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Fuzzy Queuing Approach for Designing Multi Supplier Systems (Case: SAPCO Company)

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ABSTRACT

The importance of reliable supply is increasing with supply chain network extension and just-in-time (JIT) production. Just in time implications motivate manufacturers towards single sourcing, which often involves problems with unreliable suppliers. If a single and reliable vendor is not available, manufacturer can split the order among the vendors in order to simultaneously decrease the supply chain uncertainty and increase supply reliability. In this paper we discuss with the aim of minimizing the shortage cost how we can split orders among suppliers with different lead times. The (s,S) policy is the basis of our inventory control system and for analyzing the system performance we use the fuzzy queuing methodology. After applying the model for the case study (SAPCO), the result of the developed model will be compared in the single and multiple cases and finally we will find that order splitting in optimized condition will conclude in the least supply risk and minimized shortage cost in comparison to other cases.

1. Introduction

The objective of just-in-time (JIT) is having a single and reliable supplier. However, in spite of global sourcing increasing, this may not always occur. Although companies are trying to move towards a single supplier policy, many companies have only reduced their supplier base, using just a few suppliers. Honda Company, one of Just in time purchasing pioneers, uses two suppliers for 44% of its parts, three suppliers for 16%, and four or five for 4%. In addition to above instances it's noticeable in the competitive global sourcing, response speed to the needs is one of the most important factors for vendor's growth and survival. Therefore companies try to reduce material receipt delay by different techniques. One of the most usable techniques is Order Splitting among two or more suppliers. The increase of suppliers' multiplicity cause effective lead time, the duration between making an order and receipt of that order or the duration of between two receipts decrease which causes reduction of inventory level, so inventory holding cost and shortage cost will decrease in conclusion. Using multiple supplier cause more fixed order costs so this increase must analyzed with cost trade off to find out if using multi sourcing is economy or not.

Most of studies in inventory management have focused on inventory control in stochastic environments. Although demand uncertainty is the most significant source of uncertainty for many systems, other uncertainties are obvious as well. In particular, there may exist more supply uncertainties. Uncertainty in the order delivery lead time is one of common problems in all industries. Many companies have performed re-

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engineering efforts to improve the efficiency of their supply chains with the aim of better matching supply with demand so as to reduce the inventory costs and the customer service times. To contract with the suppliers who have the shortest lead time is one strategy that complies with this aim. In most of inventory control models, an item is purchased from a single supplier.

However, in many situations more than one supplier may be necessary to sustain a desirable service standard or to diminish the total system cost. When delivery times are stochastic, multi-supplier strategies are more robust against the interruptions in supply and can lead to reducing inventory shortage and holding costs. This is because of the shortage chance reduction and therefore reduced reorder and replenishment inventory levels. Clearly, multi-supplier policies can be more costly in the presence of economies of scale due to the increase in ordering costs. However, in most practical situations, these incremental ordering costs can be outweighed by the savings in holding and shortage costs. Furthermore, multi-supplier policies potentially conclude in competition among suppliers that can force the suppliers to provide faster delivery.

In this paper at first we analyze multi-supplier strategies as one of the basic elements of a supply chain: the operational relationships between an end-producer and his direct suppliers.

A simple queuing model is created based on the assumptions of a Poisson external demand for end-products, immediate delivery to the customer from the manufacturer stock, and an exponentially distributed service time for each supplier. Within the context of traditional queuing theory, the arrival times and service times are required to follow certain probability distributions However, in many practical applications, the statistical information may be obtained subjectively; i.e., describing the arrival pattern and service pattern by linguistic terms such as fast, slow, or moderate are more suitably rather than by probability distributions. Therefore, fuzzy queues [1] are much more realistic than the traditionally used crisp queues. If the usual crisp queues with can be extended to fuzzy queues, queuing models would have even wider applications.

Buckley [2] investigated elementary multiple-server queuing systems with finite or infinite capacity and source population, in that the arrivals and departures are followed by possibility distributions; in addition, recently with other two scholars, he applied the previous results to a machine serving problem and a queuing decision problem [3].

On the basis of Zadeh's extension principle [4], the possibility concept, and fuzzy Markov chains [5], Li and Lee [6] have derived analytical solutions for two fuzzy queues, namely, $M/F/1$ and $FM/FM/1$, where F denotes fuzzy time and FM denotes fuzzified exponential time. However, as commented by Negi and Lee [7], their approach is very complicated and is generally unsuitable to computational purposes. Furthermore, as commented by Kao et al. [8], for other more complicated queuing systems, Li and Lee's solution is hardly possible to obtain analytical results.

