Stability Analysis of Network Controlled Temperature Control System with Additive Delays

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Abstract: This paper presents, using Lyapunov-Krasovskii functional technique combined with reciprocal convex lemma is considered for a networked control temperature control system with additive time-varying-delays. In the stability analysis, a new LK functional is assumed, and take the time-derivative of the (LK) functional, using reciprocal convex combination technique was employed to obtain less conservative stability criteria. Finally, the proposed stability analysis culminates into a stability criterion in the LMI (linear matrix inequalities) framework. The results obtained are in accordance with the theoretically obtained in the temperature control system and they are closer to the standard benchmark temperature-control system.

Keywords: Delay-dependent stability, time-varying additive delays, temperature control, heat exchanger, lyapunov-krasovskii functional, linear matrix inequality, reciprocal convex lemma.

1 Introduction

In this paper, stability of network controlled heat exchanger system, also known as temperature control system that maintains output temperature of a fluid within specified limits is analyzed. When heat exchanger is employed for transfer of heat between one or more fluids, closed loop control is very essential and stability of the closed loop system is significant. The fluids in a typical heat exchanger system may be separated by a solid wall to prevent mixing or they may be in direct contact also [Kakac and Liu (2002); Saunders (1988)]. The heat exchangers are widely used in space heating, refrigeration, air conditioning, power generations, chemical industries, petrochemical and petroleum industries, Oil-industries, gas-industries and sewage treatment [Kakac and Liu (2002); Saunders (1988); Stephanopoulas (2003)]. If the heat exchanger system is controlled through a master control center located at remote, then the feedback loop of the closed-loop control scheme will be completed through communication networks. Such a typical network controlled temperature control scheme will consist of heat exchanger, sensor, valve, and a regulator (embedding PI control logic) placed at the remote control center [Sharma, Gupta and Kumar (2011)]. The temperature control scheme will be pressed into service in the event of any unexpected deviation in the temperature control system output variable like sudden excessive withdrawal of steam by output loads. Under such

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perturbed operating condition, the temperature sensor sense the output temperature. This signal is feedback to the controller unit through a communication network (as digitally processed data packets). In the controller, the error between the reference signal and actual signal (delayed by communication network) is computed and processed by proportional-integral (PI) control scheme. The PI controller subsequently offers a suitable input signal through the communication network to the final control element (the valve) thereby bringing the perturbed heat exchanger terminal temperature back to the equilibrium condition. The heart of the heat exchanger control scheme, the PI controller, is designed *apriori* assuming zero network delays. However, the use of communication network for data (signal) transfer in the closed-loop system introduces time-delays in the feedback path. This, in turn, will enable one to achieve optimal performance from the time-delayed system [Batisha and Jota (2014)]. Delay dependent stability criteria are employed to compute the stable margin for the network delays within which the temperature control system remains asymptotically stable. To the best of author's knowledge, the delay-dependent stability of heat exchanger [Yeroglu (2015)]. These two feedback-loop delays, popularly referred to as network induced delays, are practically unavoidable in a distributed control system scenario where the plant and controller are connected through a communication channel in which the data (control or/and measured) experience buffering, processing and propagation at various internodes; refer Zhao et al. [Zhao, Liu and Rees (2010)]. As a result, the network-introduced two additive delays are time-varying in nature and have dissimilar characteristics [Lam, Goa and Wang (2007)]. In general case, these network induced delays are inevitably pose serious limitations to achievable performance of the closed loop system, and in some cases, may even lead to destabilization of the temperature control system. Hence, stable delay margin (i.e. maximum delay bound which the networked controlled system can accommodate without losing asymptotic stability) for such temperature control system must be computed for various subsets of controller parameters (proportional gain K_p and integral gain K_I) through delay-dependent stability analysis procedure so that they serve as a practical guide-line for fine tuning of controller parameters at the implementation stage. System under networked control environment has not been reported in literature so far. In this paper, a generalized criterion (sufficient condition) is presented for ascertaining delay dependent stability of temperature control systems with time-varying additive feedback loop delays.

In the proposed approach, the mathematical model of the PI controlled temperature control system with time-varying feedback loop delays is first developed as a linear retarded delay-differential continuous-time equation. This state-space model is a generalized modelling framework for dynamical systems with time-delays. Subsequently, based on Lyapunov-Krasovskii functional approach combined with Reciprocal convex combination [Ko and Park (2011)], a delay-dependent stability criterion is presented in LMI framework to compute the maximum value bound of the time-delay within which the closed-loop heat exchanger (temperature control) system remains asymptotically stable in the sense of Lyapunov. The paper concludes with the presentation of simulation results of substantiate the analytical results on delay-dependent stability. The proposed stability criterion is expressed as a set of solvable linear matrix inequalities (LMIs) that can be solved using standard numerical packages [Gahinet, Nemirovskii, Laub et al. (1995)] by casting it as a convex optimization problem.

