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RESEARCH ARTICLE

Power-O Graphs of Finite Groups: Structural Properties, Algorithmic Construction, and Applications in Network and Systems Engineering

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ABSTRACT: The Power-O graph of a finite group is a canonical structural representation, that is, a representation of the group whereby each of the group members is represented by a vertex and the relationship between them is defined by the power relationships between the orders of these members. In a finite group G, the graph relates two different elements of different orders in G, i.e. $o(x) \neq o(y)$, when one of these orders is a non-trivial power of the other, i.e. $o(x) = o(y)^m$. This exploration develops the impact of algebraic framework of G to the seizing graph characteristics. We obtain such basic invariants as connectedness, diameter, multipartiteness and edge bounds of the Power -O graph. In addition, we give algorithmic methods of building such graphs and computational complexity analysis, hence offering a complete toolkit to researchers who want to use such structures in a computational framework. In addition to the theoretical advancement, this paper shows the applicability of Power-O graphs in engineering. These include network topology modelling, communication hierarchy design, distributed systems, and cryptographic constructions that use graph-based primitives, thus demonstrating the applicability of the graph concept in applied areas. Case based studies demonstrates that a specific structure of Power-O graphs is inherently related to hub-based networks, hierarchical multi-layers networks and clustered communication networks, hence highlighting the possibility of such mathematical constructs to reflect rich network behaviours that are seen in practice. Finally, the current research reinforces the conceptual continuum between the group theory and the graph theory to make the Power-O graph a mathematically sound but practical useful model to study, analyze, and optimize complex networked systems.

KEYWORDS: Power-O graphs; finite groups; network topology; distributed systems; multipartite structures; algorithmic graph construction.

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INTRODUCTION

Graph-theoretic structures derived from algebraic objects have long played a central role in the modelling of complex systems, particularly where hierarchical, modular, or layered interactions are present [1-5]. In algebraic graph theory, graphs constructed from finite groups provide insight into combinatorial behaviour and offer mechanisms to encode symmetry, order relations, and algebraic dependencies into network structures [6-8].

The existing research on divisor graphs, power graphs, order-divisor graphs, and prime graphs has demonstrated how algebraic relations can be translated into graph-theoretic properties with practical significance. The recently introduced Power-O graph extends this family by defining adjacency in terms of order-based power relations among group elements [9-12]. While the motivation for this concept is rooted in group theory, the structural outcomes of such order-driven adjacency rules mirror the architectures of several engineering systems [13].

Modern engineering, particularly communication systems, network engineering, distributed computing, cyber-physical systems, data flow models, and cryptographic system analysis, frequently employs hierarchical or multi-level networks. These systems often involve elements with discrete "capabilities" or "weights" analogous to the orders of group elements. Power-O graphs, with their order-

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based connectivity, therefore provide a natural mathematical framework to model [14].

Algebraic graph theory is a fascinating area of mathematics that combines the study of graphs with the tools and concepts of group theory. By examining the relationships between these two fields, we can uncover important properties of graphs that might not be immediately apparent [15-18]. One such property concerns the groups associated with a graph and their orders. In algebraic graph theory, we often associate graphs with groups in various ways, such as through the construction of group-based graph models or by analyzing how graphs can be represented using groups [19]. The order of a group, which is the number of elements it contains, is a key concept in group theory. When connected to a graph, the order of the associated group can give us valuable insights into the structure and characteristics of the graph itself. For example, in certain classes of graphs, the order of a related group can indicate how complex or symmetrical the graph is. A larger group order might suggest that the graph has more intricate patterns or a higher level of symmetry, while a smaller group order could imply a simpler, less regular structure.

Singh and Santhosh introduced the concept of divisor graphs in 2000 where an edge exists between a pair of vertices (x, y) iff one vertex divides the later. In 2010, P.J. Cameron introduced the power graph of a finite group, in which two vertices are adjacent if one is a power of the other. In 2015, Burness and Covato wrote a paper on the prime graph of a simple group. In 2018, Rehman, et al., defined order divisor graph of finite groups, denoted by OD(G), whose vertex set is G such that two distinct vertices a and b having different orders are adjacent provided that order of a vertex divides the order of the later vertex Further properties of the graph was discussed by X Liu X. Ma in 2020.

