Inventory Planning with Batch Ordering in Multi-echelon Multi-product Supply Chain by Queuing Approach

Ebrahim Teimoury, Ali Mazlomi, Raheleh Nadafioun, Iman G. Khondabi, and Mehdi Fathi

***Abstract*—In this paper, we apply queuing models for performance evaluation analysis in multi-product multi-echelon manufacturing supply chain network with batch ordering. The analysis is clubbed with an inventory optimization model, which can be used for designing inventory policies for each product. We consider a three-echelon supply chain: retailers, warehouses and manufacturing plants supply types of products to various retailers. Production system is MTS and we use queue operating under inventory control rule to analyze the performance of any manufacturing plant. Proposed model determines the optimal inventory level at the warehouse of each product that minimizes total expected cost. Moreover, we extend the proposed model in order to analyze the logistics process.**

***Index Terms*— Queuing system, Supply chain, production/inventory systems, order batching, inventory control.**

I. INTRODUCTION

A supply chain is an integrated manufacturing process where in raw materials are converted to final products then delivered to customers. A supply chain consists of all parties involved in fulfilling customer's demands. The supply chain includes not only the manufacturer and suppliers, but also transporters, warehouses, retailers, and even customers themselves. A supply chain is consisted of two basic integrated processes: the production planning and inventory control process and the distribution and logistics process. The supply chain is dynamic and involves the constant flow of information, product and funds between different stages. The principal purpose of any supply chain is to fulfill customer's demands and generate profit for itself. In reality, a manufacturer may receive material from several suppliers and then supply several distributors. Thus, most supply chains are actually networks. The objective of every supply chain should be to maximize the overall generated value. The value a supply chain generates is the difference between what the final product is worth to the customer and the costs the supply chain incurs in filling the customer's demands.

Also, a manufacturing supply chain can be viewed as a network of suppliers, manufacturing sites, distribution centers, and customer locations, through which components and products flow. Throughout these networks, there are different sources of uncertainties, including supply (availability and quality), process (machine breakdown, operator variation), and demand (arrival time and volume).

Also, these variations will transmit from upstream stages to downstream stages and will lead longer cycle time and lower fill-rates.

One of the challenges in supply chain management is to control the capital in inventories. The objective of inventory control is therefore to balance conflicting goals like keeping stock levels down to have cash available for other purposes and having high stock levels for the continuity of the production and for providing a high service level to customers. A good inventory management system has always been important in the workings of an effective manufacturing supply chain.

Queuing systems are the natural models when dealing with problems where the main characteristics are congestion and jams. In this paper, we use $GI/G/1$ queue as tool for performance measures of the manufacturing supply chain and also, we use queue to analyze logistics processes.

We review the articles of inventory management in logistics chains including single-product multi-stage systems and multi-product systems (Table1).

Table1. Related articles in inventory management in logistics chains

Reference article(s)	Article's code (problem definition/constraints/outputs/objective functions)
[1] and [2]	Queueing, Single Production, Decomposition Method
[3]	Queueing, Assemble, Single Production, Stochastic, $M/M/1$, Longest Path Analysis
[4]	Inventory Queueing, Single Production, Decomposition Method
[5]	Multi Production, $M/G/1$, Lead Time
[6]	Queueing, Multi Production, Decomposition Method
[7]	Inventory Queueing, Multi Production, Continuous, $M^X/G/\infty$
[8]	Queueing, Single Production, Batch, $GI^X/G/1$
[9]	Production Inventory, Multi Production, Stochastic, Batch
[10]	Production Inventory, Stochastic, Batch, Decomposition Method, Monte Carlo simulation

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The remainder of this the paper is organized as follows. In section II, we described the principal characteristics of the model. In section III, we perform the proposed model and provide computational results. Finally, we give some concluding remarks in section IV.