These adequate conditions generally indicated as stability criteria are expressed as a set of solvable LMI conditions [Xu and Lam (2008)]. The less conservatism of new delay-dependent stability criteria mainly depends on the selection of the LK functional used in the analysis, and many useful results have been reported in literature in recent times [Lam, Goa and Wang (2007); Xu and Lam (2008); Gao, Chng and Lam (2008); Wu, Liao, Feng et al. (2009); Ramakrishnan and Ray (2009, 2011b); Dey, Ray, Ghosh et al. (2010); Shao and Han (2012); Shao, Zhu, Zhang et al. (2013); Li and Tian (2012); Ko, Lee and Park (2011); Jiao (2013); Chaibi, Tisser and Hmamed (2013); Hamed, Chaabane and Kacem (2011)]. However, many useful results existing in the literature, to the best of our knowledge, there are no results available for the delay-dependent stability of temperature control system with additive time-varying delays.

2 Temperature control model with additive time-varying delays

The linear model of temperature control system is used to analyze the system dynamics and to design a controller [Sharma, Gupta and Kumar (2011)]. Fig. 1 shows the block diagram of a temperature control with including network-induced time-delays. Now that each component of the system namely heat exchanger, sensor, valve, the controller is modeled by a first-order transfer function [Sharma, Gupta and Kumar (2011)]. They repeated here for convenience:

$$G_V(s) = \frac{K_V}{T_V s+1}; \quad G_H(s) = \frac{K_H}{T_H s+1}; \quad G_F(s) = \frac{K_F}{T_F s+1}; \tag{1}$$

where K_V, K_H and K_F are the gains of a valve, heat exchanger and sensor respectively, and T_V, T_H and T_F are the corresponding time constants. The transfer function of the PI controller is given below:

$$G_C(s) = K_P + \frac{K_I}{s} \tag{2}$$

where K_P and K_I are the PI controller gains respectively; the proportional term controls the rate of temperature rise after initial transients. The integral controller gain adds a pole at the origin and increases the system type by one and hence reduces the steady-state error [Ramakrishnan and Ray (2016)]. The combined effect of the PI controller gains will structure the response of the temperature control system with reduced overshoot and faster settling time.

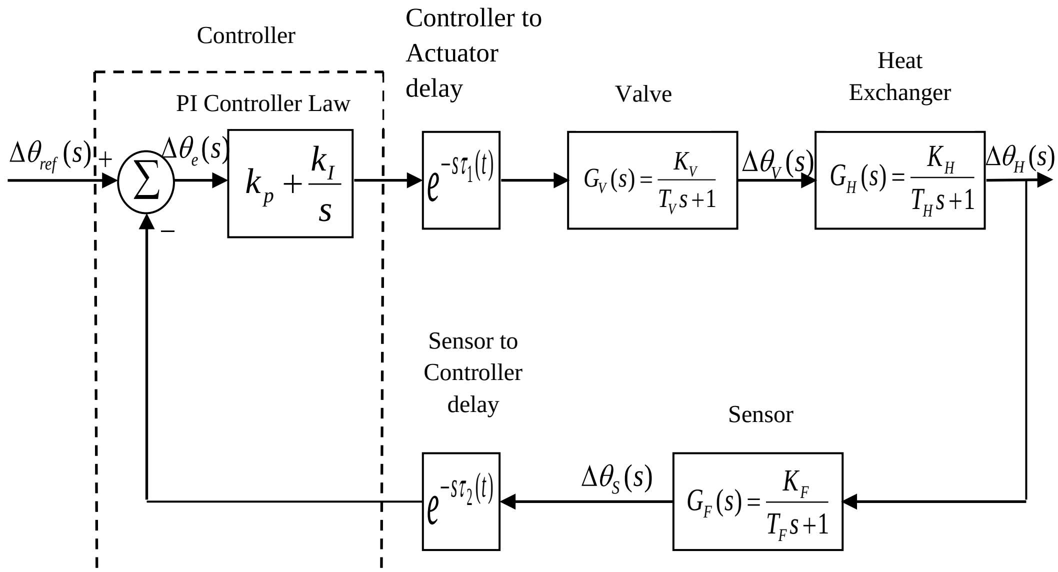

Figure 1: Block diagram of closed loop temperature control system with additive time-varying delays

In the Fig. 1 $\tau_1(t)$ portrays sensor to controller delay (measuring delay) and $\tau_2(t)$ portrays controller to actuator delay (processing delay).