The primary objective of this article is to present a class of graphs linked to finite groups. We define the Power-O graph of G, denoted by $\lambda(G)$ as a graph whose vertex set is the group G and in which two vertices x and y are adjacent iff $o(x) = o(y)^{m}$ or $o(y) = o(x)^{m}$, m being any non-negative integer other than 1. Subsequently, we will delve into the characteristics of these graphs.

The Power-O graph of a finite group will be a structure of algebraic graph theory where plates become vertices, and two different plates are related if a non-trivial power of an order is a deriving power of a different order of a plate. The main purpose of it is to encode the internal ordering structure of a finite group into a more combinatorial structure, providing structural analysis tools, including connectivity, diameter, multi-partiteness, and bounds on the number of edges.

The graph is never disconnected, always has at most a diameter of 2, usually has a star or multipartite structure, and is in a cyclic group of prime-power order, with a layered complete multipartite structure. In addition to having theoretically important applications in the relationship between group theory and graph theory, the Power-O graph has been used in network topology modelling, hierarchical communication systems, distributed computing architectures, cryptographic constructions, and multi-layer engineering systems, with naturally ordered elements representing capability levels or ranks in a hierarchy [11]. Therefore, it offers a mathematically and practically viable structure for examining structured and layered complex structures.

2. Basic definitions and notations

A graph G is a pair $G = (V, E)$ consisting of a finite set V (of vertices) and a set E (edges) of 2-element subsets of V. Two vertices $u, v \in V$ are said to be adjacent if an edge join u and v and two edges are adjacent if they have a common vertex. An edge that joins a vertex to itself is called a loop. A graph with no loops or multiple edges is called a simple graph. A graph is said to be connected if there is a path between every pair of distinct vertices. A walk $v_0 e_1 v_1 \dots e_k v_k$ is said to be a cycle if $v_0 = v_k$ and the vertices $v_0 v_1 \dots v_{k-1}$ are distinct from each other. For a connected graph, we define the diameter of G denoted by $d(G)$ as $d(G) = max_{u,v \in N(G)} d(u, v)$. A multi-partite graph or a k −partite graph is a graph whose vertices set can be partitioned into k sets such that there exists an edge between any two vertices in different sets, but there is no edge between any two vertices in the same set. A complete multipartite graph is a simple graph whose vertices can be portioned into sets so that $u \leftrightarrow v$ iff u and v belong to different sets of the portioned.[1].

Let G be a set together with a binary operation (usually called multiplication) that assigns to each ordered pair (a, b) of elements of G an element in G denoted by ab. We say G is a group under this operation if the operation is associative and there exist an identity element and an inverse. The order of an element g in a group G is the smallest positive integer n such that $g^n = e$. (In additive notation, this would be $ng = 0$. An abelian group is called an elementary abelian group if it is a p-group of order p for some prime p. [2].

In this paper we obtain we obtain the following results. Power-O graph of a finite group with order n is always a connected graph with diameter atmost 2. The Power-O graph of an elementary abelian group is a star graph.Power $-$ O graph is a star graph iff $G = p_1 p_2 \ldots p_k$ for distinct p_i, $i = 1, 2, \ldots, k$. We also find an lower bound for the number of edges of the Power-O graph of a finite group of order n with no self-inverse element. We investigate when a $\lambda(G)$ is multipartite and completely multipartite. Also, if G is a finite graph of order p^2. Then $\chi(\lambda(G)) = 3$. We denote by $G_0 \diamond G_1 \diamond G_2 \diamond \ldots \diamond G_k$ the sequential join of graphs $G_0, G_1, G_2, \ldots, G_k$ where $G_i \diamond G_{i+1}$ for all $1 \leq i \leq k-1$ i.e by adding an edge from each vertex of G to each vertex $G_{i+1}, 1 \leq i \leq k-1$. We get for a cyclic group of order p^2 where p is a prime number, $\lambda(G)$ is a sequential join $(G_1 \diamond G_2) \diamond K_1$, where $G_1 \cong (p_1 - 1)K_1, G_2 \cong p(p-1)K_1$.

