20×10^{-6}	5.0	2.0.	0.125	20	10
5	5000	2000	3000	0.1	5×10^{-6}	5.0	2.0	0.25	-5	10
6	1300	100	18000	0.1	5×10^{-6}	0.5	2.0	0.25	20	10
7	1300	2000	3000	0.3	5×10^{-6}	0.5	1.0	0.25	20	10
8	1300	2000	18000	0.1	20×10^{-6}	0.5	1.0	0.125	20	10
9	5000	2000	18000	0.3	20×10^{-6}	5.0	2.0	0.25	20	10
10	5000	100	3000	0.1	5×10^{-6}	0.5	1.0	0.125	-5	10
11	1300	2000	3000	0.3	20×10^{-6}	0.5	2.0	0.125	20	10
12	1300	100	18000	0.1	20×10^{-6}	5.0	1.0	0.25	20	10
13	2500	1000	10000	0.2	12×10^{-6}	2.0	1.2	0.185	0	10

Then the productivity index for each run is evaluated assuming time (t) to be I hour and the results were analysed using the Plackett-Burman design to identify the most significant factors affecting PI. Each run was calculated using the developed productivity index equation 4.

5. Results

RUN	K	h	φ	*Ct*	*μ*	B	rw	S	T	PI
1	2000	18000	0.3	5×10^{-6}	5.0	1.0	0.125	-5	10	9.326531261
2	100	18000	0.3	20×10^{-6}	0.5	2.0	0.125	-5	10	3.097594719
3	100	3000	0.3	20×10^{-6}	5.0	1.0	0.25	-5	10	0.064342476
4	100	3000	0.1	20×10^{-6}	5.0	2.0.	0.125	20	10	0.001539043
5	2000	3000	0.1	5×10^{-6}	5.0	2.0	0.25	-5	10	0.209232619
6	100	18000	0.1	5×10^{-6}	0.5	2.0	0.25	20	10	0.340943642
7	2000	3000	0.3	5×10^{-6}	0.5	1.0	0.25	20	10	2.200372219
8	2000	18000	0.1	20×10^{-6}	0.5	1.0	0.125	20	10	12.96251738
9	2000	18000	0.3	20×10^{-6}	5.0	2.0	0.25	20	10	0.182991942
10	100	3000	0.1	5×10^{-6}	0.5	1.0	0.125	-5	10	0.192779812
11	2000	3000	0.3	20×10^{-6}	0.5	2.0	0.125	20	10	1.10018611
12	100	18000	0.1	20×10^{-6}	5.0	1.0	0.25	20	10	0.072862378
13	1000	10000	0.2	12×10^{-6}	2.0	1.2	0.185	0	10	1.356835981

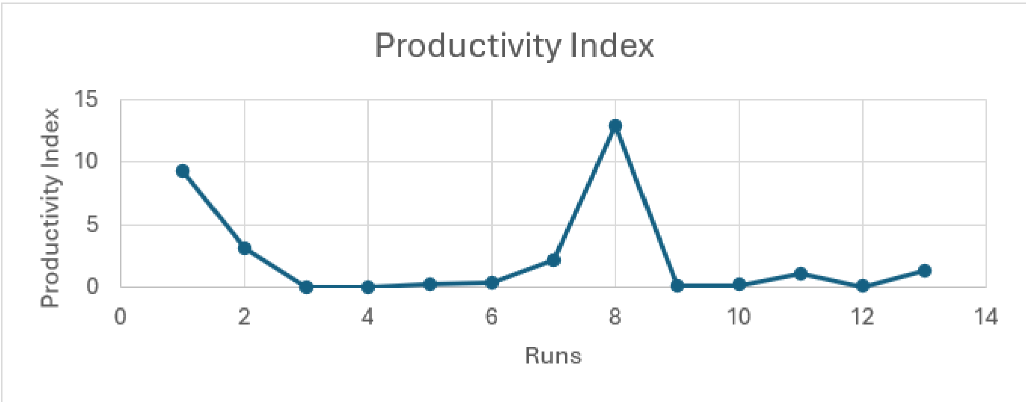

Fig.1 productivity index from each placket-Burman`s Run

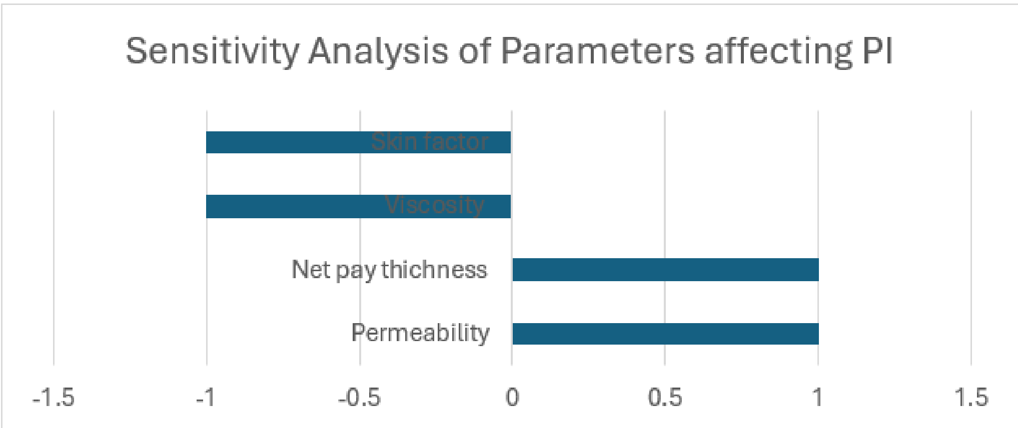

Fig 2. Sensitivity Analysis of influencing parameters of productivity index

6. Discussion

The results of fig 2. indicate that permeability and net pay thickness have a positive impact on Productivity, while skin factor and viscosity have a negative impact. The PI is time-dependent, and its value changes as the well produces over time. By analysing the PI equation, we can see that, higher permeability increases production index. Thicker net pay zones increase productivity index. Lower skin factor increases petroleum index. Production index decreases over time due to the logarithmic term.

7. Conclusions

1. Analysing early-time data makes the understanding of well behaviour easier.
2. 2.The uunderstanding of reservoir properties is crucial in optimizing production strategies.
3. Optimizing well completion design minimizes skin factor. By applying transient analysis, operators can gain valuable insights into the behaviour of infinite-acting wells and optimize production strategies.

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