3 System description and problem statement

The general state-space model of the additive state-delayed system is given by:

$$\dot{x}(t) = Ax(t) + A_d x\big(t - \tau_1(t) - \tau_2(t)\big)$$

$$x(t) = \emptyset(t),\ t \in [-(\bar{\tau}_1 + \bar{\tau}_2, 0)]. \tag{3}$$

where $x(t) \in \mathbb{R}^{n\times 1}$ is the state vector, $A \in \mathbb{R}^{n\times n}$ are system matrices, $\emptyset(t)$ is the initial condition of the system for, $t \in [-(\bar{\tau}_1 + \bar{\tau}_2, 0)]$.

Consider the system (3) with the following system matrices are:

$$A = \begin{bmatrix} -\frac{1}{T_V} & 0 & 0 & 0 \\ \frac{K_H}{T_H} & -\frac{1}{T_H} & 0 & 0 \\ 0 & \frac{K_F}{T_F} & -\frac{1}{T_F} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix};\quad A_d = \begin{bmatrix} 0 & 0 & \frac{-K_P K_V}{T_V} & \frac{-K_I K_V}{T_V} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The time-varying additive delays $\tau_1(t)$ and $\tau_2(t)$ are satisfying the following conditions:

$$0 \le \tau_1(t) \le \bar{\tau}_1, 0 \le \tau_2(t) \le \bar{\tau}_2, \tag{4}$$

$$\dot{\tau}_1(t) \le \mu_1 \le \infty, \dot{\tau}_2(t) \le \mu_2 \le \infty, \tag{5}$$

where $\bar{\tau}_1$ and $\bar{\tau}_2$ are the upper bound of the time-varying additive delays $\tau_1(t)$ and $\tau_2(t)$; μ_1 and μ_2 are the derivatives of the delay respectively. To develop a robust stability criterion in LMI framework to get to know the delay-dependent stability of the system (3) subject to the condition satisfying (4) and (5) using LK functional approach and reciprocal convex combination lemma [Ko and Park (2011)]. For driving the new stability criterion, following lemmas are of the utmost importance.

Lemma 1 [Zhu and Yang (2008)]: For any positive symmetric constant matrix $N \in \mathbb{R}^{n\times n}$, scalars η_1 and η_2 satisfying $\eta_1 < \eta_2$, a vector valued function $\omega: [\eta_1, \quad \eta_2] \to \mathbb{R}^n$ such that the integration concerned are well defined, then following inequality holds:

$$\left(\int_{\eta_1}^{\eta_2} \omega(s)ds\right)^T N \left(\int_{\eta_1}^{\eta_2} \omega(s)ds\right) \leq \eta_{12} \int_{\eta_1}^{\eta_2} \omega^T(s) N\, \omega(s)ds \tag{6}$$

where

$$\eta_{12} = \eta_2 - \eta_1$$

*Lemma 2: F*or a continuous function $f(x)$ and its derivative $f'(x)$ are defined on $[a, \quad b]$, then

$$\int_a^b f'(x)dx = f(b) - f(a). \tag{7}$$

Lemma 3 [Ko and Park (2011)]: For any vectors $\varepsilon_1, \varepsilon_2$, matrices $P > 0, Q$ and real numbers $\alpha_1 > 0, \alpha_2 > 0$ with $\alpha_1 + \alpha_2 = 1$ satisfying

$$\begin{bmatrix} P & Q \\ Q^T & P \end{bmatrix} \geq 0, \tag{8}$$

The following inequality condition holds:

$$-\frac{1}{\alpha_1}\varepsilon_1^T P \varepsilon_1 - \frac{1}{\alpha_2}\varepsilon_2^T P \varepsilon_2 \leq -\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}^T \begin{bmatrix} P & Q \\ Q^T & P \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{9}$$

Proof: The condition (8) implies that

$$\begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}}\varepsilon_1 \\ -\sqrt{\frac{\alpha_2}{\alpha_1}}\varepsilon_2 \end{bmatrix}^T \begin{bmatrix} P & Q \\ Q^T & P \end{bmatrix} \begin{bmatrix} \sqrt{\frac{\alpha_2}{\alpha_1}}\varepsilon_1 \\ -\sqrt{\frac{\alpha_2}{\alpha_1}}\varepsilon_2 \end{bmatrix} \geq 0, \tag{10}$$

which gives

$$\frac{\alpha_2}{\alpha_1}\varepsilon_1^T P \varepsilon_1 + \frac{\alpha_1}{\alpha_2}\varepsilon_2^T P \varepsilon_2 \geq \varepsilon_1^T Q \varepsilon_2 + \varepsilon_2^T Q \varepsilon_1 \tag{11}$$

By using the relationship $\alpha_1 + \alpha_2 = 1$ on (11) and by rearranging the terms, we deduce the inequality condition (9).

4 Main results

In this section, the new stability criterion for the temperature control system with additive time-varying delay is presented in the following theorem.