Result 2.1. *If G is a finite group and p is a prime number dividing the order of G (the number of elements in G), then G contains an element of order p.* [2].

3. The Power-O Graph

Definition 3.1. *Let G be a finite group of order n and λ be a graph with vertex set = G. Let x and y be any two distinct vertices of λ with distinct order. Then there exists an edge between x and y iff $o(x) = o(y)^m$ or $o(y) = o(x)^m$, for any integer m ≥ 2.*

Example 1.

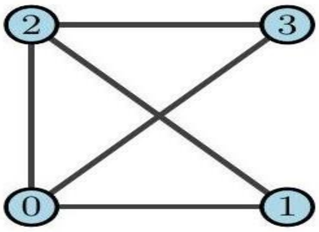

Fig 1: Graph of λ (Z_4)

Remark:

- $\lambda(G)$ is a simple graph, so there are no loops and multiple edges.
- For $|G| = n$, the vertex corresponding to the identity element e of $\lambda(G)$ has a degree $n - 1$.
- For any non-self-inverse element, a of a finite group G, a and a^{-1} are non-adjacent in $\lambda(G)$.

Theorem 3.1. If $|G| \geq 2$, $\lambda(G)$ is always a connected graph with diameter atmost 2.

Proof. Clearly the vertex associated with the identity element is adjacent to each vertex. Therefore, for any two vertices x and y, there exists a path $x - e - y$. Hence $\lambda(G)$ is always a connected graph of diameter atmost 2.

Theorem 3.2. If $|G| \geq 3$, then $\lambda(G)$ cannot be a cycle.

Proof. If $|G| = 3$, it has 2 elements of order 3 which cannot be adjacent.

If $|G| > 3$, then the vertex associated with identity element is adjacent to each other vertex. Therefore if $|G| \geq 3$, then $\lambda(G)$ cannot be a cycle.

Theorem 3.3. If G is an elementary abelian group, then $\lambda(G)$ is a star graph.

Proof. Suppose that G is an elementary abelian group. Consider 2 distinct non-identity elements x and y. Since G is elementary abelian group $o(x) = o(y)$. Then the existence of an edge is not possible. Similarly, if every non-identity element of G has prime order, then $\lambda(G)$ is a star graph.

Example 2.

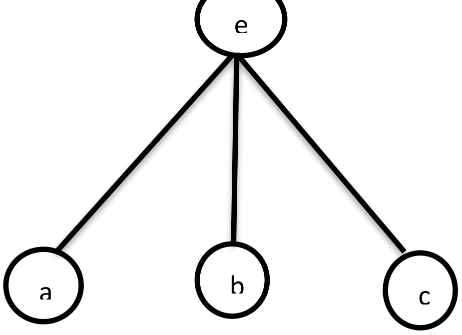

Fig 2: Graph of λ(V_4)

Theorem 3.4. λ (G) is complete if $G = Z_1$ or Z_2

Proof. Clearly if $G = Z_1$ or Z_2 , λ (G) is complete.

Conversely if λ (G) is complete. Assume order of $|G| \geq 3$.

Case(i): If G has one non-self inverse element then $\lambda(G)$ cannot be complete.

Case (ii): Suppose every element is a self-inverse element. Let $x, y \in G$ and $x, y \neq e$. Then $o(x) = o(y)$. Thus x and y cannot be adjacent. There arises a contradiction

Theorem 3.5. λ (G) is a star graph iff $G = p_1 p_2 \ldots p_k$ for distinct p_i, $i = 1, 2, \ldots, k$.