Therefore make the membership functions of the performance measures for fuzzy queues by Kao et al. [8], adopting parametric programming, and successfully apply to four simple fuzzy queues with one or two fuzzy variables, namely, $M/F/1$, $F/M/1$, $F/F/1$, and $FM/FM/1$. It seems that the fuzzy bulk service queues could be analyzed with their approach. Clearly fuzzy bulk service queuing systems are more complicated than the above four fuzzy queues, so the solution procedure for the fuzzy bulk service queue is not explicitly known and more investigation must be done.

In this paper we will demonstrate a fuzzy queuing model in which D denotes arrival rate with Poisson distribution, in other words D shows the material needs of producer or producer orders. And the service time is shown by $\tilde{T}$ which denotes fuzzy service time or fuzzy delivery time. The goal of this model is to allocate the optimized order size for each supplier in order to have the least shortage cost. The percentages of orders is shown by *x,* it means each supplier is responsible for $x_i D$ of total orders.

In this study we focus on only one basic item of the end-products and suppliers are supposed equivalent in terms of quality and cost. They only differ by their average service time the potential usefulness of the model for the producer is in the a priori determination of his ''optimal'' inventory level and of the volumes (or frequency) of his orders to suppliers, based on a priori evaluation of their average delivery time. To optimize the supplier's inventory level and the ordering procedure, it is essential to combine the effects of random fluctuations on demand flows, and delays of deliveries from suppliers.

In the existing models of inventory control, Random demands have often been considered, although random delays in part deliveries have not usually been investigated but we can find some scholars who have worked on this field i.e. the work of Dolgui and Louly [9], in which several vendors with random delivery delays are considered. It is hard to find many serious studies on fuzzy delays except the studies which we have already talked about (Kao et al. [8]). The case with different suppliers and different fuzzy delivery times has never been studied and we would show it in this paper.

The outline of the paper is as follow. We review order splitting concepts in Section 2. In Section 3, the problem is defined more precisely and describes the case study and motivation of applying fuzzy theory. Section 3 formulates the optimal inventory and ordering problem for one producer and several suppliers. Then Section 4 solves optimally the order dispatching problem in the particular make to order

(MTO) case and the proposed model is tested with real data. The performance of the approximate solution is comparatively evaluated Section 4. Finally, we give some concluding remarks in Section 5.

2. Order Splitting

In many papers, much attention is paid to order splitting models (also known as multiple sourcing). The main goal of order splitting is to reduce lead time uncertainties by splitting the replenishment orders over more than one supplier. In order splitting every time replenishment is placed, each supplier is involved. When a company works under single supplier strategy, in many occasions production may halt because the capacity of the single supplier gets destroyed. Sometimes, a supplier is able to fulfill the buyer's requirements only partially. Obviously single sourcing creates a great dependency between company and supplier and, therefore, increases supply risks (on the other hand, it involves many advantages). Most of the studies have focused on the analysis of the advantages and disadvantages of the sourcing strategies, which are summarized in Table 1, and focused on qualitative models of decision-making (Spekman [10]; Ramsay and Wilson [11]; Agrawal and Nahmias [12]; Burke et al.[13]). Only few researchers have proposed quantitative models that support decision-making in risky and uncertain situations.

Tab. 1. Advantages and disadvantages of multiple and single sourcing strategy (Costantino and Pellegrino [14])

	Single sourcing	Multiple sourcing
ADVANTAGES	• Partnership between buyers and suppliers allows cooperation, shared benefits and long-term relationship based on high levels of trust	• Alternative sources of materials in case of delivery stoppage by a supplier
	• Reduction of risk of opportunistic behavior	• Reduced probability of bottlenecks due to insufficient production capacity to meet peak demand
	• Large commitment of the supplier that is willing to invest in new facilities or new technology	• Increased competition among suppliers leads to better quality, price, delivery, product innovation and buyer's negotiation power
	• Lower purchase price resulting from reduced production costs, due to better knowledge of the manufacturing process by supplier and achieved economies of scale	• More flexibility to react to unexpected events that could endanger supplier's capacity
DISADVANTAGES	• Great dependency between the buyer and the supplier	• Reduced efforts by supplier to match buyer's requirements
	• Increased vulnerability of supply	• Higher costs for the purchasing organization (greater number of orders, telephone calls, records, and so on)
	• Increased risk of supply interruption, especially for asset specific products	