Theorem: Given non-negative scalars $\bar{\tau}_1, \bar{\tau}_2, \mu_1, \mu_2$ the system (1) satisfying the conditions (4) and (5) is asymptotically stable, if there exists real, positive, symmetric matrices $Z, R, Q_i, i = 1, 2, 3$; free matrices S_{12}, S_{13}, S_{23} of appropriate dimensions such that following LMI conditions hold:

$$\begin{bmatrix} Z & S_{12} & S_{13} \\ * & Z & S_{23} \\ * & * & Z \end{bmatrix} \geq 0 \tag{12}$$

$$Z > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R > 0; \tag{13}$$

$$\Xi < 0, \tag{14}$$

where

$$\Xi = e_1 P e_5^T + e_5 P e_1^T + e_1(Q_1 + Q_2 + Q_3)e_1^T + e_2\big(-Q_1(1-\mu_1)\big)e_2^T + e_3\big(-Q_2(1-\mu)\big)e_3^T + e_4(-Q_3)e_4^T + e_5 U e_5^T - \begin{bmatrix} (e_1 - e_2)^T \\ (e_2 - e_3)^T \\ (e_3 - e_4)^T \end{bmatrix}^T \begin{bmatrix} Z & S_{12} & S_{13} \\ * & Z & S_{23} \\ * & * & Z \end{bmatrix} \tag{15}$$

with

$$e_1 = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T,$$
$$e_2 = \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T,$$
$$e_3 = \begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}^T,$$
$$e_4 = \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T,$$
$$e_5 = \left(A e_1^T + A_d e_3^T\right)^T,$$
$$U = \bar{\tau}^2 Z$$

where

$\bar{\tau} = \bar{\tau}_1 + \bar{\tau}_2$ and $\mu = \mu_1 + \mu_2$.

Proof: consider the LK functional $V(t) = \sum_{i=1}^{3} V_i(t)$ with

$$V_1(t) = x^T(t) P x(t), \tag{16}$$

$$V_2(t) = \int_{t-\tau_1(t)}^{t} x^T(s) Q_1 x(s)\, ds + \int_{t-\tau(t)}^{t} x^T(s) Q_2 x(s)\, ds + \int_{t-\bar{\tau}}^{t} x^T(s) Q_3 x(s)\, ds, \tag{17}$$

$$V_3(t) = \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z \dot{x}(s)\, ds d\theta, \tag{18}$$

where P, Q_i, $i = 1, 2, 3$, and Z are real symmetric positive definite matrices, and $\tau(t) = (\tau_1(t) + \tau_2(t))$. Define $\eta(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_1(t)) & x^T(t-\tau(t)) & x^T(t-\bar{\tau}) \end{bmatrix}^T$.

The time-derivative of the functional $V_i(t), i = 1, 2, 3$, is computed along the trajectory of (3) as follows: The time-derivative of $\dot{V}_1(t)$ is given by

$$\dot{V}_1(t) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t)$$

$$\dot{V}_1(t) = \eta^T(t)\big(e_1 P e_5^T + e_5 P e_1^T\big)\eta(t) \tag{19}$$

The time-derivative of $\dot{V}_2(t)$ is given by

$$\dot{V}_2(t) = x^T(t)(Q_1 + Q_2 + Q_3)x(t) - (1 - \dot{\tau}_1(t))x^T(t - \tau_1(t))Q_1 x(t - \tau_1(t)) - (1 - \dot{\tau}(t))x^T(t - \tau(t))Q_2 x(t - \tau(t)) - x^T(t - \bar{\tau})Q_3 x(t - \bar{\tau}) \tag{20}$$

Since in NCSs, $\dot{\tau}_1(t) \leq \mu_1 < 1$ and $\dot{\tau}(t) \leq \mu < 1$, using Eq. (20) can be expressed as an inequality as follows:

$$\dot{V}_2(t) \leq x^T(t)(Q_1 + Q_2 + Q_3)x(t) - (1-\mu_1)x^T(t-\tau_1(t))Q_1x(t-\tau_1(t)) - (1-\mu)x^T(t-\tau(t))Q_2x(t-\tau(t)) - x^T(t-\bar{\tau})Q_3x(t-\bar{\tau}). \quad (21)$$

In other words, Eq. (21) can be revealed in terms of $\eta(t)$ as follows:

$$\dot{V}_2(t) \leq \eta^T(t)(e_1(Q_1+Q_2+Q_3)e_1^T + e_2(-(1-\mu_1)Q_1)e_2^T + e_3(-(1-\mu)Q_2)e_3^T + e_4(-Q_3)e_4^T)\eta(t) \quad (22)$$

The time-derivative $\dot{V}_3(t)$ is given by

$$\dot{V}_3(t) = \dot{x}^T(t)\bar{\tau}^2Z\dot{x}(t) - \bar{\tau}\int_{t-\bar{\tau}}^{t}\dot{x}^T(s)Z\dot{x}(s)ds. \quad (23)$$