Proof. Clearly if $G = p_1 p_2 \ldots p_k, \lambda(G)$ is a star graph. Conversely assume that $p_1{}^{a_1}, p_2{}^{a_2}, \ldots, p_k{}^{a_k}$ and Λ (G) is a star graph. Let x, y are elements such that $o(x) = d_1$ and $o(y) = d_2$. Since $\lambda(G)$ is a star graph $o(x) \neq (o(y))^m$. But this violates Cauchy's theorem. Hence the proof.

Theorem 3.6: Let *G* be a finite group of order n with no self inverse element and q be the number of edges in $\lambda(G)$. Then $q \leq \frac{(n-1)^2}{2}$

Proof: Clearly deg(e) in $\lambda(G)$ is $n - 1$, where e is the identity element of G. Since G has no self inverse element, for all $x \in G - e$, $\deg(x) \leq n - 2$. From this we getsum of the degrees of vertices $\leq (n-1) + (n-1)(n-2) = (n-1)^2$. Hence $q \leq \frac{(n-1)^2}{2}$

Theorem 3.7. λ (G) is a multi partite graph if and only if G is a finite of order p^n, where p is a prime number and n is a perfect square.

Proof. Let G be a group and p is a prime number. Consider the subgroup G_i where $G_i = \{a \in G$ such that $o(a) = p^i\}$. Now $G = G_0 \cup G_1 \cup \ldots \cup G_n$. An element of order p^i for $i > 1$ may be non-existent. But by Cauchy's theorem there exist an element of order p. Therefore G_0 and G_1 are non-empty i.e $G = G_0 \cup G_1 \cup \ldots \cup G_n$ is non-empty. Clearly for some $x, y \in G_i, xy \neq E(\lambda(G))$, for some $0 \leq i \leq n$. Suppose $x \in G_i$ and $y \in G_j$ where $i \neq j$. Then $o(x) = p^i$ and $o(y) = p^j$. If $i < j$, then $p^i = (p^j)^k$, for some integer k and if $j < i$, then $p^j = (p^i)^k$, for some integer k. Therefore $xy \in E(\lambda(G))$ and hence λ (G) is a multipartite graph.

Corollary 3.8: If G is a finite graph of order p^2. Then $\chi(\lambda(G)) = 3$

Theorem 3.9: Let G be a finite group of order p^2. Then $\lambda(G)$ is a complete multipartite graph. Also $\lambda(G) \cong K_{1,(p-1),p(p-1)}$, a 3-partite graph

Proof: Let p be a prime and $A_i = \{x \in G | o(x) = p^i\}$. Consider $G = A_0 \cup A_1 \cup A_2$. It is not necessary that G has elements of order p^i for $i > 1$ and by Cauchy's theorem, G must have an element of order p. Therefore for some $i > 1$, A_i may be empty but A_0 and A_1 must be non-empty in the union $A_0 \cup A_1 \cup A_2$. Clearly st not an element of edge set of $\lambda(G)$ if $s, t \in A_i$ for some $0 \leq i \leq 2$. Suppose $s \in A_i$ and $t \in A_j$, where $i \neq j$. Then $o(s) = p^i$ and $o(t) = p^j$. If $i < j$ then p^i strictly divides p^j and if $i > j$, then p^j strictly divides p^i. Therefore $st \in E(\lambda(G))$ and hence $\lambda(G)$ is complete multipartite graph.

Now let $A_0 = \{x \in G: o(x) = 1\}, A_1 = \{x \in G: o(x) = p\}, A_2 = \{x \in G: o(x) = p^2\}$. Consider $G = A_1 \cup A_2 \cup A_3$. Since G is cyclic, by [1,Theorem 4.4], $|A_0| = 1, |A_1| = \phi(p) = p - 1$ and $|A_2| = \phi(p^2) = p(p-1)$, where ϕ is the Euler function. Note that st is an element in edge set of $\lambda(G)$ if $s, t \in A_i$ for any $0 \leq i \leq 2$. Suppose that $s \in A_i$ and $t \in A_j$ where $i \neq j$. then $o(s) = p^i$ and $o(t) = p^j$. If $i < j$, then p^i strictly divides p^j and if $j < i$, then p^j strictly divides p^i. Therefore $st \in \lambda(G)$ and hence $\lambda(G) \cong K_{1,(p-1),p(p-1)}$.