II. Problem Description and formulation

We consider a three echelon supply chain network including n retailers, L warehouses and L manufacturing plants as shown in Fig.1. This network offers L types of product to the customers arrived into retailers' node. Customers' demands enter to the retailers and the whole demand accumulation for each product is forwarded to warehouses of that product. We apply production authorization (PA) system to produce each production. The PA system is a generalized pull-based production control system. We assume that products in inventories are stored in batches for each product j, and there is a PA card attached to each batch. In this paper, we consider the case when the number of PA cards is the same as the number of batches. The PA system operates in the following way whenever Q_j units are depleted from a batch in the inventory; the corresponding PA card is transmitted to the manufacturing plant. And also serve as new production orders that trigger the manufacturing plant to begin its production process. In general, the manufacturing plant uses a FCFS discipline to produce these orders. Once the manufacturing plant produces Q_j units, the finished units and the PA card are sent to the warehouse. In the event when a customer places an order and there is no production inventory available, we assume that this customer wait until the product becomes available. In our model, we assume that the set-up time is incurred when the processor begins its production for each batching order.

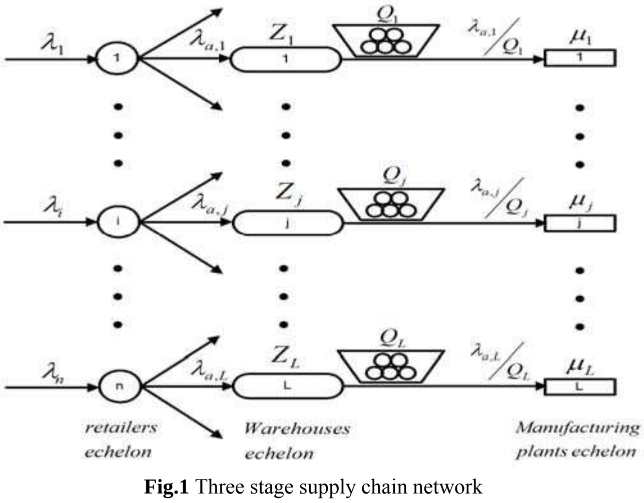

Fig.1 Three stage supply chain network

A. Assumptions

Assumptions of the developed model are as follow:
Customers' demand includes all types of products. We assume the orders of retailer i are as an independent renewal process with a constant rate $\lambda_i \geq 0$ and coefficient of variation C_i^2. The probability vector $q_i = (q_{i1}, q_{i2}, ..., q_{iL})$ defines customers' demand from each kinds of product at retailer $i\,(\sum_{j=1}^{L} q_{ij} = 1, \quad 0 \leq q_{ij} \leq 1; \quad i = 1,2,...,n)$. The orders of warehouse j are as an independent renewal process with a constant rate $\lambda_{a,j} = \sum_{i=1}^{n} \lambda_i q_{ij}$ and coefficient of variation $C_{a,j}^2 = \sum_{i=1}^{n} \lambda_i q_{ij} C_i^2 / \lambda_{a,j}$. In our problem it is assumed that each warehouse hold one type product in batch size Q_j which maximum number of batches is K_j. Therefore, maximum capacity of warehouses for each product is given by $Z_j = K_j \times Q_j$. Production policy is MTS strategy for warehouses. In the following, we use $GI/G/1$ queue operating under $(K_j - 1, K_j)$ inventory control rule to analyze the performance of the single-product type j.

We assumed that unit production times at manufacturing plants for product j are i.i.d. generally distributed random variable, denoted by B_j, with $1/\mu_j \equiv E(B_j)$ and coefficient of variation C_j^2 .Thus, mean production time for batch product j is Q_j/μ_j and coefficient of variation C_j^2/Q_j.

In our model, each manufacturing plant produces one type product, in other words, each warehouse have single sourcing constraint from manufacturing plants.

Arriving orders from different retailers deplete the on-hand inventory at warehouses, if any. Otherwise, (in a stock-out situation) the arriving orders have to wait to be fulfilled; the waiting process consists of manufacturing the desired units to be produced at the manufacturing plant of product j and being shipped to the warehouse of product j. We refer this situation as back order. In our problem, we assume that back orders can be infinite. Warehouse of product j places orders to manufacturing plant of product j when one bucket (Q_j units) is depleted from the inventory at the warehouse. Thus, arrival process of product j is characterized by $\lambda_{a,j}/Q_j$, $C_{a,j}^2/Q_j$. ($j = 1,2,..., L$) We assume that all manufacturing plants have infinite waiting line capacity.

The system incurs a holding cost h_j per unit of inventory of product j per unit time, a backordering cost b_j per unit of product j backordered per unit time and order set up cost for product j (\$ per set up) C_{s_j}.