3. An Illustrative Case Study: SAPCO Company

3.1. Problem Description

SAPCO is one of Iran-Khodro's chief holding corporations who we choose it as our case study. SAPCO's mission is to supply automotive material and parts for Iran-Khodro. SAPCO works as one of the advanced firms who succeeded to implement supply chain concepts and patterns in Iran. SAPCO was established in 1993 holding the idea of supplying national automotive parts. Today there are more than 150,000 employees in 500 automotive part making companies and more than 100 supporting firms are the members of this multi echelon supply chain which SAPCO acts as the head of it. To supply thousands of parts for more than 600,000 cars in 10 different models yearly is the current activity of this corporation. Suppliers are divided to two main groups; suppliers who supply end products and the others who send their products to first group suppliers. In this paper the only focus is on the first group. SAPCO's superior criteria for ordering, receiving and holding parts include some main items. Warehouse space limits is one of the common problems in most of industries, In addition they must have a rapid stocking system for fast response to production line. Keeping the inventories in the minimum level is the best solution. Less holding cost and less wastage are other benefits of low inventory level. In other hand low inventory level increases stock out risk. To reduce the effect of this problem they have decided to implement multi sourcing strategy, and make contracts with more than one supplier for every strategic and high important product; having low inventories with low stock out risk both simultaneously is the supreme usefulness of this strategy. For instance the automotive part Axle is purchased from two companies; *Mehvar-Sazan-co* and *Farasanat-co*. Because of different manufacturing technology, different human resources and many other factors these factories have different lead times. How to share demands among these factories is a vital decision, with the first delay in delivery and facing shortage in material production line would stop and it impose a lot of losses to manufacturer. The applied inventory policy in this study is one of the most popular continuous review policies; (*s,S*) policy, in which s stands for the inventory position reorder point and S for the inventory position replenishment level. The base stock policy can be considered as a variant of the (*s,S*) policy, for which an order is placed whenever a demand comes, so as to permanently maintain the inventory position *S*. The (*s,S*) policy seems to be optimal for the problems which involves independent items and stationary stochastic demand, whenever the cost criterion only depends on the inventory position and has a single local minimum (Axsater [15]). Moreover, under unitary demands the optimal base-stock policy reduces to the policy (*s,S*) with *s=S-1*: This policy is denoted the reference inventory policy. We can interpret such a base-stock control policy as Kanban mechanism. This policy is so useful for the companies who must control so many items continuously and use high tech computer systems. In every point of time the cumulative quantity of on hand inventory and backlog orders must be constant (S). At time *t*, the current inventory level of the product denoted $I(t)$. The number of placed replenishment orders which are not yet delivered denoted $u(t)$. $P(t)$ is notation of the global state of the system which is characterized by the inventory position:

$$P(t) = I(t) + u(t) \tag{1}$$

In this inventory policy several kinds of cost could be considered; purchasing cost, fixed order cost, holding cost and shortage cost or lost opportunity cost. In this case the purchasing cost in the global supply chain is fixed and fixed order cost is venial. From SAPCO viewpoint, the cost function to be minimized is the sum of the average holding cost and the average stock-out cost. But for simplicity the base-stock level is supposed equal to zero (MTO). Iran-khodro's demand is denoted (*D*). Every supplier portion of demand is $x_i D$. *i* denotes the number of suppliers as follows:

$$\sum_{i=1}^{N} x_i = 1, 0 \le \left\{ x \middle| \sum_{i=1}^{N} x_i = 1 \right\} \le 1$$

Delivery time (lead time) include duration between making an order until receiving that order (back log order become on hand inventory). Suppliers lead times is denoted by t_i. We assume the supplier delivery time is exponentially distributed with mean service time $1/T$, satisfying the stability condition $\rho = D/T < 1$. Under the (δ_1, δ) base stock policy, the inventory position is a constant with value S and the number of uncompleted orders, *u(t)*, represents the queue length of orders for the supplier. It is a simple *M/M/*1 system with birth-death coefficients (*D,T*).

3.2. Motivation of Applying Fuzzy Theory and the Basic Definitions

In this section we intend to discuss the motivation for using fuzzy set theory in an order optimizing model and present some basic definitions from fuzzy theory.

Most of traditional approaches for formal modeling and computing have crisp, deterministic, and precise characteristic. When we talk about Precision we mean that the parameters of a model represent exactly our perception of the case modeled or the aspects of the real system that has been modeled. Of one of the leading researchers in the area of fuzzy theory [16], believed that real situations are very often not crisp and deterministic, and they cannot be described precisely. Therefore we find out that real situations are very often uncertain and vague so predicting the future of a system in lack of information is impossible or hardly conceivable.

There are different approaches for modeling uncertainty, such as probability theory and fuzzy theory. In probabilistic approach, we can fit probability distributions on the basis of the stochastic experiments and the recorded data. We can estimate parameters of the model by this approach, so the structure of the model could be achieved. In order splitting problems for instance we can assume the probability distribution for the demand rate at each period and the service time are known. Arda and hennet [17] have adopt this approach to develop a order split optimizing model that is described in Section 3.3.2.

