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Numerical investigations in the behaviour of one-dimensional bubbly flow in hydrodynamic cavitation

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Abstract

Hydrodynamic cavitation has been attempted as an alternative to acoustic cavitation over past few years and this form of cavitation has been proved to hold higher potential for industrial scale processes. In this work, we have tried to investigate the dynamics of the bubbly flow in hydrodynamic cavitation with the help of numerical simulations. A continuum bubbly mixture model is used to study the one-dimensional cavitating flow through a venturi. The nonlinear dynamics of the bubbles has been modeled by Rayleigh–Plesset equation. The results of the simulations show that the bubble/bubble and bubble/flow interaction through the hydrodynamics of the flow has important effect on the behavior of the bubbly flow. Effect of parameters such as downstream recovery pressure, venturi to pipe area ratio, initial bubble void fraction in liquid and initial bubble size in the flow on the dynamics of the flow has been studied. Based on the results of the simulations, recommendations have been made for an effective design and scale-up of a hydrodynamic cavitation reactor.

Keywords: Cavitation; Sonochemistry; Bubble dynamics

1. Introduction

Use of cavitation and the energy associated with it has been pursued by several researchers for improving chemical processing over past several years. Although ultrasound has been the most popular means of generating cavitation in the laboratory scale studies, ultrasonic reactors have suffered from several shortcomings on industrial scale applications. Over past few years hydrodynamic cavitation has been explored as an alternative to the acoustic cavitation. In hydrodynamic cavitation, the cavities or bubbles are generated by the flow of liquid under controlled conditions through simple geometries such as a venturi or an orifice plate. When the pressure downstream of the mechanical constriction such as venturi or orifice falls below the vapor pressure of the liquid a number of cavities are generated, which subsequently collapse with the recovery of pressure downstream of the mechanical constriction. Although this phenomenon has been studied extensively over past several years, the main emphasis of the research in this area has been to suppress the occurrence of cavitation due to its destructive nature in the liquid transportation. Focus of our research is to generate the hydrodynamic cavitation under controlled conditions and harness the energy associated with it for enhancing the efficiency of the chemical processes. A number of earlier experimental studies such as hydrolysis of fatty oils (Joshi & Pandit, 1993), microbial cell disruption (Save, Joshi & Pandit, 1997, 1994; Shirgaonkar, Lothe & Pandit, 1998), polymerization and depolymerization of aqueous polymeric solutions (Chivate & Pandit, 1993) have proved that the hydrodynamic cavitation is far more energy efficient than its acoustic counterpart.

In our earlier theoretical studies (Moholkar, 1996; Moholkar & Pandit, 1997; Moholkar, Kumar & Pandit, 1999; Shah, Pandit & Moholkar, 1999) we have reported dynamics of a single, isolated bubble in the hydrodynamic cavitation ignoring the bubble/liquid and bubble/bubble interaction and the effect of the bubble content on the two phase flow. In this work, we have presented the dynamics of the bubbly flow in the hydrodynamic cavitation taking into consideration the

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bubble/bubble and bubble/flow interaction using the nonlinear continuum bubbly mixture model coupled with Rayleigh–Plesset equation for the bubble dynamics (van Wijngaarden, 1968, 1972). We expect this analysis to form a useful basis for the effective design and scale-up of the hydrodynamic cavitation reactors.