The goal of modeling such supply network is to minimize supply chain total cost in order to find optimal values of K_j, Q_j. Costs contain inventory holding cost (h_j), back ordering cost (b_j) and order set up cost (C_{s_j}).

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B. Notations

The notations used in this paper are as follow:

- Q_j Number of units in one bucket of product j; $j = 1,2,...,L$
- K_j Total number of buckets at warehouse of product j
- Z_j Maximum inventory at warehouse of product j; $K_j Q_j$
- λ_i Demand arrival rate at retailer i
- A_j Number of orders arrived at manufacturing plant of product j
- μ_j Production rate of manufacturing plant of product j units/unit time
- I_j Inventory level at warehouse of product j
- N_j Number of orders at manufacturing plant of product j being processed
- B_j Number of back orders at warehouse of product j
- R_j Number of orders arrived at warehouse of product j, but after the last batch was released for processing
- h_j Inventory holding cost for product j ($ per unit per unit time)
- b_j Back order cost for product j ($ per unit per unit time)
- C_{s_j} Order set up cost for product j ($ per set up)
- ρ_j Intensity of the manufacturing plant of product j
- l_{ji} Lead time of logistics for retailer *i* to receive items from warehouses of product j, $l_{j1} = l_{j2} = ... = l_{jn} = l_j$
- Γ_j Expected number of orders in the queue $M^{C_j}/M/\infty$ in steady state
- ξ Service rate of logistics process
- ρ'_j Intensity of the logistics hub $\lambda_{a,j}/\xi < 1$
- W_j Expected waiting time at warehouse of product j due to back ordering alone
- L_{ji} Mean lead time (including back ordering delay) for an order of items from retailer *i* to be filled from warehouses of product j, $L_{j1} = L_{j2} = ... = L_{jn} = L_j$
- θ_{ji} Expected demand for product j during replenishment lead time for item at retailer i ($\theta_{ji} = \lambda_{a,j} L_{ji}$)

C. Problem formulation

In this paper, we would like to minimize the expected total cost at the warehouses, i.e. *Minimize Total Cost = Expected inventory holding cost + Expected back ordering cost + Expected ordering set up cost*

Mathematically, we can express,

$$Min \sum_{j=1}^{L} TC(Q_j, K_j) = \sum_{j=1}^{L} (E[I_j]h_j + E[B_j]b_j + C_{s_j}(\lambda_{a,j}/Q_j)) \quad (1)$$

$$s.t. K_j, Q_j \in Z^+$$

The goal of modeling such a supply chain is minimizing the total cost where results can be used to obtain the optimal values of Q_j, Z_j.

For computing inventory and back orders, we need to develop stochastic equations which capture the properties of the system as in [11]. Observe that,

$$R_j = A_j - \left\lfloor \frac{A_j}{Q_j} \right\rfloor Q_j, \quad j = 1,2,...,L \quad (2)$$

$$B_j = \max[N_j Q_j + R_j - K_j Q_j, 0], \quad j = 1,2,...,L \quad (3)$$

$$I_j = \max[K_j Q_j - N_j Q_j - R_j, 0], \quad j = 1,2,...,L \quad (4)$$

The corresponding steady state probability distribution for R_j, N_j, B_j, I_j are as follows:

R_j is uniformly distributed from 0 to $Q_j - 1$. Thus,

$$P\{R_j = n\} = \frac{1}{Q_j}, \quad n = 0,1,...,Q_j - 1 \quad (5)$$

Characterizing the probability distribution of queue size in a GI/G/1 is difficult in general. Therefore, we use a development described in [11] to approximate the probability distribution of batches in the system using a geometric distribution of the following form:

$$P\{N_j = n\} = P_{N_j}(n) \approx \begin{cases} 1 - \rho_j & n = 0 \\ \rho_j (1 - \sigma_j) \sigma_j^{n-1} & n = 1,2,... \end{cases} \quad (6)$$