In situations where we have no reliable recorded data to estimate model parameters, we can estimate them imprecisely on the basis of our perceptions. It means instead of gathering data for statistical estimation of parameters by spending time and cost, we can develop and analyze the model on the basis of the imprecise data.

In this case fuzzy set theory can help us to formulate the model by incorporating the linguistic variables which demonstrates people feelings and perceptions. For instance in order splitting problem the delivery time of a supplier can be estimated "approximately 2 days". For comparing probability theory and fuzzy set theory we can refer to Zimmermann [16 p.125] who proposed that they are not substitutable, but they complement each other. Also he believed that fuzzy set theory seems to be more adaptable to different contexts. Now we adduce some basic definitions [16] that are basic to understanding this paper.

Definition 3.2.1. if X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}(x)}) | x \in X\}$$

Where the symbol x denotes the element of the set X and $\mu_{\tilde{A}(x)}$ is called the membership function or the degree of membership of X in $\tilde{A}$ that maps X to the membership space [0,1].

Definition 3.2.2. A fuzzy set $\tilde{A}$ is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, x_1, x_2 \in X, \lambda \in [0,1]$$

Definition 3.2.3. if $\sup_x \mu_{\tilde{A}(x)} = 1$,the fuzzy set $\tilde{A}$ is called normal

Definition 3.2.4. a fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$

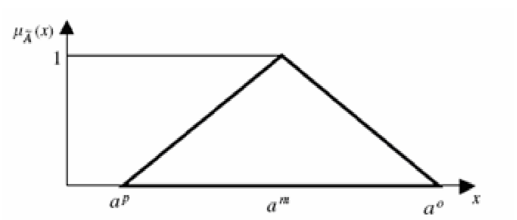
Definition 3.2.5. the membership function $\mu_{\tilde{C}(x)}$ of the intersection $\tilde{C} = \tilde{A} \cap \tilde{B}$ is point wise defined by

$$\mu_{\tilde{C}}(x) = Min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X$$

Or

$$\mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x)\mu_{\tilde{B}}(x), \quad x \in X$$

Definition 3.2.6. Approximate numbers can be defined as triangular fuzzy number , such as "approximate 5" that would normally be defined by a triangular fuzzy number $\{3,5,7\}$ where the membership degree of 5 is 1, while for 3 and 7 it is zero. For the other real numbers between 3 and 5, the membership degrees, and between 5 and 7 are between zero and 1. In general, suppose $\tilde{A}$ is triangular fuzzy number that is defined as (Fig. 1):



$$\tilde{A} = (a^p, a^m, a^o)$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a^m - x}{a^m - a^p} & a^p \leq x \leq a^m \\ 1 - \dfrac{x - a^m}{a^o - a^m} & a^m \leq x \leq a^o \\ 0 & otherwise \end{cases}$$

Fig. 1. A triangular fuzzy number $\tilde{A}$

Definition 3.2.7. suppose $\tilde{A} = (a^p, a^m, a^o)$ and $\tilde{B} = (b^p, b^m, b^o)$ are triangular fuzzy numbers so the arithmetic operation on them can be shown as:

$$\tilde{A} + \tilde{B} = (a^p + b^p, a^m + b^m, a^o + b^o)$$
$$\tilde{A} - \tilde{B} = (a^p - b^p, a^m - b^m, a^o - b^o)$$
$$\tilde{A} * \tilde{B} = (a^p * b^p, a^m * b^m, a^o * b^o)$$
$$\tilde{A} / \tilde{B} = (a^p / b^p, a^m / b^m, a^o / b^o)$$

3.3. Mathematical Formulation

3.3.1. Notation

The following notation is used throughout the paper:

Indices and parameters:

k	index set of supplier $k = \{1, 2, ..., K\}$
h	Holding cost per unit
$I(t)$	The current inventory level of the product considered at time t
b	Shortage cost per unit
n_k	n unit of orders which supplier k must deliver them
N	orders waiting in the suppliers queue
D	Manufacturer demand rate
T_k	The delivery time for supplier k

Variables

x_k	Optimum assignment for supplier k
S	Maximum level of replenishment

3.3.2. Mathematical Model

The goal of model in the MTO case is finding the optimized percentages of each supplier proportion of total demand. In other words we must find minimized total cost (holding cost and shortage cost):

$$TC(S, x_1, x_2, ..., x_N) = E\left[h(I)^+ + b(I)^-\right] = h\sum_{N=0}^{S}(S-N)P_N + b\sum_{N=S+1}^{\infty}(N-S)P_N \quad (2)$$

