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SOLUTIONS OF NEGATIVE PELL EQUATION INVOLVING CHEN PRIME

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Abstract

In this manuscript, we look for non-trivial integer solution to the equation $x^2 = 29y^2 - 7^t$, $t \in N$ for the singular choices of particular by (i) $t = 1$, (ii) $t = 3$, (iii) $t = 5$, (iv) $t = 2k$, (v) $t = 2k + 5$, $\forall k \in N$. Additionally, reappearance relations on the solutions are obtained.

I. Introduction

It is well known the Pell equation $x^2 - Dy^2 = 1 (D > 0$ and square free) has at all times positive integer solutions. When $N \neq 1$, the Pell equation $x^2 - Dy^2 = -N$ possibly will not boast at all positive integer solutions. In favor of instance, the equations $x^2 = 3y^2 - 1$ and $x^2 = 7y^2 - 4$ comprise refusal integer solutions.

This manuscript concerns the negative Pell equation $x^2 = 29y^2 - 7^t$, where $t > 0$ and infinitely numerous positive integer solutions are obtained

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for the choices of$t$ known by (i) $t = 1$, (ii) $t = 3$ (iii) $t = 5$ (iv) $2k$ and (v) $t = 2k + 5$. A few fascinating relationships surrounded by the solutions are obtainable. Supplementary reappearance relationships on the solutions are consequent.

II. Preliminary

The Pell equation is a Diophantine equation of the form $x^2 - dy^2 = 1$. Given d, we would like to find all integer pairs (x, y) that satisfy the equation. Since any solution (x, y) yields multiple solutions $(\pm x, \pm y)$, we may restrict our attention to solutions where x and y are nonnegative integers.

We usually take d in the equation $x^2 - dy^2 = 1$ to be a positive nonsquare integer. Otherwise, there are only uninteresting solutions: if $d < 0$, then $(x, y) = (\pm 1, 0)$ in the case $d < -1$, and $(x, y) = (0, \pm 1)$ or $(\pm 1, 0)$ in the case $d = -1$; if $d = 0$, then $x = \pm 1$ (y arbitrary); and if d is a nonzero square, then dy^2 and x^2 are consecutive squares, implying that $(x, y) = (\pm 1, 0)$.

Notice that the Pell equation always has the trivial solution $(x, y) = (1, 0)$. We now investigate an illustrate case of Pell's equation and its solution involving recurrence relations.

Theorem 1. *Let p be a prime. The negative Pell's equation*

$$x^2 - py^2 = -1$$

is solvable if and only if $p = 2$ *or* $p \equiv 1 \pmod 4$.

III. Method of Analysis

In favor of this meticulous equation, we think about the prime $p = 29$. Given that p satisfies all the setting of Theorem 1, we can terminate to the negative Pell equation $x^2 = 29y^2 - 7^t$, $t \in N$ is solvable in integers.

3.1 Choice 1. $t = 1$

The Pell equation is

$$x^2 = 29y^2 - 7. \tag{1}$$

Assent to (x_0, y_0) be the primary key of (1) known by

$$x_0 = 43;\ y_0 = 8.$$

In the direction of finding the additional solutions of (1), think about the Pell equation

$$x^2 = 29y^2 + 1$$

whose initial solution $(\tilde{x}_n, \tilde{y}_n)$ is given by

$$\tilde{x}_n = \frac{1}{2} f_n$$

$$\tilde{y}_n = \frac{1}{2\sqrt{29}} g_n$$

where

$$f_n = (9801 + 1820\sqrt{29})^{n+1} + (9801 - 1820\sqrt{29})^{n+1}$$

$$g_n = (9801 + 1820\sqrt{29})^{n+1} - (9801 - 1820\sqrt{29})^{n+1},\ n = 0, 1, 2, \ldots$$