Where

$\sigma_j = (\hat{N}_j - \rho_j)/\hat{N}_j$,

$\hat{N}_j = \lambda_{a,j}(B) w_{0j} + \rho_j$ and

$$w_{0j} = \hat{W}_{GI/G/1}\left[\frac{\lambda_{a,j}}{Q_j}, \frac{Q_j}{\mu_j}, \frac{C_{a,j}^2}{Q_j}, \frac{C_j^2}{Q_j}\right].$$

$$w_{0j} = \left\{\frac{\rho_j^2 (1 + \frac{C_j^2}{Q_j})}{1 + \rho_j^2 \frac{C_j^2}{Q_j}}\right\} \left\{\frac{\frac{C_{a,j}^2}{Q_j} + \rho_j^2 \frac{C_j^2}{Q_j}}{2 \frac{\lambda_{a,j}}{Q_j}(1 - \rho_j)}\right\}$$

$$= \left\{\frac{\rho_j^2 (Q_j + C_j^2)}{Q_j + \rho_j^2 C_j^2}\right\} \left\{\frac{C_{a,j}^2 + \rho_j^2 C_j^2}{2 \lambda_{a,j} Q_j (1 - \rho_j)}\right\} \quad (7)$$

To obtain the steady state utilization of the production system, which we denote by ρ_j, as follows

$$\rho_j = \frac{\lambda_{a,j}}{\mu_j} < 1$$

In continue, we can obtain steady state probability distributions I_j, B_j as follow:

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$$P\{B_j = n\} = \frac{1}{Q_j} P_{N_j}\left(\left\lfloor \frac{Z_j + n}{Q_j} \right\rfloor\right); \; n = 1,2,... \quad (8)$$

$$P\{I_j = n\} = \frac{1}{Q_j} P_{N_j}\left(\left\lfloor \frac{Z_j - n}{Q_j} \right\rfloor\right) \quad n = 1,2,...,K_j Q_j \quad (9)$$

And also, we can calculate $E\lfloor I_j \rfloor$ and $E\lfloor B_j \rfloor$ as,

$$E\lfloor I_j \rfloor = \sum_{i=1}^{Z_j} \frac{i}{Q_j} P_{N_j}\left(\left\lfloor \frac{Z_j - i}{Q_j} \right\rfloor\right) \quad (10)$$

$$E\lfloor I_j \rfloor = \frac{\rho_j(1-\sigma_j)\sigma_j^{-2}}{2(1-\sigma_j^{-1})}[(Q_j+1)\sigma_j^{K_j} + \frac{2Q_j(\sigma_j^{K_j-1}-1)}{1-\sigma_j^{-1}} - [(2K_j-1)Q_j+1]] \quad (11)$$

$$E\lfloor B_i \rfloor = \sum_{i=0}^{\infty} \frac{i}{Q_j} P_{N_j}\left(\left\lfloor \frac{Z_j + i}{Q_j} \right\rfloor\right) \quad (12)$$

$$E\lfloor B_j \rfloor = \rho_j[\frac{Q_j - 1}{2}]\sigma_j^{K_j-1} + \rho_j[\frac{Q_j}{1-\sigma_j}]\sigma_j^{K_j} = \rho_j\sigma_j^{K_j}(\frac{Q_j-1}{2\sigma_j} + \frac{Q_j}{1-\sigma_j}) \quad (13)$$

D. Performance measure of warehouses

The stock-out probability at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is zero and is obtained as follows:

$$P\{I_j = 0\} = P\{Z_j \le N_j Q_j + R_j\} = \frac{1}{Q_j}\rho_j\sigma_j^{k-1} + (1 - \frac{1}{Q_j})\rho_j\sigma_j^{k-2} \quad (14)$$

And also, the fill rate at warehouse of product j is the fraction of time that the on-hand inventory at warehouse of product j is greater than zero:

$$P\{I_j > 0\} = P\{Z_j > N_j Q_j + R_j\} = 1 - P\{I_j = 0\}$$
$$= 1 - \frac{1}{Q_j}\rho_j\sigma_j^{k-1} + (1 - \frac{1}{Q_j})\rho_j\sigma_j^{k-2} \quad (15)$$

Also the lead time of product j at its manufacturing plant is given by

$$W_{s_j} = \frac{(Q_j - 1)}{2}(1/\lambda_{a,j}) + w_{0j} + (Q_j/\mu_j) \quad (16)$$

Where $\frac{(Q_j-1)}{2}(1/\lambda_{a,j})$ is batch forming time of product j and Q_j/μ_j is mean production time for product j batch.