The probability of having n_k orders in queue k is given by:

$$P(n_k) = (\frac{x_k D}{T_k})^{n_k}(1 - \frac{x_k D}{T_k}) \quad (3)$$

The necessary and sufficient condition for stability of queue k is $x_k \rho_k < 1$ with $\rho_k = D / T_k$. The probability for the k number of queues is given by:

$$P(n_1, n_2, ..., n_N) = \prod_{k=1}^{K}(x_k \rho_k)^{n_k}(1 - x_k \rho_k) \quad (4)$$

Then the generating function of sum $n_1 + ... + n_K$ is obtained as follows (Arda and Hennet [18]):

$$G_{n_1+n_2+...+n_K}(z) = \prod_{k=1}^{K} (x_k\rho_k)^{n_k}(1 - x_k\rho_k) \quad (5)$$

$$G_{n_1+n_2+...+n_K}(z) = \sum_{k=1}^{K} \frac{A_k}{1 - x_k\rho_k z} \quad (6)$$

$$A_k = H(K) \times (x_k\rho_k)^{K-1} \times \prod_{\substack{j=1 \\ j\neq k}}^{K} b_{kj} \quad (7)$$

with

$$H(K) = \prod_{k=1}^{K} (1 - x_k\rho_k)$$

$$b_{nj} = \frac{1}{x_n\rho_n - x_j\rho_j}$$

$$G_{n_1+n_2+...+n_K}(z) = H(K) \times \sum_{k=1}^{K} ((\prod_{\substack{j=1 \\ j\neq k}}^{K} b_{ij}) \times \sum_{N=0}^{K} x_k^{K+N-1}\rho_k^{K+N-1} z^N)$$

and finally:

$$P_N = H(K) \times \sum_{k=1}^{K} (\prod_{\substack{j=1 \\ j\neq k}}^{K} b_{kj})(x_k\rho_k)^{K+N-1} \quad (8)$$

The mean value of number of pending orders is denoted *Z*, with:

$$Z = E[u] = \sum_{N=0}^{\infty} NP_N \quad (9)$$

total cost expression (2) can be re-written:

$$TC(S, x_1, x_2, ..., x_K) = (h+b)\sum_{N=0}^{S} (S-N)P_N + b(Z - S) \quad (10)$$

and the following expression is obtained:

$$TC(S, x_1, x_2, ..., x_K) = (h+b) \times H(K) \times \sum_{k=1}^{K} ((\prod_{\substack{j=1 \\ j\neq k}}^{K} b_{kj}) \times (\frac{Sx_k^{K-1}\rho_k^{K-1}}{1 - x_k\rho_k} - \frac{x_k^K\rho_k^K(1 - x_k^S\rho_k^S)}{(1 - x_k\rho_k)^2}) + b(\sum_{k=1}^{K} \frac{x_k\rho_k}{1 - x_k\rho_k} - S) \quad (11)$$

in the MTO case because of keeping no inventory, the base stock level is equal to zero:

$$TC(x_1, x_2, ..., x_K) = bZ = b\sum_{k=1}^{K} \frac{x_k D}{T_k - x_k D} \quad (12)$$

b is the unit shortage cost and $\frac{x_k D}{T_k - x_k D}$ Shows the number of orders in the supplier *k* queue. Suppliers can be rated base on their service time $T_1 > T_2 > ... > T_K > 0$. The problem constraints are stabled on following conditions:

$$0 \le x_k \le 1, \quad k = 1, ..., K \quad (13)$$

$$\sum_{k=1}^{K} x_k = 1 \quad (14)$$

$$\frac{x_k D}{T_k} < 1, \quad k = 1, ..., K \quad (15)$$

$$D < \sum_{k=1}^{K} T_k \quad (16)$$

If we replace (14) in (15) and (13):

$$0 \le x_k \le \min(1, \frac{T_k}{D}), \quad k = 1, ..., K \quad (17)$$

The MTO optimization problem takes the following form:

$$\underset{x_1,...,x_K}{Min} \sum_{k=1}^{K} \frac{x_k}{T_k - x_k D}$$

$E(T)$ is a convex function because :

$$\frac{d^2E(T)}{d^2x_k} > 0$$

$$\frac{d^2E[T]}{d^2x_k} = \frac{d}{dx_k}(\frac{T_k}{(T_k - x_k D)^2}) = \frac{2T_k D(T_k - x_k D)}{(T_k - x_k D)^4} = \frac{2T_k D}{(T_k - x_k D)^3}$$

The lagrangean of the relaxed problem can be written as follow:

$$L = (\sum_{k=1}^{K} \frac{x_k}{T_k - x_k D}) - \gamma(\sum_{i=1}^{K} x_k - 1)$$