Applying Brahma Gupta lemma connecting (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the progression of non-zero dissimilar integer solutions to (1) is obtained as

$$x_{n+1} = \frac{1}{2}[43f_n + 8\sqrt{29}g_n] \tag{2}$$

$$y_{n+1} = \frac{1}{2\sqrt{29}}[8\sqrt{29}f_n + 43g_n]. \tag{3}$$

The reappearance relation fulfilled by the solutions of (1) is specified by

$$x_{n+2} - 19602x_{n+1} + x_n = 0$$

$$y_{n+2} - 19602y_{n+1} + y_n = 0.$$

Choice 2. $t = 3$

The Pell equation is

$$x^2 = 29y^2 - 7^3. \tag{4}$$

Allow (x_0, y_0) be the primary solution of (4) specified by

$$x_0 = 11; \; y_0 = 4.$$

Applying Brahma Gupta connecting (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the progression of non-zero dissimilar integer solutions to (4) is obtained as

$$x_{n+1} = \frac{1}{2}[11f_n + 4\sqrt{29}g_n] \tag{5}$$

$$y_{n+1} = \frac{1}{2\sqrt{29}}[4\sqrt{29}\, f_n + 11g_n]. \tag{6}$$

The reappearance relationships satisfied by the solutions of (4) are specified by

$$x_{n+2} - 19602x_{n+1} + x_n = 0$$

$$y_{n+2} - 19602y_{n+1} + y_n = 0.$$

Choice 3. $t = 5$.

The Pell equation is

$$x^2 = 13y^2 - 7^5. \tag{7}$$

Agree to (x_0, y_0) be the primary key of (7) specified by

$$x_0 = 677; \; y_0 = 128.$$

Applying Brahma Gupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the sequence of non-zero distinct integer solution to (7) are obtained as

$$x_{n+1} = \frac{1}{2}[677f_n + 128\sqrt{29}g_n] \tag{8}$$

$$y_{n+1} = \frac{1}{2\sqrt{29}}[128\sqrt{29}\, f_n + 677g_n]. \tag{9}$$

The reappearance relationships satisfied by the solutions of (7) are given by

$$x_{n+2} - 19602x_{n+1} + x_n = 0$$

$$y_{n+2} - 19602y_{n+1} + y_n = 0.$$

Choice 4. $t = 2k,\ k > 0.$

The Pell equation is

$$x^2 = 29y^2 - 7^{2k},\ k > 0. \tag{10}$$

Let (x_0, y_0) be the primary key of (10) specified by

$$x_0 = 7^k(70);\ y_0 = 7^k(13).$$

Applying Brahma Gupta lemma connecting (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the progression of non-zero separate integer solution to (10) are obtained as

$$x_{n+1} = \frac{7^k}{2}[70f_n + 13\sqrt{29}g_n] \tag{11}$$

$$y_{n+1} = \frac{7^k}{2\sqrt{29}}[13\sqrt{29}\,f_n + 70g_n]. \tag{12}$$

The reappearance relationships fulfilled by the solutions of (10) are specified by

$$x_{n+2} - 19602x_{n+1} + x_n = 0$$

$$y_{n+2} - 19602y_{n+1} + y_n = 0.$$

Choice 5. $t = 2k + 5,\ k > 0.$

The Pell equation is

$$x^2 = 29y^2 - 7^{2k+5},\ k > 0. \tag{13}$$

Let (x_0, y_0) be the primary key of (13) given by

$$x_0 = 7^{k-1}(45571);\ y_0 = 7^{k-1}(8464).$$

Applying Brahma Gupta lemma connecting (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the progression of non-zero dissimilar integer way out to (13) is obtained as

$$x_{n+1} = \frac{7^{k-1}}{2}[45571f_n + 8464\sqrt{29}g_n] \tag{14}$$

$$y_{n+1} = \frac{7^{k-1}}{2\sqrt{29}}[8464\sqrt{29}\, f_n + 45571 g_n]. \tag{15}$$

The reappearance relationships fulfilled employing the solutions of (13) are convinced utilizing

$$x_{n+2} - 19602 x_{n+1} + x_n = 0$$

$$y_{n+2} - 19602 y_{n+1} + y_n = 0.$$

IV. Conclusion

Solving a Pell's equation employing the on top of technique technology authoritative instrument for discover solutions to equations of parallel form. Neglecting in the least time bearing in mind it is probable by present methods to find out the solvability of Pell-like an equation.

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