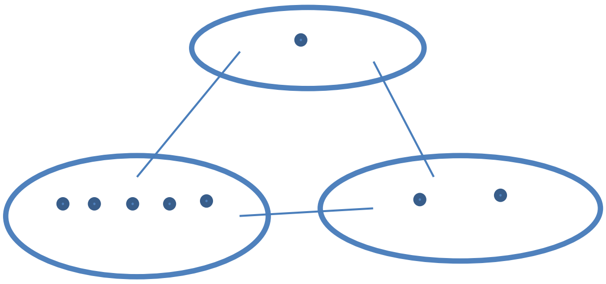

Fig 3: Graph of $\lambda(Z_{3^2})$:The line joining two independent sets represent set of edges joining every vertex of one set to every vertex of other set

We denote by $G_0 \diamond G_1 \diamond G_2 \diamond \ldots \diamond G_k$ the sequential join of graphs $G_0, G_1, G_2, \ldots, G_k$ where $G_i \diamond G_{i+1}$ for all $1 \leq i \leq k-1$ i.e by adding an edge from each vertex of G to each vertex $G_{i+1}, 1 \leq i \leq k-1$

Theorem 3.10: Let G be a cyclic group of p^2 where p is a prime number. Then $\lambda(G)$ is a sequential join $(G_1 \diamond G_2) \diamond K_1$, where $G_1 \cong (p_1 - 1)K_1, G_2 \cong p(p-1)K_1$.

Proof : The divisor of p^2 are $1, p$ and p^2. We make a partition of the vertex set Gas $G = A_p \cup A_{p^2} \cup A_1$, where $A_p = \{x \in G : o(x) = p\}, A_{p^2} = \{x \in G : o(x) = p^2\}, A_1 = \{x \in G : o(x) = 1\}$. [By [1], theorem 4.4], $|A_p| = \phi(p_1) - p_1 - 1, |A_{p^2}| = p(p-1), |A_1| = 1$ where ϕ is the Euler's phi function. Since p strictly divides p^2, so each vertex in A_p is adjacent to each vertex in A_{p^2}. Clearly the single vertex in A_1 is adjacent to every vertex in A_1 and every vertex in A_{p^2}. Also note that the vertices in A_p and A_{p^2} are independent. Hence $\lambda(G) = (G_1 + G_2) + K_1$ where $G_1 \cong (p-1)K_1$ and $G_2 \cong p(p-1)K_1$

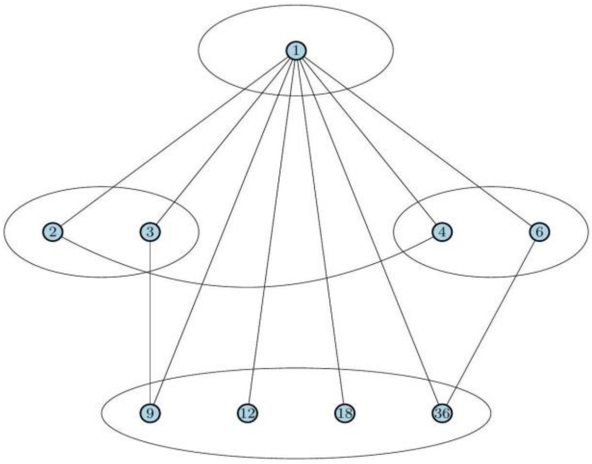

Fig 4: Graph of $\lambda(Z_{36})$

4.Generalized Divisor Sum T_k – function of Power-O Graph

Theorem 4.1: Generalized divisor sum $\mathbf{T_k}$ − function $\boldsymbol{\lambda}(\mathbf{Z_n})$ when $\mathbf{n = p_1 p_2 \ldots p_k}$ where p is a prime number is

$$\mathbf{T_k(\lambda(Z_p)) = \left(\prod_{i=1}^{t} \frac{p_i^{(a_i+1)k} - 1}{p_i^{k} - 1}\right) + (n-1)} \text{ where } \mathbf{(n-1) = p_1^{a_1} p_2^{a_2} \ldots p_t^{a_t}}$$