Where γ is the Lagrange parameter. Then the optimal solution of the relaxed problem satisfies the following set of conditions:

$$\frac{dL}{dx_k} = \frac{T_k}{(T_k - x_k^* D)^2} - \gamma = 0, k = 1, 2, ..., K \quad (18)$$

$$\sum_{k=1}^{K} x_k^* = 1 \quad (19)$$

For any pair (x_k, x_j) the above condition can be re-written:

$$T_j - x_j^* D = \frac{\sqrt{T_j}\,(T_k - x_k^* D)}{\sqrt{T_k}} \tag{20}$$

$$\sum_{j=1}^{K} (T_j - x_j^* D) = \sum_{j=1}^{K} \left(\frac{\sqrt{T_j}\,(T_k - x_k^* D)}{\sqrt{T_k}}\right)$$

$$\sum_{j=1}^{K} T_j - D = \frac{T_k - x_k^* D}{\sqrt{T_k}} \sum_{k=1}^{K} \sqrt{T_k}$$

The optimal percentage of order for each supplier is obtained:

$$x_k^* = \frac{1}{D}(T_k - \tau_K \sqrt{T_k}), k = 1, 2, ..., K \tag{21}$$

Where:

$$\tau_K = \frac{\sum_{j=1}^{K} T_j - D}{\sum_{j=1}^{K} \sqrt{T_j}}$$

3.3.3. Fuzzy Model

The fuzzy sets are defined as follows:

$$\tilde{T}_1 = \{(t_1, \mu_{\tilde{T}_1}(t_1)) \mid t_1 \in T_1\}$$
$$\tilde{T}_k = \{(t_k, \mu_{\tilde{T}_k}(t_k)) \mid t_k \in T_k\} \qquad \forall k = 1, ..., K \tag{22}$$

Where:

$\tilde{T}_1, ..., \tilde{T}_k$	Fuzzy sets of service rates
$\tilde{t}_1, ..., \tilde{t}_k$	Fuzzy service rates
$\mu_{\tilde{T}_1}, ..., \mu_{\tilde{T}_k}$	Membership function of Fuzzy service rates
$T_1, ..., T_k$	General crisp sets

From now on we show the percentage of allocated order for each supplier with $f(t_k)$, so the specification function is:

$$f(t_k) = \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right) \tag{23}$$

$$\mu_{f(t_k)}(z) = \sup_{t_k \in T_k} \min\{\mu_{\tilde{T}_k}(t_k) \mid z = f(t_k)\}$$
$$\mu_{f(t_k)}(z) = \sup_{t_k \in T_k} \min\{\mu_{\tilde{T}_1}(t_1), \mu_{\tilde{T}_2}(t_2), ..., \mu_{\tilde{T}_K}(t_K) \mid z = f(t_k)\} \tag{24}$$

Membership function of objective function is:

$$\mu_{\tilde{f}(t_k)}(z) = \sup_{t_k \in T_k} \min\left\{\mu_{\tilde{T}_k}(t_k) \mid z = \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right)\right\} \tag{25}$$

a-cuts of the function can be written as:

$$T_k(\alpha) = \{t_k \in T_k \mid \mu_{\tilde{T}_k}(t_k) \geq \alpha\} \quad \forall k = 1, ..., K \tag{26}$$

$$T_k(\alpha) = [t_k^L(\alpha), t_k^U(\alpha)] = \left[\min_{t_k \in T_k}\{t_k \mid \mu_{\tilde{T}_k}(t_k) \geq \alpha\}, \max_{t_k \in T_k}\{t_k \mid \mu_{\tilde{T}_k}(t_k) \geq \alpha\}\right]$$
$$\{T_k(\alpha) \mid 0 < \alpha \leq 1\} \qquad \forall k = 1, ..., K \quad 0 < \alpha \leq 1 \tag{27}$$

$$t_k^L(\alpha) = \min \mu_{\tilde{T}_k}^{-1}(\alpha) \quad t_k^U(\alpha) = \max \mu_{\tilde{T}_k}^{-1}(\alpha) \quad \forall k = 1, ..., K \tag{28}$$

Because of hard imagination of the membership function $\mu_{\tilde{f}(t_k)}(z)$ shape, we consider K cases (Chuan Ke and Horng Lin [19]),

$$Case(1) = (\mu_{\tilde{T}_1}(t_1) = \alpha, \mu_{\tilde{T}_2}(t_2) \geq \alpha, ... \mu_{\tilde{T}_K}(t_K) \geq \alpha)$$
$$\vdots$$
$$Case(k) = (\mu_{\tilde{T}_1}(t_1) \geq \alpha, ... \mu_{\tilde{T}_{k-1}}(t_{k-1}) \geq \alpha, \mu_{\tilde{T}_k}(t_k) = \alpha, \mu_{\tilde{T}_{k+1}}(t_{k+1}) \geq \alpha, ... \mu_{\tilde{T}_K}(t_K) \geq \alpha)$$
$$\vdots$$
$$Case(K) = (\mu_{\tilde{T}_1}(t_1) \geq \alpha, \mu_{\tilde{T}_2}(t_2) \geq \alpha, ... \mu_{\tilde{T}_K}(t_K) = \alpha) \tag{29}$$