2. Mathematical formulation

The flow model used in this analysis is a nonlinear continuum bubbly mixture model coupled with Rayleigh–Plesset equation for the bubble dynamics. This was first proposed by van Wijngaarden (1968, 1972). Since then it has been extensively used for studying the steady and transient shock wave propagation in the bubbly liquids without acceleration of the mean flow (for example, Noordzij & van Wijngaarden, 1974; Kameda & Matsumoto, 1995) and the one-dimensional cavitating flow through a converging–diverging nozzle (Wang & Brennen, 1998). Earlier studies based on the space-averaged equations for the mixture in the absence of relative motion between the two phases (Tangren, Dodge & Seifert, 1949) did not consider the bubble dynamics effect. More recently, Ishii, Umeda, Murata and Shishido (1993) have proposed a bubbly flow model to study the steady-state flow through a nozzle. However, by assuming that the gas pressure inside the bubbles is equal to the ambient fluid pressure, they neglected the bubble radial dynamics effect, which is the dominant mechanism in the cavitating flow. Other studies in two-phase flow with the continuum model include investigations of d'Agostino and Brennen (1983, 1989) in the linearized dynamics of the spherical bubble clouds. A more general model for two-phase flows with small gas bubbles which includes the effect of bubble dynamics and the relative motion between the phases has been presented by Biesheuvel and van Wijngaarden (1984). Omta (1987) linearized homogeneous flow equations of Biesheuvel and van Wijngaarden (1984) to obtain the solution to the flow in a spherical bubble cloud. Another perspective on the subject of two-phase models and dynamics of bubble clusters is given by Mørch and co-workers (Mørch, 1980, 1981, 1982; Hansson & Mørch, 1980; Hansson, Kedrinskii & Mørch, 1981). They speculated that the collapse of cavity or bubble clusters in a cavitating flow involves formation and inward propagation of a shock wave. Computational results of Wang and Brennen (1994, 1995) based on the continuum model confirm the idea put forth by Mørch and co-workers. However, in a recent paper Wang & Brennen (1999) have shown that the direction of the shock wave depends on the bubble cloud interaction parameter and can be in either direction (inward or outward from the center of the bubble cloud) and not necessarily in the inward direction as proposed by Mørch.

2.1. Basic equations

Fig. 1 shows a venturi with length L and cross-sectional area $A(x)$. The flow direction is in the positive x direction and the inlet and throat of the venturi are located at $x = 0$ and $0.5L$, respectively. We make the following assumptions in the analysis:

1. The liquid is assumed to be incompressible.
2. Relative motion between the two phases is neglected.
3. Friction between the liquid and the duct wall is neglected.
4. The total upstream bubble population is uniform and there is no coalescence and further break up of the bubbles in the flow.
5. The density of gas and vapor is neglected in comparison to the density of the liquid.
6. All bubbles have assumed to have the same initial radius R_0.

We consider a gas–liquid flow in the venturi with the gas bubbles of radius $R(x,t)$ and the number density n. The mass density of the mixture is written as

$$\rho = \rho_L(1 - nV), \tag{1}$$

where $V = 4/3\pi R^3(x,t)$ is the volume of one bubble.

The continuity and momentum equations of the two-phase flow have the form (Wang & Brennen, 1998)

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{3}$$

The bubble void fraction in the flow is $\alpha(x,t) = 4/3n\pi R^3(x,t)/[1 + 4/3n\pi R^3(x,t)]$. The interaction of the bubbles with the flow is modeled by Rayleigh–Plesset equation of bubble dynamics (Knapp, Daily & Hammitt, 1970; Plesset & Prosperetti, 1977):

$$\rho_L\left[R\frac{D^2R}{Dt^2} + \frac{3}{2}\left(\frac{DR}{Dt}\right)^2\right] = \left[\left(P_0 + \frac{2\sigma}{R_0} - P_v\right)\left(\frac{R_0}{R}\right)^{3k} - P_\infty - \frac{2\sigma}{R} - \frac{4\mu}{R}\left(\frac{DR}{Dt}\right) + P_v\right], \tag{4}$$

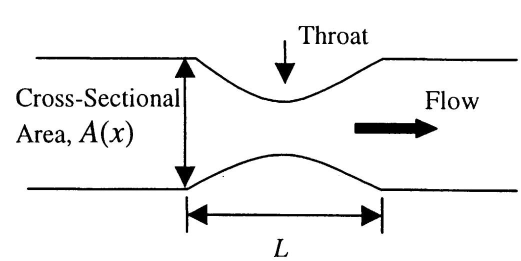

Fig. 1. Venturi for hydrodynamic cavitation.