Proof: By Theorem 3.5 $\boldsymbol{\lambda}(\mathbf{Z_n})$ when $\mathbf{n = p_1 p_2 \ldots p_k}$, is a star graph. Thus we have $\mathbf{n-1}$ vertices of degree one and one vertex of degree $\mathbf{n-1}$. Thus [using 11,Preposition 2.2] we have

$$\mathbf{T_k(\lambda(Z_p)) = \sigma_k(n-1) + (n-1)}$$

$$\mathbf{= \left(\prod_{i=1}^{t} \frac{p_i^{(a_i+1)k} - 1}{p_i^{k} - 1}\right) + (n-1)}$$

where $\mathbf{(n-1) = p_1^{a_1} p_2^{a_2} \ldots p_t^{a_t}}$

Theorem 4.2: Generalized divisor sum $\mathbf{T_k}$ − function of $\boldsymbol{\lambda}(\mathbf{G})$,when order of $\mathbf{G}$ is $\mathbf{p^2}$, is

$$\sum_{i=1}^{n} \left(m_i * \left(\prod_{i=1}^{t} \frac{p_i^{(a_i+1)k} - 1}{p_i^{k} - 1} \right) \right)$$

Proof:

$$T_k(K_{m_1,m_2,m_3}) = \sum_{v \in V(K_{m_1,m_2,m_3})} \sigma_k(deg(v))$$

$$= \sigma_k(deg(v_1)) + \sigma_k(deg(v_2)) + \cdots + \sigma_k(deg(v_n))$$

$= \sum_{i=1}^{n} \left(m_i * \left(\prod_{i=1}^{t} \frac{p_i^{(a_i+1)k} - 1}{p_i^{k} - 1} \right) \right)$ [using 11,Preposition 2.2]

Comparison of Power-O Graph, Divisor Graph and Order Divisor Graph

Graph Type	Vertex Set	Adjacency Condition	Number of Edges	Diameter	Clique Number	Special Remarks
Power-O Graph P_O(G)	G	o(x)=o(y)^m or o(y)=o(x)^m, m≠1	≥ n−1; depends on order structure	≤ 2 (always connected)	Depends on order structure	
Divisor Graph D(G)	G	o(x) ∣ o(y) or o(y) ∣ o(x)	Depends on divisibility of orders	≤ 3	Depends on longest divisibility chain	Introduced by Singh & Santhosh (2000)
Order Divisor Graph OD(G)	G	Different orders with one dividing the other	Depends on order distribution	≤ 3	Depends on comparable orders	Defined by Rehman et al. (2018)

Power-O graphs based on finite groups give a mathematically rigorous and structurally efficient modeling framework of complex systems in Computer Science and Engineering. Among the most notable uses are in network topology design, where it is possible to design low-latency communication networks and to design highly efficient networks with the properties of Power-O graphs, including guaranteed connectivity and the ability to have a diameter of at most two (highlighted). In these models, network nodes are modeled by the vertex and possible communication links by the order-power relationships are the edges. The identity element of the group is also a natural beholder of the central hub, thus reminiscent of star or hub-and-spoke architectures typical of the present-day data centers and high-speed communication network. This topological attribute has a low hop count between nodes, and therefore Power-O graphs are a perfect fit for next-generation network technologies such as 5G and beyond [20].

Power-O graphs can also serve as a useful abstraction for hierarchical and multi-level computing environments in distributed systems. The hierarchy of elements of a finite group may be understood as distinct computational abilities or levels of priority in a distributed architecture. The higher-order nodes can be mapped onto the more powerful computer units or controllers whereas the lower-order elements are worker nodes. The proximity established by power relations makes it possible to delegate tasks and propagate them efficiently within the system. This renders Power-O graphs especially helpful in modeling orchestration systems like microservices and container-based systems where hierarchical relationships are crucial to the task scheduling and resource allocation.