With use of parametric nonlinear programming, upper and lower limits of *a-cuts* can be obtained:

$$f_\alpha^{L_1}(t_k) = \min \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right)$$
$$S.t: \quad t_1^L(\alpha) \leq t_1 \leq t_1^U(\alpha),\ t_k \in T_k(\alpha) \ \ \forall k = 1, ..., K$$
$$f_\alpha^{U_1}(t_k) = \max \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right)$$
$$S.t: \quad t_1^L(\alpha) \leq t_1 \leq t_1^U(\alpha),\ t_k \in T_k(\alpha) \ \ \forall k = 1, ..., K \tag{30}$$

To find the membership function $\mu_{\tilde{f}(t_k)}$:

$$f_\alpha^L(t_k) = \min \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right)$$
$$S.t: t_k^L(\alpha) \leq t_k \leq t_k^U(\alpha) \qquad \forall k = 1, ..., K$$
$$f_\alpha^U(t_k) = \max \frac{1}{D}\left(t_k - \frac{\sum_{j=1}^{K} t_j - D}{\sum_{j=1}^{K} \sqrt{t_j}} \sqrt{t_k}\right)$$
$$S.t: t_k^L(\alpha) \leq t_k \leq t_k^U(\alpha) \qquad \forall k = 1, ..., K \tag{31}$$

Finally with use of *a-cuts* intervals in K case and $[f_\alpha^L(t_k), f_\alpha^U(t_k)]$, we can find membership function of $\mu_{\tilde{f}(t_k)}$:

$$L(z) = (f_\alpha^L(t_k))^{1}$$
$$R(z) = (f_\alpha^U(t_k))^{1}$$
$$0 < \alpha_2 < \alpha_1 \leq 1 \ \ f_{\alpha_1}^L(t_k) \geq f_{\alpha_2}^L(t_k) \qquad f_{\alpha_1}^U(t_k) \leq f_{\alpha_2}^U(t_k) \tag{32}$$
$$\mu_{\tilde{f}(tk)}(z) = \begin{cases} L(z) & z_1 \leq z \leq z_2 \\ R(z) & z_2 \leq z \leq z_3 \end{cases}$$

The following algorithm shows the summery of solution procedure. Input delivery rates for k suppliers which are trapezoidal fuzzy number represented by $(t_{k1}, t_{k2}, t_{k3}, t_{k4})$. Output the numbers $t^L_{k(\alpha)}, t^U_{k(\alpha)}, f^L_\alpha, f^U_\alpha$:

Step 1: $for\, \alpha = 0\ to\ 1\ step\, \Delta;$

Step 2:

$t^L_{k(\alpha)} = (t_{k2} - t_{k1})\alpha + x_1;\ t^U_{k(\alpha)} = t_{k4} - (t_{k4} - t_{k3})\alpha;$

Step 3: $for\ t_k = t^L_{k(\alpha)}\ to\, t^U_{k(\alpha)};$

Step 4:

$f^L_\alpha = \arg\{\min f(t_k)\};\quad f^U_\alpha = \arg\{\max f(t_k)\};$

Step 5: $Output\ t^L_{k(\alpha)}, t^U_{k(\alpha)}, f^L_\alpha, f^U_\alpha$

Step 6: STOP

The numerical solutions of f^L_α and f^U_α at different α levels can be gathered to approximate the shape of $L(z)$ and $R(z)$. Also the membership function can be constructed from these shapes.

4. Case Study

In this section we demonstrate how this model can be applied to analyze the case study. Because of complexity of fuzzy variables and analytical solution we use the software *Matlab 7.0* to solve the problem and to find the shape of $\mu_{\tilde{f}(t_k)}$ for a given α. Here we count 11 values of α : 0,0.1,0.2,…,1.0. The Fig. 2 displays the rough shape $\mu_{\tilde{f}(t_k)}$ from 22 values (f^L_α, f^U_α) for these 11 α values. As we mentioned in Section 3.1 each supplier has different delivery time because of many reasons. To put it simple we name Mehvar-sazan-co as supplier 1, and Farasanat-co as supplier 2. Our fuzzy delivery rate numbers are based on expert opinion, but the customer demand is obtained from historical data. The customer demand is approximated 100 axles per day. The economic quantity for every delivery is 10 numbers therefore we can consider *100/10=10* number as standard batch, in other words $D = 10$. We assume delivery rates for supplier 1 and 2 are trapezoid numbers. As expert proposed the supplier 1 can dispatch (12,15,18,19) batches and the supplier 2 can dispatch (11,12,14,16) batches per day. We want to find out how to split orders between tow suppliers in order to have minimized shortage cost. After performing numerical solutions for different α values, the shape of corresponding membership function can be approximated. The rough shape seems quite well and looks like a continuous function. The values of variables at different possibility levels and supplier 1 proportion are shown in Table 2. Table 3 and Fig. 3 are result of supplier 2 solution procedure.