The Eq. (23) can be rewritten by expanding the integral part as follows:

$$\dot{V}_3(t) = \eta^T(t)\left(e_5Ue_5^T\right)\eta(t) - \bar{\tau}\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{x}^T(s)Z\dot{x}(s)ds - \bar{\tau}\int_{t-\tau(t)}^{t-\tau_1(t)}\dot{x}^T(s)Z\dot{x}(s)ds - \bar{\tau}\int_{t-\tau_1(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds \quad (24)$$

Which, for applying Lemma 1, is rewritten on Eq. (24), we get

$$\dot{V}_3(t) \leq \eta^T(t)\left(e_5Ue_5^T\right)\eta(t) - \frac{\bar{\tau}}{(\bar{\tau}-\tau(t))}\left[(\bar{\tau}-\tau(t))\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{x}^T(s)Z\dot{x}(s)ds\right] - \frac{\bar{\tau}}{(\tau(t)-\tau_1(t))}\left[\left(\tau(t)-\tau_1(t)\right)\int_{t-\tau(t)}^{t-\tau_1(t)}\dot{x}^T(s)Z\dot{x}(s)ds\right] - \frac{\bar{\tau}}{\tau_1(t)}\left[\left(\tau_1(t)\right)\int_{t-\tau_1(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds\right]. \quad (25)$$

Now, by applying Lemma 1 on Eq. (25), we get

$$\dot{V}_3(t) \leq \eta^T(t)\left(e_5Ue_5^T\right)\eta(t) - \frac{1}{\delta_1}\left(\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{x}(s)ds\right)^T Z\left(\int_{t-\bar{\tau}}^{t-\tau(t)}\dot{x}(s)ds\right) - \frac{1}{\delta_2}\left(\int_{t-\tau(t)}^{t-\tau_1(t)}\dot{x}(s)ds\right)^T Z\left(\int_{t-\tau(t)}^{t-\tau_1(t)}\dot{x}(s)ds\right) - \frac{1}{\delta_3}\left(\int_{t-\tau_1(t)}^{t}\dot{x}(s)ds\right)^T Z\left(\int_{t-\tau_1(t)}^{t}\dot{x}(s)ds\right) \quad (26)$$

where $\delta_1 = \frac{(\bar{\tau}-\tau(t))}{\bar{\tau}}$, $\delta_2 = \frac{\left(\tau(t)-\tau_1(t)\right)}{\bar{\tau}}$, $\delta_3 = \frac{\left(\tau_1(t)\right)}{\bar{\tau}}$

Now, we can express Eq. (26) as follows:

$$\dot{V}_3(t) \leq \eta^T(t)\left(e_5Ue_5^T\right)\eta(t) - \frac{1}{\delta_1}\eta^T(t)(e_3-e_4)^TZ(e_3-e_4)\eta(t) - \frac{1}{\delta_2}\eta^T(t)(e_2-e_3)^TZ(e_2-e_3)\eta(t) - \frac{1}{\delta_3}\eta^T(t)(e_1-e_2)^TZ(e_1-e_2)\eta(t) \quad (27)$$

Since $\delta_1 + \delta_2 + \delta_3 = 1, \forall t$, the reciprocal convex combination technique of Eq. (27) is dealt using Lemma 3 as follows:

$$\dot{V}_3(t) \leq \eta^T(t)\left(e_5Ue_5^T\right)\eta(t) - \begin{bmatrix}(e_1-e_2)^T\\(e_2-e_3)^T\\(e_3-e_4)^T\end{bmatrix}^T\begin{bmatrix}Z & S_{12} & S_{13}\\ * & Z & S_{23}\\ * & * & Z\end{bmatrix}\begin{bmatrix}(e_1-e_2)^T\\(e_2-e_3)^T\\(e_3-e_4)^T\end{bmatrix} \quad (28)$$

Provided

$$\begin{bmatrix}Z & S_{12} & S_{13}\\ * & Z & S_{23}\\ * & * & Z\end{bmatrix} \geq 0 \quad (29)$$

holds with S_{12}, S_{13} and S_{23} being free matrices of any dimensions.

By adding $\dot{V}_1(t), \dot{V}_2(t)$, and $\dot{V}_3(t)$ are expressions from (19), (22) and (28) respectively, we obtain the following quadratic bounding relation.

$$\dot{V}(t) \leq \eta^T(t)\Xi\eta(t) \tag{30}$$

If the condition $\Xi < 0$ and relation (29) holds simultaneously, which the additive time-delay system (3) with satisfying the condition (4) and (5) is asymptotically stable.

5 Results and discussions

In this discussion, the effectiveness of the stability criterion is presented for temperature control system with additive time-varying delays is illustrated. The maximum allowable delay margin for additive time-varying delays $\bar{\tau}_1$ and $\bar{\tau}_2$ for stability for a pick range of PI controller gains is computed. The gains and time constants of the temperature control used in the analysis are as follows [Ko and Park (2011)].