Cryptography is another important area of use for Power-O graphs, where algebraic structures are typically used to implement secure communication protocols. The order-based adjacency condition presents a non-linear and non-trivial relational form of structure which can be used to build graph-based cryptographic primitives. Cryptographic keys can be used as vertex, and permitted transformations or relation between keys as edges. The deterministic though complex form of such graphs provides an opportunity to come up with secure key exchange mechanisms and encryption schemes, especially in group-based and post-quantum cryptography. The mathematical strength of the Power-O graphs guarantees that they can be used to develop systems that are more resistant to cryptanalytic attacks [21].

The graphs are also used in designing and optimizing graph algorithms and data structures using the power-O graphs. Their foreseeable structural characteristics like multipartite decomposition and limited diameter permit the creation of special algorithms to traverse, cluster and compute shortest paths. The search and routing algorithms can be used to achieve near-constant-time performance in most cases because any two vertices are connected by at most two edges. It can also be used to partition vertices into independent sets as doing so enables efficient graph compression and storage methods, which are important in working with large scale information structures in computing systems.

Power-O graphs provide a natural dependence model and hierarchy of execution of a task in parallel computing environments. The computational tasks may be represented as each vertex; the dependency relationships are based on order-constrained dependency relationships. This hierarchy means that jobs will be arranged in a way that conflicts are reduced and stalemates are avoided meaning that they will run in parallel. The natural hierarchy also minimizes synchronization cost and enables scalable execution models and Power-O graphs can be used in high performance computing, in GPGUs and workflow management systems.

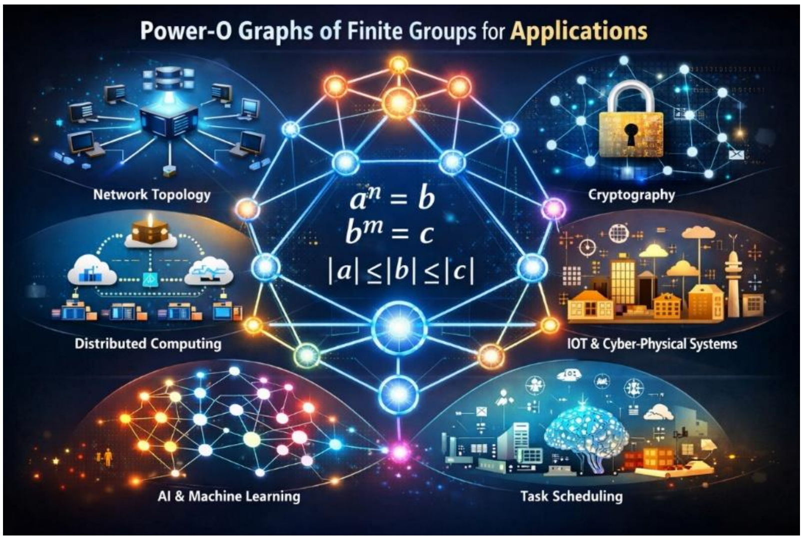

Fig 5: Power-O graphs in technology applications

The applicability of Power-O graphs can also be applied to artificial intelligence and machine learning, in graph-based learning paradigms. Such graphs can be used as organized priors in Graph Neural Networks (GNNs), in which the hierarchical structural relationship between nodes can be used to improve feature representation and learning. The ranking could be understood as the importance of features or importance of their order whereas the meaningful interaction between features is represented by the edges. It results in non-deterministic models that can be more easily interpreted and better at examples of knowledge representation, semantic analysis, and pattern recognition [22].

Power-O graphs are an effective modeling tool of layered architecture of heterogeneous devices in cyber-physical systems and Internet of Things (IoT) networks. Elements of different orders may express devices with different computational and communication abilities, whereas the adjacency relation may reflect possible communication patterns. The simple sensors can be represented by low-order elements, and those at the edge devices or cloud servers can be represented by higher-order elements. This hierarchical model facilitates effective data aggregation, routing and processing thus promoting the performance and scalability of IoT ecosystems.