where $D/Dt = [\partial/\partial t + u\partial/\partial x]$ is the Lagrangian derivative. P_0 is the ambient pressure in the liquid at the point of generation of the bubbles, which is the throat of the venturi in the present case. P_∞ is the pressure in the bulk liquid far from the bubble. k is the effective polytropic exponent of the bubble content, which can be gas or vapor or mixture of both. Plesset and Prosperetti (1977) have calculated how k depends on the (thermal) Péclet number, $P\acute{e} = R_0^2\omega/\kappa$, which gives the ratio between the bubble length scale R_0 and the thermal diffusion length $\sqrt{\kappa/\omega}$. However, Hilgenfeldt, Lohse and Brenner (1996) have argued that during the bubble motion the conditions for which $P\acute{e}(t) \gg 1$ (which implies that the bubble motion is adiabatic) holds for very small time intervals (less than a nanosecond or so) and hence the global dynamics of the bubble is not affected by setting $k = 1$ uniformly in time. The assumption of no slip between the two phases allows replacing the term P_∞ in Eq. (4) by p, the average pressure in the gas–liquid mixture.

Eqs. (2)–(4) now represent a simple model of one-dimensional two-phase flow with the nonlinear bubble dynamics. A number of papers have appeared in the past which study the dynamics of the two-phase flow by linearizing the above model (for example, Biesheuvel & van Wijngaarden, 1984; Omta, 1987; d'Agostino & Brennen, 1983, 1989). In a venturi flow the mean flow parameters change rapidly in space and time due to the acceleration of the liquid. Therefore, the approach of simple linearization as used by previous researchers is not valid in the present case. Therefore, for the solution of equation system (2)–(4) we use the approach of Wang and Brennen (1998) based on the assumption of steady-state cavitating flow for a constant mass flow rate.

2.2. Steady-state solutions

Under the assumption of steady-state conditions, all the partial time derivative terms in Eqs. (2)–(4) disappear. Now Eqs. (2)–(4) are transformed into a system of ordinary differential equations with only one independent variable, x:

$$\rho_L\left(1 - \frac{4}{3}\pi n R^3\right) u A = \text{constant}, \tag{5}$$

$$u\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{\rho_L(1 - 4/3\pi n R^3)}\frac{\mathrm{d}p}{\mathrm{d}x}, \tag{6}$$

$$p = P_v + \left(P_0 + \frac{2\sigma}{R_0} - P_v\right)\left(\frac{R_0}{R}\right)^{3k} - \frac{2\sigma}{R} - \frac{4u\mu}{R}\left(\frac{\mathrm{d}R}{\mathrm{d}x}\right) - \rho_L\left[R\left(u^2\frac{\mathrm{d}^2R}{\mathrm{d}x^2} + u\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}R}{\mathrm{d}x}\right) + \frac{3u^2}{2}\left(\frac{\mathrm{d}R}{\mathrm{d}x}\right)^2\right]. \tag{7}$$

The area of the venturi, $A(x)$, is written as

$$A(x) = \left\{1 - \frac{1}{2}\delta\left[1 - \cos\left(\frac{2\pi x}{L}\right)\right]\right\}^{-1/2} \quad \text{for } 0 \leqslant x \leqslant L. \tag{8}$$

The constant in Eq. (5) can be determined by the initial conditions at the throat of the venturi, which are, $x = L/2$, $R = R_0$, $u = u_0$ and $\alpha = \alpha_0$. u_0 is the velocity at the throat of the venturi and is determined by the cavitation number

$$C_i = \frac{P_2 - P_v}{1/2\rho_L u_0^2}, \tag{9}$$

where P_2 is the downstream recovery pressure.

Elimination of p from Eqs. (6) and (7) (using the assumption of no slip between the two phases) gives the following system of differential equations with one independent variable, x, and two dependent variables, R and u:

$$\frac{\mathrm{d}R}{\mathrm{d}x} = a, \tag{10}$$

$$\frac{\mathrm{d}^2R}{\mathrm{d}x^2} = b, \tag{11}$$

$$\frac{\mathrm{d}^3R}{\mathrm{d}x^3} = \frac{(1 - nV)(\mathrm{d}u/\mathrm{d}x)}{uR} + \frac{1}{\rho_L u^2 R}\left\{\left(P_0 + \frac{2\sigma}{R_0} - P_v\right) \times \frac{R_0^{3k}}{R^{3k+1}}a + \frac{2a\sigma}{R^2} - 3\rho_L\left[u^2ab + ua^2\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)\right] - \rho_L\left[2buR\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right) + abu^2 + u\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)a^2 + auR\left(\frac{\mathrm{d}^2u}{\mathrm{d}x^2}\right) + aR\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 + bRu\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)\right]\right\}. \tag{12}$$