Table 1: Parameter values of temperature control system

System	Gain	Time-constant
Heat exchanger	$K_H = 34$	$T_H = 30$
Valve	$K_V = 1.25$	$T_V = 3$
Sensor	$K_F = 0.08$	$T_F = 2$

For the aforesaid system parameters with delay free condition, the controller capability curve can be obtained easily; it is illustrated in Fig. 2. From the Fig. 2, it is clear that for all values of K_P and K_I lying below the controller capability curve, the closed-loop system is asymptotically stable. From the curve we choose $K_P = 2$ and $K_I = 0.15s^{-1}$, then the closed loop system converges asymptotically to equilibrium point as illustrated in Fig. 3 for a unit step perturbation in temperature control system output variable (from its equilibrium value).

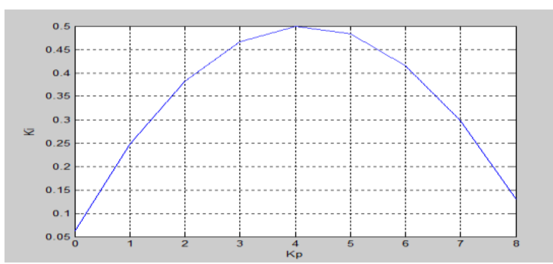

Figure 2: Maximum value of K_I for different values of K_P

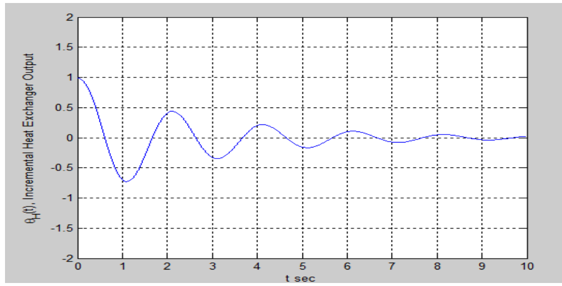

Figure 3: Evolution of $\Delta\theta_H(t)$ less than zero delay condition

when the network delay τ is made zero, the system Eq. (3) becomes

$$\dot{x}(t) = (A + A_d)x(t) \tag{31}$$

Now, it is observed that the Eigenvalues of the system state matrix and input matrix $(A + A_d)$ depend on the controller parameters K_P and K_I. Tab. 2 given the maximum values of $K_I(s^{-1})$ for a fixed value of K_P for which the temperature control system is on the verge of instability i.e. having one complex conjugate pair of eigenvalues on the $j\omega$ axis. It is observed from the Fig. 3 that there is a tendency for the curve to decrease gradually with the increase of K_I.

Table 2: Maximum integral controller gain for a given K_P

K_P	$K_I(s^{-1})$	Eigen values of $(A + A_d)$
0	0.062	-0.4329±0.009j; -0.0004; -0.0790
1	0.248	-0.0000±0.167j; -0.5801; -0.2864
2	0.382	-0.0001±0.223j; -0.6415; -0.2250
3	0.466	-0.0000±0.267j; -0.6885; -0.1781
4	0.499	-0.0000±0.305j; -0.7282; -0.1385
5	0.482	-0.0000±0.339j; -0.7631; -0.1036
6	0.415	-0.0001±0.370j; -0.7946; -0.0719
7	0.298	-0.0000±0.398j; -0.8236; -0.0430
8	0.130	-0.0000±0.425j; -0.8506; -0.0160

Now, when this PI controller operates atop a networked environment, the temperature control system will be subjected to additive network induced time-varying delays $\tau_1(t)$ and $\tau_2(t)$. Let the time-derivative of the additive time-varying delays be bounded as $\dot{\tau}_1(t) = 0.1$ and $\dot{\tau}_2(t) = 0.2$. Let choose the controller parameter $K_P = 2$ and $K_I = 0.15s^{-1}$, in the presence of network induced delays, according to the stability criterion are presented in theorem, the temperature control system is stable up to $\tau_1 = 1$ second $\tau_2 = 1.3305$ seconds, and if network induced delays is increased far beyond this value, the system output becomes unbounded evolution. Hence, this analysis clearly brings to the impact of the network induced delays on stability and performance of the system. Let $\mu_1 = 0.1$, and $\mu_2 = 0.2$. for different values of K_P and K_I, from the capability curve, the maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$is given in Tabs. 3-7. Likewise, Tabs. 8-12 give the maximum allowable delay bound $\bar{\tau}_1$ for a given $\bar{\tau}_2$. From the tables, it is clear that as the various values of K_P and K_I, the maximum delay bound that system can accommodate without losing stability decreases. The corresponding stability margin presented in Fig. 4 clearly indicates lessening.