Another highly important field where Power-O graphs have proven to be of practical value is fault tolerance and system resilience. These graphs have high connectivity and limited diameter, thus, being highly robust to either node or edge failures. In case some parts do not work, the whole system is linked and there is no disconnection in operations. Power-O graphs are especially useful in creating resilient systems in important applications and include aerospace, defense, healthcare infrastructure, and industrial automation [15].

Moreover, Power-O graphs may be applied to the social networks and information flow systems, where hierarchical influence is considered to be important. The nodes may symbolize people or objects, and the rank that the node has is a symbol of influence, authority, or significance. Adjacency relation is a capturing of the capacity of one node to have influence of another by the hierarchical relationships. These models have their use in information dissemination, viral marketing and organizational behavior where the system dynamics are structured by pattern of influence.

Lastly, Power-O graphs are used in compiler design and analysis of programs as a structure to represent instructions or modules dependencies. The hierarchy that is based on orders allows the effective identification of sequence of executions and optimality. These graphs can help a compiler code generate better code, resulting in higher compilation and runtime performance by reducing the number of dependency chains, as well as by providing a structured flow of execution.

In general, the interdisciplinary intersection between the fields of abstract algebra and practical system design as reflected in the integration of the Power-O graphs with Computer Science and Engineering is particularly strong. The intrinsic characteristics that are developed in the mathematical framework are directly translated to the real world, providing scalable, efficient and robust solutions to various areas [22].

CONCLUSION

The Power-O graph presented in this paper offers a structurally large and versatile framework of finite group analysis using an adjacency criterion based on order. The set of theoretical findings made, which include connectivity, diameter bounds, multipartite structures, edge estimations, and star and complete graphs conditions proves that Power-O graphs have clear and predictable architectural characteristics. In addition to being algebraically important, these properties are also consistent with the structural patterns of engineering systems like hierarchical communication networks, wireless architectures, distributed computing layers and order-dependent cryptographic structures.

Using algorithmic approaches to construction and the computational interpretation, the study demonstrates that Power-O graphs can be effectively created and analyzed, which eases their integration into a realistic modelling setting. The shown similarities between the structures of Power-O graphs and physical engineering topologies underscore their possible application as analytical network design, fault analysis, on optimization and multi-layer systems modelling tools. Altogether, combining the group-theoretic roots with engineering applicability, the Power-O graphs are a potential opener to the crossroads of abstract algebra and practical network science. The future work can also build on this framework with dynamic networks, probabilistic group actions, or algorithmic optimization problems, and thereafter generalize it to more fields in mathematics and engineering.

Here the Power-O graph of a finite group is a rigorous definition through the use of order-based power relations, and some structural results are achieved: the graph is always connected and is of diameter at most two; structural necessary and sufficient conditions are obtained when the graph is a star, complete, multipartite, or complete multipartite; and a lower bound to the number of edges taken in the specified conditions of the group. Characterizations for elementary abelian groups and cyclic groups of prime power order are given explicitly, including structural decompositions in the form of sequential joins of independent sets. These findings explain the use of algebraic properties, especially element orders and their relations of divisibility to define graph architecture.

Nevertheless, although possible engineering uses are mentioned (e.g., hierarchical networks, distributed systems and cryptographic structures), these relationships are only abstract. None of the simulations, empirical data sets or performance measures are given to confirm the effectiveness of modelling and no formal correspondence theorem is drawn between algebraic invariants and quantifiable behaviour of engineering networks. Similarly, though the algorithmic construction steps are described, no complexity bounds, scalability analysis or comparisons with other existing algorithms to generate graphs are provided meaning that the analysis of computational advantage is not easily made. The theoretical structural classification of the work and the graph characterizations in terms of orders, then, come to be seen as the major contribution of the work, whereas the applied implications are to be thought of as the potential directions for further research instead of the experimentally confirmed results.

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Conflict of interest

The authors of this manuscript have no conflict of interest to declare.

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