Tab. 2. $\alpha - Cuts$ of delivery rates and obtained proportion of supplier 1

α	t_1^L	t_1^U	t_2^L	t_2^U	f_1^L	f_1^U
0	12	19	11	16	0.3646	0.7642
0.1	12.3	18.9	11.1	15.8	0.3816	0.7577
0.2	12.6	18.8	11.2	15.6	0.3985	0.7512
0.3	12.9	18.7	11.3	15.4	0.4155	0.7447
0.4	13.2	18.6	11.4	15.2	0.4324	0.7382
0.5	13.5	18.5	11.5	15	0.4494	0.7317
0.6	13.8	18.4	11.6	14.8	0.4663	0.7252
0.7	14.1	18.3	11.7	14.6	0.4831	0.7186
0.8	14.4	18.2	11.8	14.4	0.5	0.7121
0.9	14.7	18.1	11.9	14.2	0.5168	0.7055
1	15	18	12	14	0.5336	0.699

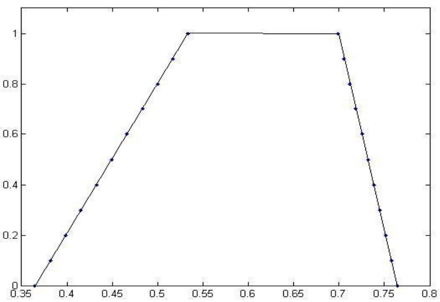


Fig. 2. The membership function for fuzzy proportion for supplier 1

Tab.3. *a-cuts* **of delivery rates and obtained proportion of supplier 2**

α	t_1^L	t_1^U	t_2^L	t_2^U	f_2^L	f_2^U
0	12	19	11	16	0.2358	0.6354
0.1	12.3	18.9	11.1	15.8	0.2423	0.6184
0.2	12.6	18.8	11.2	15.6	0.2488	0.6015
0.3	12.9	18.7	11.3	15.4	0.2553	0.5845
0.4	13.2	18.6	11.4	15.2	0.2618	0.5676
0.5	13.5	18.5	11.5	15	0.2683	0.5506
0.6	13.8	18.4	11.6	14.8	0.2748	0.5337
0.7	14.1	18.3	11.7	14.6	0.2814	0.5169
0.8	14.4	18.2	11.8	14.4	0.2879	0.5
0.9	14.7	18.1	11.9	14.2	0.2945	0.4832
1	15	18	12	14	0.301	0.4664

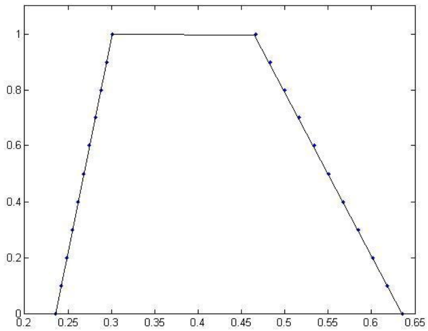


Fig. 3. The membership function for fuzzy proportion for supplier 2

By replacing the above results in expression 12 we can obtain the fuzzy total cost, which can be defuzzied and get changed to crisp form. In this case after defuzzifying, total cost is obtained 1.01. Now we can compare this number with two different cases in order to ensure that the solution result is optimized. If we solve the problem in situations which just one of suppliers is considered for dispatching orders we will obtain two numbers. By considering supplier 1 proportion equal to 100% we obtain 2.50 as total cost. And similarly for supplier 2 we obtain 4.97. Therefore we can confidently propose that the solution is optimized.

5. Conclusion

In this paper we have studied on a new framework of order splitting problems in uncertain situations. We have analyzed the application of a fuzzy order splitting in a two level supply chain with purpose of minimizing the total cost. This study has shown that in the case of random demands from customers and fuzzy delivery delays from suppliers, it is generally useful to split the orders between several suppliers than to allocate all the replenishment orders to a single one and in such uncertain situation that we cannot approximate delivery rates base on historical data, fuzzy theory is a useful approach to be utilized. More specifically we solved the problem in a 2-suppliers case study (SAPCO) in MTO status to determine the percentages of orders to be allocated to each supplier; however the introduced technique has been proposed to solve the general N-suppliers case. We validate this technique by comparing the optimal solution in 2 supplier case with single sourcing case.

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