Table 3: Maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$-1

K_P	K_I	$\bar{\tau}_1 =$ 0	$\bar{\tau}_1 =$ 0.2	$\bar{\tau}_1 =$ 0.4	$\bar{\tau}_1 =$ 0.6	$\bar{\tau}_1 =$ 0.8	$\bar{\tau}_1 =$ 1.0	$\bar{\tau}_1 =$ 1.2	$\bar{\tau}_1 =$ 1.4	$\bar{\tau}_1 =$ 1.6	$\bar{\tau}_1 =$ 1.8	$\bar{\tau}_1 =$ 2	$\bar{\tau}_1 =$ 2.2	$\bar{\tau}_1 =$ 2.331
2	0.15	2.3305	2.1305	1.9305	1.7305	1.5305	1.3305	1.1305	0.9305	0.7305	0.5305	0.3305	0.1305	0

Table 4: Maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$-2

K_P	K_I	$\bar{\tau}_1 =$ 0	$\bar{\tau}_1 =$ 0.02	$\bar{\tau}_1 =$ 0.04	$\bar{\tau}_1 =$ 0.06	$\bar{\tau}_1 =$ 0.08	$\bar{\tau}_1 =$ 0.1	$\bar{\tau}_1 =$ 0.2	$\bar{\tau}_1 =$ 0.4	$\bar{\tau}_1 =$ 0.6	$\bar{\tau}_1 =$ 0.8	$\bar{\tau}_1 =$ 1.0	$\bar{\tau}_1 =$ 1.1408
2	0.25	1.1408	1.1208	1.1008	1.0808	1.0608	1.0408	0.9408	0.7408	0.5408	0.3408	0.1408	0

Table 5: Maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$-3

K_P	K_I	$\bar{\tau}_1 =$ 0	$\bar{\tau}_1 =$ 0.02	$\bar{\tau}_1 =$ 0.04	$\bar{\tau}_1 =$ 0.06	$\bar{\tau}_1 =$ 0.08	$\bar{\tau}_1 =$ 0.1	$\bar{\tau}_1 =$ 0.2	$\bar{\tau}_1 =$ 0.4	$\bar{\tau}_1 =$ 0.6	$\bar{\tau}_1 =$ 0.8	$\bar{\tau}_1 =$ 1.0	$\bar{\tau}_1 =$ 1.0983
4	0.1	1.0983	1.0783	1.0583	1.0383	1.0183	0.9983	0.8983	0.6983	0.4983	0.2983	0.0983	0

Table 6: Maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$-4

K_P	K_I	$\bar{\tau}_1 =$ 0	$\bar{\tau}_1 =$ 0.02	$\bar{\tau}_1 =$ 0.04	$\bar{\tau}_1 =$ 0.06	$\bar{\tau}_1 =$ 0.08	$\bar{\tau}_1 =$ 0.1	$\bar{\tau}_1 =$ 0.12	$\bar{\tau}_1 =$ 0.14	$\bar{\tau}_1 =$ 0.16	$\bar{\tau}_1 =$ 0.18	$\bar{\tau}_1 =$ 0.2	$\bar{\tau}_1 =$ 0.2611
6	0.1	0.2611	0.2411	0.2211	0.2011	0.1811	0.1611	0.1411	0.1211	0.1011	0.0811	0.0611	0

Table 7: Maximum allowable delay bound $\bar{\tau}_2$ for a given $\bar{\tau}_1$-5

K_P	K_I	$\bar{\tau}_1 =$ 0	$\bar{\tau}_1 =$ 0.02	$\bar{\tau}_1 =$ 0.04	$\bar{\tau}_1 =$ 0.06	$\bar{\tau}_1 =$ 0.08	$\bar{\tau}_1 =$ 0.1	$\bar{\tau}_1 =$ 0.12	$\bar{\tau}_1 =$ 0.14	$\bar{\tau}_1 =$ 0.16	$\bar{\tau}_1 =$ 0.1644
2.5	0.4	0.1644	0.1444	0.1244	0.1044	0.0844	0.0644	0.0444	0.0244	0.0044	0

Table 8: Maximum allowable delay bound $\bar{\tau}_1$for a given $\bar{\tau}_2$-1

K_P	K_I	$\bar{\tau}_2 =$ 0	$\bar{\tau}_2 =$ 0.2	$\bar{\tau}_2 =$ 0.4	$\bar{\tau}_2 =$ 0.6	$\bar{\tau}_2 =$ 0.8	$\bar{\tau}_2 =$ 1.0	$\bar{\tau}_2 =$ 1.2	$\bar{\tau}_2 =$ 1.4	$\bar{\tau}_2 =$ 1.6	$\bar{\tau}_2 =$ 1.8	$\bar{\tau}_2 =$ 2	$\bar{\tau}_2 =$ 2.2	$\bar{\tau}_2 =$ 2.331
2	0.15	2.3305	2.1305	1.9305	1.7305	1.5305	1.3305	1.1305	0.9305	0.7305	0.5305	0.3305	0.1305	0