Using Eq. (5) u can be expressed as a function of R and x. With substitution of u in terms of R and x Eqs. (10)–(12) become system of three simultaneous differential equations and can be solved using Runge–Kutta fourth-order method with adaptive step size control (Press, Teukolsky, Flannery & Vetterling, 1992). The bubbles are assumed to collapse when during the radial motion the ratio R/R_0 becomes less than 0.05. This is also the end point of the simulations. The physical properties of the flow are $\rho_L = 1000\ \mathrm{kg\,m^{-3}}$; $\sigma = 0.072\ \mathrm{N\,m^{-1}}$; $\mu = 10$ cP.

3. Results and discussion

Simulations were carried out to investigate the effect of the following parameters on the bubbly flow in hydrodynamic cavitation.

1. Downstream recovery pressure (P_2).
2. The venturi to pipe area ratio (β).
3. Initial bubble void fraction in the flow (α_0).
4. Initial size of the bubbles (R_0).

For brevity only representative solutions are given in this section. Effect of the above parameters was assessed mainly on two aspects of the bubbly flow downstream of the venturi:

1. Variation in the bubble void fraction in the flow.
2. The pressure pulse resulting from the collective bubble collapse.

3.1. *Effect of recovery pressure*

The conditions for simulation are given in caption of Fig. 2. Fig. 2 shows that the variation of bubble void fraction in the flow downstream of the throat of the venturi indicates that with an increase in the recovery pressure the distance that the bubble clusters travel from the point of inception (throat of the venturi) before collapse decreases. No significant variation is seen in the pressure pulse resulting from the collapse of the clusters with variation in the recovery pressure. The physical picture of this effect is clear: With increasing recovery pressure, the bubbles are subjected to increasing "annihilation" which results in the reduction in the life of the bubbles. This effect is similar to the effect of acoustic pressure amplitude on life of a single bubble in ultrasonic cavitation.

3.2. *Effect of venturi to pipe area ratio*

Simulations were performed for two different venturi to pipe area ratios (β) under otherwise similar conditions. The parameters for the simulations are listed in the caption of Fig. 3. It can be seen that the higher the value of β, the higher is the distance that the bubble clusters travel downstream of the throat of the venturi. This effect is a consequence of the different rates of pressure recovery downstream of the throat for different β values. A higher β value results in a faster pressure recovery downstream of the venturi. Therefore, the bubble clusters are subjected to a lesser rate of annihilation resulting in the increase in the life of an individual bubble and hence the distance that the bubble cluster travel downstream of the throat of the venturi.

3.3. *Effect of initial bubble radius*

The simulation conditions are given in the caption of Fig. 4. The sizes of the bubbles were chosen to cover the widest possible size distribution expected in the hydrodynamic flow situation (Yan & Thorpe, 1990). It is evident from the results that the clusters comprising of the bubbles of larger initial radius have higher life and hence the distance they travel from the throat of the venturi is also higher. But the pressure pulse arising out of the collapse of the clusters consisting of larger bubbles is of a smaller magnitude than in the case of the clusters comprising of the bubbles with smaller initial size. Therefore, it is evident that the contribution to the cavitational