E. The squared coefficient of variation of the inter-departure times is produced from the warehouses

We use the approximation of squared coefficient of variation (SCV) of the inter-departure times for the batches from the warehouse of product j with batch setups in the $GI/G/1$ queue, given in [11] as shown in (17):

$$C^2_{d,j}(B) = (1-\rho_j^2)\left\{\frac{C^2_{a,j} + \rho_j^2 C_j^2}{Q_j(1+\rho_j^2\frac{C_j^2}{Q_j})}\right\} + \rho_j^2\frac{C_j^2}{Q_j} \quad (17)$$

And also, from [12], we use the following approximation of the squared coefficient of variation of the inter-departures of individuals from the warehouse of product j:

$$C^2_{d,j}(I) = Q_j C^2_{d,j}(B) + Q_j - 1 \quad (18)$$

Where fixed batches size of product j is Q_j. When a product departs from the warehouse of product j, there is a probability q_{ij}, that the product will be routed to retailer i. Therefore, the mean inter-arrival time and squared coefficient of variation for arrivals to retailer i is given by

$$\lambda_{a,ji} = \lambda_i q_{ij} \quad (19)$$

$$C^2_{a,ji} = q_{ij}C^2_{d,j}(I) + 1 - q_{ij} \quad (20)$$

F. Logistics process

In continue, we extend the model by adding logistics processes. we assume that there is some logistics time to supply products from warehouses to retailers .We model the logistics process of product j by using $M/M^{c_j}/\infty$ queue in continuous time, where c_j is vehicle capacity which is deterministic and logistics time is exponential. We assume that the logistics process is depending on the demands of customers for its arrival process.

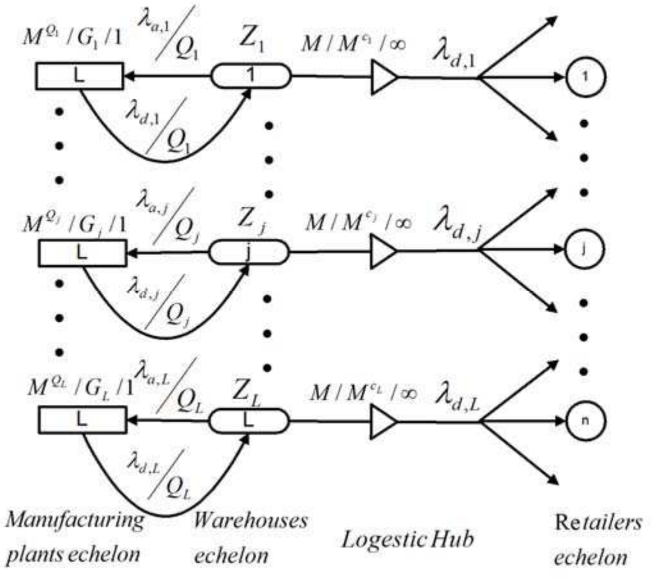

Fig.2 three stage supply chain network with logistics hub and co-located retailers

For the performance analysis of $M/M^{c_j}/\infty$ queue, we use the results of [13] and [14].

We obtain mean lead time of product j from warehouse of product j at retailer i, $L_{ji} = l_{ji} + W_j$ by using Little's law as following on:

$$W_j = \frac{E\lfloor B_j \rfloor}{\lambda_{a,j}} \quad (21)$$

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$$L_{ji} = l_{ji} + W_j \quad (22)$$

$$\Gamma_{ij} = \frac{q_{ij}\lambda_{a,j}}{\xi} C_j = \frac{1}{\xi}(q_{ij}\lambda_{a,j}C_j) \quad (23)$$

$$l_{ji} = \frac{\Gamma_{ij}}{q_{ij}\lambda_{a,j}} = \frac{C_j}{\xi} \quad (24)$$

We can compute expected demand of product j at retailer i during replenishment lead time as

$$\theta_{ji} = \lambda_{a,j,i} L_{ji} \quad (25)$$

III. NUMERICAL EXAMPLE

In this section, we analyze the model by a numerical example. We consider a supply chain network which produces three products. The supply chain includes two retailers which their exponential arrival demands are characterized by $\lambda_1 = 0.6$, $\lambda_2 = 0.8$ and $C_{a,1}^2 = C_{a,2}^2 = 1$. The probability vectors $q_1 = (0.2,0.3,0.5)$, $q_2 = (0.4,0.3,0.3)$ define customers' demands for three products at two retailers. Information of three manufacturing plants to produce the products and costs of three warehouses is showed in Table 2 and information of logistics processes is showed in Table 3:

Table2. Information of manufacturing plants

Product type	μ_j	C_j^2	h_j	b_j	C_{S_j}
1	0.5	0.7	10	100	6
2	0.6	0.8	12	120	10
3	0.8	0.6	14	140	12

Table3. Information of logistics processes

Product type	l_j	c_j	ξ_j
1	1	3	5
2	2	5	6
3	3	4	7

We solved the problem with Matlab7 software. We obtain optimum value K_j by varying values of batch sizing Q_j for three products. In the condition that $\frac{b_j}{h_j} = 1$, we increase Q_j, and optimum maximum inventory level and total cost of three products are increasing. The results imply that if back order costs are greater than holding costs, system tends to hold more inventories (Table4).

Table4. Total cost variation by increasing Q_j if $b_j / h_j = 1$

Product type	Q_j	K_j^*	$E[I_j]$	$E[B_j]$	TC^*
	2	3	71.7856	2.6298e-004	55.3690
	4	2	150.2265	2.4949e-009	52.7148
	6	2	227.9333	1.6411e-013	57.9050
	8	2	305.4215	5.7578e-017	66.4439
1	12	2	460.1604	2.2435e-022	87.2627
	16	2	614.7741	1.6795e-026	110.0539
	…	…	…	…	…
	26	1	e+0031.0011	7.9296e-034	161.1810
	…	…	…	…	…
	36	1	1.3874e+003	6.5868e-039	210.4378
	3		85.3011	6.2813e-011	36.1863
	9		358.6008	3.7625e-024	65.2599
2	15	1	431.5110	7.6391e-031	99.7485
	21		604.3593	3.1365e-035	135.0883
	27		777.1865	1.7767e-038	170.7179
	5		243.5091	5.7664e-030	49.4682
	15		732.0308	1.4908e-054	114.8010
3	25	1	1.2203e+003	5.7596e-066	183.8347
	35		1.7086e+003	2.2884e-073	253.4170
	45		2.1969e+003	7.7460e-079	323.1841

In condition that $\frac{b_j}{h_j} = 10$, we increase Q_j, and optimum number of batches (K_j^*) are obtained. (System does not tend to hold more inventories) and optimum maximum inventory level and total cost of three products are increasing (Table5).

By comparing Table5 and Table6, we show that optimum maximum inventory level and total cost of three products in Table5 are greater than Table6 (To decrease total costs).

Table5. Total cost variation by increasing Q_j if $b_j / h_j = 10$

Product type	Q_j	K_j^*	$E[I_j]$	$E[B_j]$	TC^*	W_j
	2	10	71.7856	2.6298e-004	178.5769	18.6698
	6	4	227.9333	1.6411e-013	183.9680	27.4243
1	10	3	382.8155	7.0952e-020	209.3437	39.0557
	16	3	614.7741	1.6795e-026	275.5858	57.3316
	26	2	1.0011e+003	7.9296e-034	342.2329	88.3362
	36	2	1.3874e+003	6.5868e-039	395.5933	119.5374
	3	3	85.3011	6.2813e-011	109.7122	11.1878
	9	2	258.6008	3.7625e-024	156.1350	27.1409
2	15	2	431.5110	7.6391e-031	205.3305	44.0215
	21	2	604.3593	3.1365e-035	259.5529	61.0493
	27	2	777.1865	1.7767e-038	315.3358	78.1274
	5	3	243.5091	5.7664e-030	143.8693	12.0201
	10		487.8361	2.2056e-045	182.9201	22.5258
3	20		976.1911	5.2530e-061	289.8666	44.0905
	30	2	1.4645e+003	5.4790e-070	402.7081	65.7838
	40		1.9527e+003	2.8003e-076	516.9135	87.5098
	50		2.4410e+003	4.0622e-081	631.6475	109.2490

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In the conditions that $\frac{b_j}{h_j} = 10$ we increase C_{S_j}, optimum maximum inventory levels of three produces are obtained and only total cost of three products are increasing (Table6).