Table 9: Maximum allowable delay bound $\bar{\tau}_1$for a given $\bar{\tau}_2$-2

K_P	K_I	$\bar{\tau}_2 = 0$	$\bar{\tau}_2 = 0.02$	$\bar{\tau}_2 = 0.04$	$\bar{\tau}_2 = 0.06$	$\bar{\tau}_2 = 0.08$	$\bar{\tau}_2 = 0.1$	$\bar{\tau}_2 = 0.2$	$\bar{\tau}_2 = 0.4$	$\bar{\tau}_2 = 0.6$	$\bar{\tau}_2 = 0.8$	$\bar{\tau}_2 = 1.0$	$\bar{\tau}_2 = 1.0983$
4	0.1	1.0983	1.0783	1.0583	1.0383	1.0183	0.9983	0.8983	0.6983	0.4983	0.2983	0.0983	0

Table 10: Maximum allowable delay bound $\bar{\tau}_1$for a given $\bar{\tau}_2$-3

K_P	K_I	$\bar{\tau}_2 = 0$	$\bar{\tau}_2 = 0.02$	$\bar{\tau}_2 = 0.04$	$\bar{\tau}_2 = 0.06$	$\bar{\tau}_2 = 0.08$	$\bar{\tau}_2 = 0.1$	$\bar{\tau}_2 = 0.2$	$\bar{\tau}_2 = 0.4$	$\bar{\tau}_2 = 0.6$	$\bar{\tau}_2 = 0.8$	$\bar{\tau}_2 = 1.0$	$\bar{\tau}_2 = 1.1408$
2	0.25	1.1408	1.1208	1.1008	1.0808	1.0608	1.0408	0.9408	0.7408	0.5408	0.3408	0.1408	0

Table 11: Maximum allowable delay bound $\bar{\tau}_1$for a given $\bar{\tau}_2$-4

K_P	K_I	$\bar{\tau}_2 = 0$	$\bar{\tau}_2 = 0.02$	$\bar{\tau}_2 = 0.04$	$\bar{\tau}_2 = 0.06$	$\bar{\tau}_2 = 0.08$	$\bar{\tau}_2 = 0.1$	$\bar{\tau}_2 = 0.12$	$\bar{\tau}_2 = 0.14$	$\bar{\tau}_2 = 0.16$	$\bar{\tau}_2 = 0.18$	$\bar{\tau}_2 = 0.2$	$\bar{\tau}_2 = 0.26$
6	0.1	0.2611	0.2411	0.2211	0.2011	0.1811	0.1611	0.1411	0.1211	0.1011	0.0811	0.0611	0

Table 12: Maximum allowable delay bound $\bar{\tau}_1$for a given $\bar{\tau}_2$-5

K_P	K_I	$\bar{\tau}_2 = 0$	$\bar{\tau}_2 = 0.02$	$\bar{\tau}_2 = 0.04$	$\bar{\tau}_2 = 0.06$	$\bar{\tau}_2 = 0.08$	$\bar{\tau}_2 = 0.1$	$\bar{\tau}_2 = 0.12$	$\bar{\tau}_2 = 0.14$	$\bar{\tau}_2 = 0.16$	$\bar{\tau}_2 = 0.1644$
2.5	0.4	0.1644	0.1444	0.1244	0.1044	0.0844	0.0644	0.0444	0.0244	0.0044	0

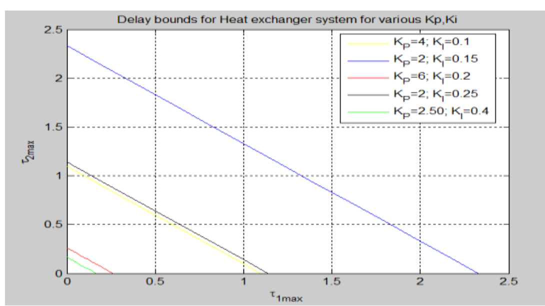

Figure 4: Stability margin for temperature control of heat exchanger

Simulation study of the closed-loop temperature control system was carried out for zero network delay is confined to the stable region shown in Fig. 3; it is easy to infer the influence of network delays on the stability of the system. As network delays increase, the response of the temperature control system gets malformed and for higher delays, the closed-loop system becomes unstable.

6 Conclusion

In this research, the problem of delay-dependent stability of temperature control systems with additive time-varying delay has been addressed using Lyapunov-Krasovskii functional technique. In the stability analysis, the temperature control system is mathematically modeled in terms of state space model, and reciprocal convex combination technique is employed in the stability analysis to make it less number of decision variables. The proposed results are validated benchmark temperature control system with time-varying additive delays. The deduced results of this paper impart more realistic operating condition in a real-time temperature control system.

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