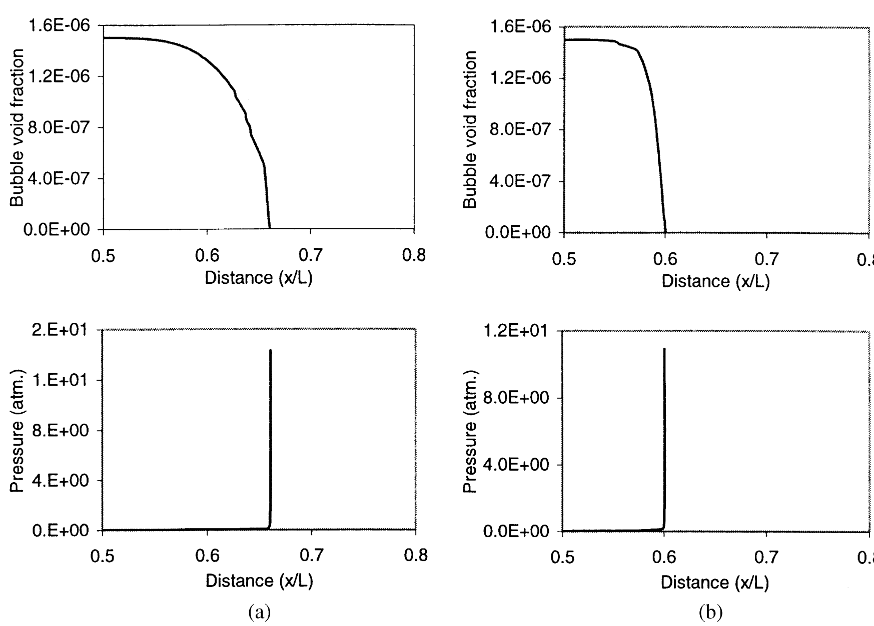

Fig. 2. Variation in the bubble void fraction and pressure in the bubbly flow: effect of recovery pressure downstream of the venturi: (A) $P_2 = 1$ atm, (B) $P_2 = 3$ atm; Other parameters for simulations: $\alpha_0 = 1.5 \times 10^{-6}$, $\beta = 0.71$, $R_0 = 100$ μm, $C_i = 1$.

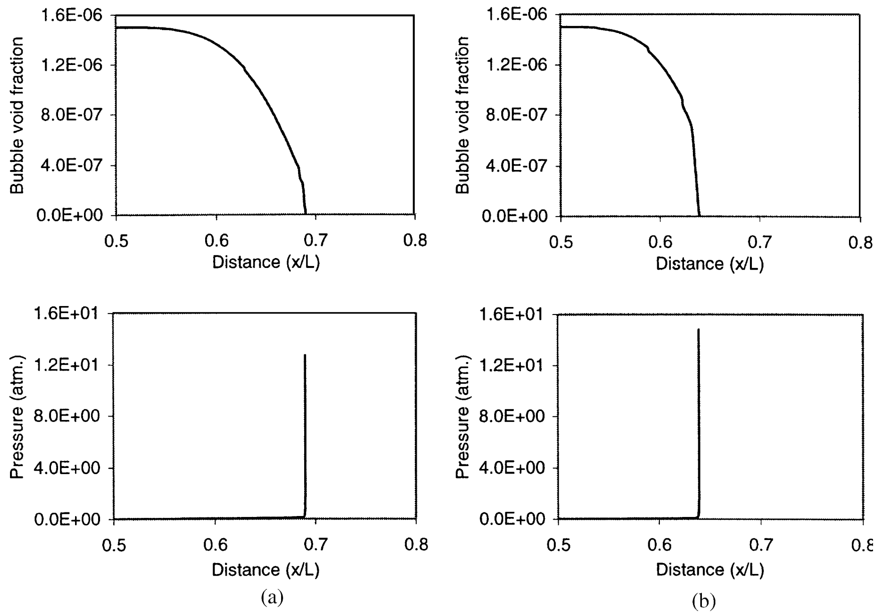

Fig. 3. Variation in the bubble void fraction and pressure in the bubbly flow: effect of venturi throat to pipe area ratio: (A) $\beta = 0.82$, (B) $\beta = 0.58$; Other parameters for simulations: $P_2 = 1$ atm, $\alpha_0 = 1.5 \times 10^{-6}$, $R_0 = 100$ μm, $C_i = 1$.