Table6. Total cost variation by increasing C_{S_j} and Q_j if

$$b_j / h_j = 10$$

Product type	C_{S_j}	Q_j	K_j^*	$E[I_j]$	$E[B_j]$	TC^*
1	0	2	10	71.7856	2.6298e-004	186.2569
	6					178.5769
	16					189.7769
	0	10	3	382.8155	7.0952e-020	209.0797
	6					209.3437
	16					209.7837
	0	26	2	1.0011e+003	7.9296e-034	342.1314
	6					342.2329
	16					342.4021
	0	36	2	1.3874e+003	6.5868e-039	395.5200
	6					395.5933
	16					395.7156
2	0	3	3	85.3011	13e-0116.28	108.3122
	8					109.4322
	18					110.8322
	0	9	2	258.6008	3.7625e-024	155.6683
	8					156.0417
	18					156.5083
	0	18	2	517.9391	3.1706e-033	231.9102
	8					232.0969
	18					232.3302
	0	30	2	863.5960	7.8844e-040	343.3697
	8					343.4817
	18					343.6217
3	0	5	3	243.5091	5.7664e-030	142.5733
	10					143.6533
	20					144.7333
	0	15	2	732.0308	1.4908e-054	234.4384
	10					234.7984
	20					235.1584
	0	25	2	1.2204e+003	5.7596e-066	345.7510
	10					345.9670
	20					346.1830
	0	40	2	1.9527e+003	2.8003e-076	516.7515
	10					516.8865
	20					517.0215

IV. CONCLUSION

In this paper, we presented a model for the analysis of a three-layer supply chain which produces more than one product. We used $GI/GI/1$ queue operating under $(K_1 - 1, K_j)$ inventory control rule to analyze the performance of warehouses. We obtained performance of measures such as stock-out probability, fill-rate and lead time of warehouses in proposed model. In the model, we used $M/M^{c_j}\infty$ queue for analyze logistics process. In this paper, we surveyed the effect of order batching in multi-product multi-echelon supply chains. In future researches, we can consider a central warehouse that in the stock-out condition in each warehouse, customers' demands are satisfied (adding transmittal cost). Also, pricing problem can be added to the presented model.

REFERENCES

[1] Y.J. Lee, P.H. Zipkin,"Tandem queues with planned inventories." Oper Res, 1992, 40:936–947.
[2] Y.J. Lee, P.H. Zipkin,"Processing networks with inventories: Sequential refinement systems", Oper Res 43(6), 1995, 1025–1036.
[3] A. Azaron, H. Katagiri, K. Kato, M. Sakawa," Modelling complex assemblies as a queueing network for lead time control", Eur J Oper Res 174, 2006,150–168.
[4] L.M. Liu, X.M. Liu, D.D. Yao, "Analysis and optimization of a multistage inventory-queue system", Manage Sci 50(3), 2004,365–380
[5] U.W. Thonemann, J.R. Bradley,"The Effect of Product Variety on Supply Chain-Performance", European Journal of Operational Research, 2002.
[6] G.R. Bitran, D. Tirupati," Multi-product queueing networks with deterministic routing : decomposition approach and the notion of interference", Manage Sci34(1), 1988, 75–100.
[7] M. Ettl, G.E. Feigin, G.Y. Lin, D.D. Yao,"A supply network model with base-stock control and service requirements", Oper Res48, 2000, 216–232.
[8] M. Dong,"Inventory planning of supply chains by linking production authorization strategy to queueing models", Prod Plan Control 14(6), 2003, 533–541.
[9] Y.D. Lu, J.S. Song, D.D. Yao,"Backorder minimization in multiproduct assemble-to-order systems", IIE Trans 37, 2005,763–774.
[10] Y. Zhao, D. Simchi-Levi,"Performance analysis and evaluation of assemble-to-order systems with stochastic sequential lead times", Oper Res 54(4), 2006, 706–724.
[11] J.A. Buzacott, J.G. Shanthikumar," Stochastic models of manufacturing systems. "Prentice Hall, Englewood Cliffs, NJ, 1993.
[12] G.L. Curry, R.M. Feldman,"Manufacturing Systems Modeling and Analysis." Springer-Verlag Berlin Heidelberg, 2009.
[13] B.R.K. Kashyap, L. Liu, J.G.C. Templeton," On the GI X /G/∞system", J Appl Probab 27, 1990, 671–683.
[14] P. Purdue, D. Linton,"An infinite-server queue subject to an extraneous phase process and related models." J Appl Probab 18, 1981, 236–244.

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