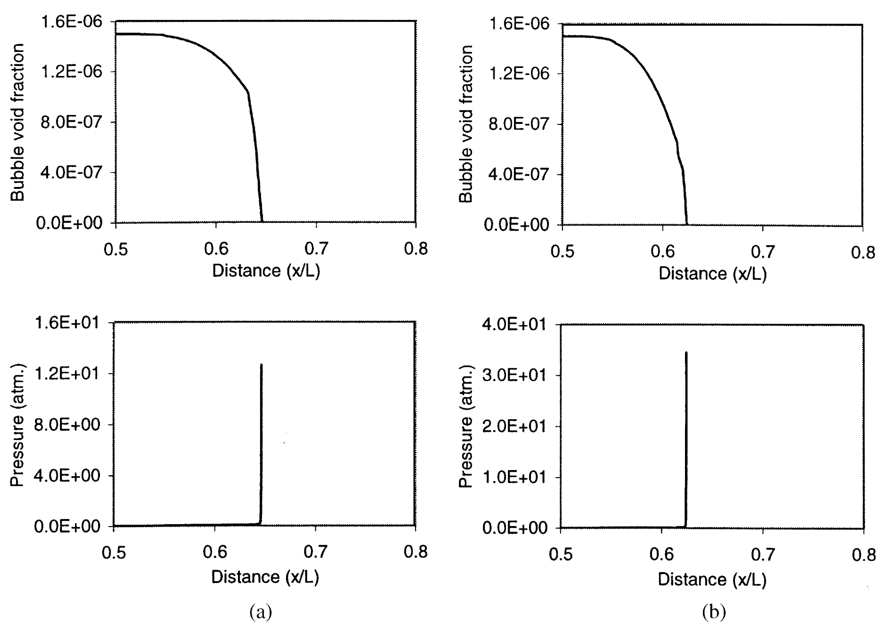

Fig. 4. Variation in the bubble void fraction and pressure in the bubbly flow: effect of initial bubble size: (A) $R_0 = 100$ μm, (B) $R_0 = 50$ μm; Other parameters for simulations: $P_2 = 1.5$ atm, $\alpha_0 = 1.5 \times 10^{-6}$, $\beta = 0.75$, $C_i = 1$.

effect is higher due to the clusters comprising of the bubbles of smaller initial size.

3.4. *Effect of the initial bubble void fraction*

For a fixed downstream recovery pressure and venturi to pipe area ratio simulations were carried out for two different values of initial bubble void fraction. The conditions for the simulation are given in the caption of Figs. 5 and 6. It can be inferred from Figs. 5 and 6 that unlike other parameters the trend observed in the effect of initial bubble void fraction on the flow behavior is not independent of the other flow conditions. Simulations shown in Fig. 5 reveal that the bubble clusters in the flow with smaller initial void fraction travel larger distance downstream of the venturi before collapse. This result is

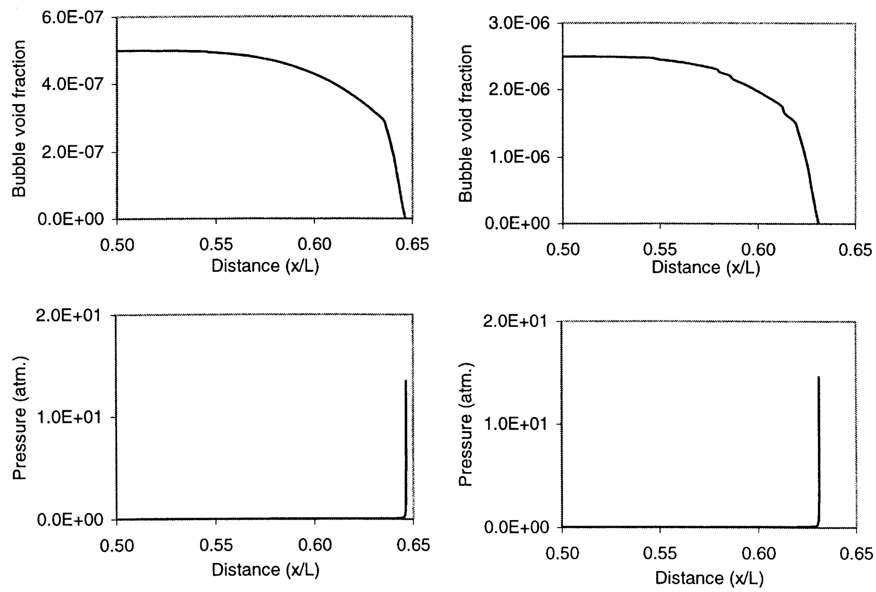

Fig. 5. Variation in the bubble void fraction and pressure in the bubbly flow: effect of initial bubble void fraction in the flow: (A) $\alpha_0 = 0.5 \times 10^{-6}$, (B) $\alpha_0 = 2.5 \times 10^{-6}$; Other parameters for simulations: $P_2 = 1.5$ atm, $R_0 = 100$ μm, $\beta = 0.67$, $C_i = 0.9$.

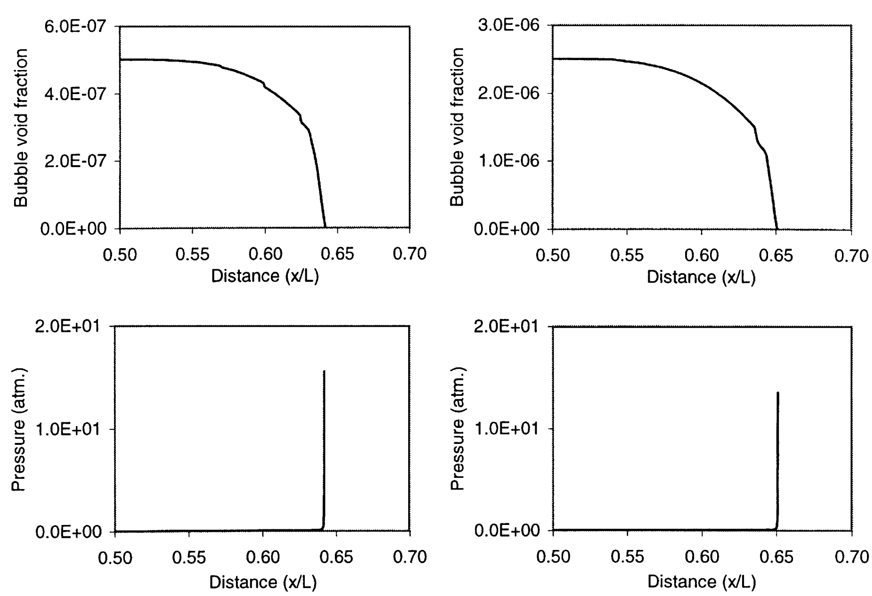

Fig. 6. Variation in the bubble void fraction and pressure in the bubbly flow: effect of initial bubble void fraction in the flow: (A) $\alpha_0 = 0.5 \times 10^{-6}$, (B) $\alpha_0 = 2.5 \times 10^{-6}$; Other parameters for simulations: $P_2 = 1$ atm, $R_0 = 100$ μm, $\beta = 0.71$, $C_i = 0.9$.

contradicted by simulations shown in Fig. 6 for different set of values of P_2 and β, where an opposite trend is observed. As mentioned earlier the two mechanisms that influence the dynamics of an individual bubble (and hence the bubble cluster as a whole) are bubble/bubble interaction and bubble/flow interaction. The simulations reported earlier to assess the effect of the parameters P_2 and β on the behavior of bubbly flow shows that these parameters manifest their effect through bubble/flow interaction. The smaller the bubble void fraction in the flow, the lesser is the bubble/bubble interaction. But the bubble/flow interaction depends on P_2 and β. Therefore, in this case the dynamics of the bubble clusters is governed by the resultant driving mechanism arising out of the combination of bubble/bubble and bubble/flow interaction. As such the alteration in the flow behavior with the initial bubble void fraction in the flow does not show a well-defined trend independent on the other flow conditions.

4. Conclusion

This paper reports the numerical simulations of a steady cavitating flow through a venturi using the continuum model of the two-phase flow. The nonlinear bubble dynamics has been coupled to the equations of motion of the flow to assess the effect of the bubble content on the flow and the bubble/bubble interaction. Despite of several assumptions involved in the model and exclusion of the non-equilibrium factors such as thermal equilibrium between the phases and the variation in the number density and size distribution of the bubbles in the flow, the results of the simulations represent some interesting physical phenomenon. It is found that the manipulations of the flow and the geometrical parameters can offer good control over the cavitational effect. The parameters studied in this study were found to affect mainly two aspects of the cavitating flow:

(a) The distance that the bubble clusters travel from the point of inception, which is the throat of the venturi.
(b) The pressure pulse generated out of the collapse of the clusters.

In hydrodynamic cavitation the bubbles formed at the throat of the venturi travel along with the flow. Therefore the higher the distance that the bubbles travel downstream of the throat, the higher is the "*active* " volume of the hydrodynamic cavitation reactor where the cavitational effect will be observed. A typical hydrodynamic cavitation reactor design is shown in Fig. 7 (Pandit & Moholkar, 1996). The results of the simulations presented in this paper provide an idea about the design and optimization of the hydrodynamic cavitation reactor for maximising the cavitational effect. Following recommendations can be made on the basis of the simulations provided in this paper.

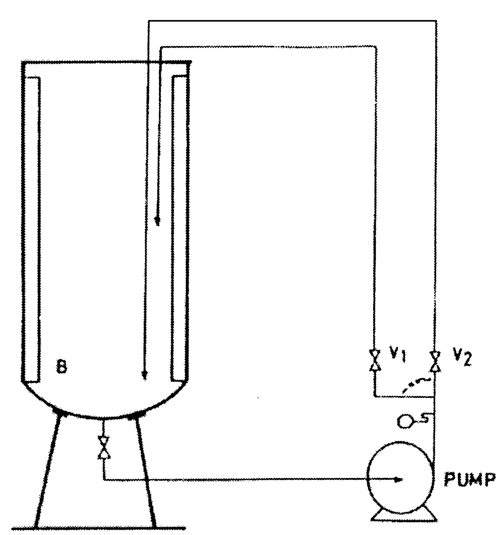

Fig. 7. Typical set-up for hydrodynamic cavitation reactor.

(1) A rise in the discharge pressure and hence the final recovery pressure results in reduced bubble cluster life and hence a decrease in the "*active*" volume downstream of the throat of the venturi where the cavitational effects can be observed. Though the cavitation intensity in this reduced volume represented by the magnitude of the pressure pulse resulting from the collapse of bubble cluster is higher. This phenomenon can be used for more stubborn reactions.
(2) Manipulation of the venturi to pipe area ratio offers a good control over the cavitational activity. An increase in the venturi to pipe area ratio increases the "*active*" volume of the hydrodynamic cavitation reactor.
(3) The contribution from the bubbles with smaller initial sizes to the overall cavitational effect higher than the contribution from the bubbles with higher initial sizes. Although there is a wide variation in the initial sizes of the bubbles that are generated downstream of the throat of the venturi, a partial control over the initial bubble size can be obtained by the exposure of the flow at the throat of the venturi to ultrasonic irradiation (Chatterjee & Arakeri, 1997).

Notation

A	area of the venturi, m^2
C_i	cavitation number, dimensionless
k	polytropic constant of the bubble contents
L	length of the venturi, m.
n	number density of bubbles in the liquid, m^{-3}
p	average pressure in the gas–liquid flow, $N\,m^{-2}$
P_∞	pressure in the bulk liquid far from bubble, $N\,m^{-2}$
P_2	recovery pressure downstream of venturi, $N\,m^{-2}$
P_i	pressure inside the bubble, $N\,m^{-2}$
P_0	ambient pressure in the liquid at the point of inception of generation of the bubbles, $N\,m^{-2}$
P_v	vapor pressure of the liquid, $N\,m^{-2}$
R	radius of the bubble, m
R_0	initial bubble radius, m
u	fluid velocity, $m\,s^{-1}$
u_0	velocity at the throat of the venturi, $m\,s^{-1}$
V	volume of an individual bubble, m^3

Greek letters

α	bubble void fraction of the gas–liquid mixture
α_0	initial bubble void fraction of the gas-liquid mixture (at the throat of the venturi)
β	throat of venturi to pipe area ratio, dimensionless
κ	thermal diffusivity, $m^2\ s^{-1}$
μ	viscosity of the liquid, Pa s
ρ	density of the gas–liquid mixture, $kg\ m^{-3}$
ρ_L	density of pure liquid, $kg\ m^{-3}$
σ	surface tension of the liquid, $N\ m^{-1}$
ω	angular frequency of the driving pressure, s^{-1}

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