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MATHEMATICAL MODELLING OF FORGING OF SINTERED PREFORM: COMPARATIVE STUDY OF OPEN AND CLOSED DIE

Parveen Kumar[1 §], R.K. Ranjan[2], Rajive Kumar[3]
[1]Department of Mathematics
BRCM, College of Engineering and Technology
Bahal, Bhiwani, INDIA
[2]Department of ME
Gyan Ganga ITS, Jabalpur, INDIA
[3]Department of Mathematics
DCRUST, Murthal, INDIA

Abstract: This paper reports the investigation of the various aspects of forging of sintered square powder performs under open die and as well as closed die forming process. The deformation of sintered square disc for open and closed die forging has been discussed simultaneously and a Mathematical model is drafted considering the various decisive factors like friction, relative density, geometry and other forging parameters. An attempt has been tried for the determination of the die pressures developed of the disc during forging by using an upper bound approach.

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1. Introduction

We are well aware that metal-powder components are being used successfully with better mechanical properties than those of wrought materials [1].Processing

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§Correspondence author

of metal powder performs is a convenient method of reducing the porosity as compare to the conventional Methods. Sintered powder performs are used in metal forming processes and has wide application in different areas [2]. Although a considerable amount of work has been reported [3], no systematic attempt has been made so far to develop the Mathematical model to study the deforming load and different characteristics during the processing of Square disc in a open and closed die together.

The flow pattern in the closed die forging is quite different than that of open die [4, 5, 6] due to constraints. Since the flow pattern goes on changing at different reductions. The load is calculated when the die is nearly full as the load is maximum at complete filling of die, as in the case of closed die case very systematic approach is required and we have to be much careful for the calculation of the die load.

In the case of sintered powder metal preforms, change in volume occurs due to porosity. Density is also not uniform throughout as it is high in the central region and low at the edges [7]. Upper bound method approach is the suitable approach for the drafting of mathematical model[8].

2. Mathematical Model

Open Die

$$\begin{aligned} P = \Bigg[1 - \frac{N(1-2\eta)\mu A \cot\alpha}{3(1+\eta)h} \Bigg\{ \Bigg(F(\alpha) + \frac{K}{2} f(\alpha) \Bigg) \\ \times \Bigg(1 + x \Bigg| 1 - \frac{r_m}{n \sec(\frac{\pi}{N})} \Bigg| \Bigg) + \frac{3x}{4n \sec(\pi/N)} \\ \times \Bigg(\frac{1}{2} \Bigg(\tan\alpha + \frac{\tan^3\alpha}{3} \Bigg) + \frac{K}{2} f_1(\alpha) \Bigg) \Bigg\} \Bigg]^{-1} \\ \times A \Bigg[\sqrt{\frac{2}{3}} \sigma_0^* N A^2 \tan\alpha \Bigg\{ \frac{(1-2\eta)^2}{2(1+\eta)^2} \Bigg(1 + \frac{K^2 N^2}{4} \Bigg) + 1 \Bigg\} \Bigg], \end{aligned}$$

$p = \dfrac{P}{NA^2 \tan\alpha}$, where $\rho_0 \phi_0 = xp$, $x = 0.1, 0.2, 0.3, \ldots$;

$A = b$ (Half of the side of square), $\alpha = \dfrac{\pi}{4}$; $f(\alpha) = 0.09680$;

$f_1(\alpha) = -1.33333$; $F(\alpha) = 0.5738$; $N = 4$ (no. of sides); $\dfrac{\partial P}{\partial K} = 0$

$$W_i = \frac{2}{\sqrt{3}} \sigma_0^* \int_v \left[\frac{1}{2} (\varepsilon_r^{\bullet 2} + \varepsilon_\theta^{\bullet 2} + \varepsilon_z^{\bullet 2} + \varepsilon_{r\theta}^{\bullet 2}) \right]^{\frac{1}{2}} dV;$$

Yield stress is $\sigma_0^* = \dfrac{\rho^k \sigma_0}{(1-2\eta)}$

$$W_i = \sqrt{2/3}\sigma_0^* \int_v \left[\frac{(1-2\eta)^2}{4(1+\eta)^2}\left\{\frac{2U^2}{h^2} + 2B^2\cos^2(N\theta) + \frac{N^2B^2\sin^2(N\theta)}{2}\right\} + \frac{U^2}{h^2}\right]^{\frac{1}{2}} dV,$$

$$W_i = \sqrt{2/3}\sigma_0^* \frac{U}{h}\int_v \left[\left\{\frac{(1-2\eta)^2}{4(1+\eta)^2}\right\}\left\{2 + 2K^2Cos^2(N\theta) + \frac{N^2K^2}{4} - \frac{(N^2K^2)\cos^2(N\theta)}{4}\right\} + 1\right]^{\frac{1}{2}} dV,$$

$$W_i = \sqrt{2/3}\sigma_0^* \frac{U}{h}\int_v \left[\frac{(1-2\eta)^2}{2(1+\eta)^2}\left\{1 + \frac{K^2N^2}{4} \times \left\{1 + \frac{\cos^2(N\theta) - (N^2/4)\cos^2(N\theta)}{(N^2/4)}\right\}\right\} + 1\right]^{\frac{1}{2}} dV$$

$\dfrac{\cos^2(N\theta) - (N^2/4)\cos^2(N\theta)}{N^2/4}$ is very very small and $B = \dfrac{KU}{h}$

$$W_i = \sqrt{2/3}\sigma_0^* \frac{U}{h}\int_V \left[\frac{(1-2\eta)^2}{2(1+\eta)^2}\left\{1 + \frac{K^2N^2}{4}\right\} + 1\right]^{\frac{1}{2}} dv;$$

$$V = NA^2h\tan\alpha,$$

$$W_i = \sqrt{2/3}\sigma_0^* UNA^2\left[\frac{(1-2\eta)^2}{2(1+\eta)^2}\left\{1 + \frac{K^2N^2}{4}\right\} + 1\right]^{\frac{1}{2}} \tan\alpha,$$

$$W_f = \int_s \tau|\Delta v|ds;\ \ \Delta v = (|U_r| + U_\theta|);$$

$$\tau = \mu\left[p + \rho_0\phi_0\left\{1 - \frac{r_m - r}{nA\sec(\pi/\mathrm{N})}\right\}\right],$$

$$W_f = \frac{N(1-2\eta)\mu UA^3}{3(1+\eta)h}\left[\left\{F(\alpha) + \frac{K}{2}f(\alpha)\right\} \times \left\{p + \rho_0\phi_0\left(1 - \frac{r_m}{4n\sec(\pi/N)}\right)\right\} + \frac{3\rho_0\phi_0}{4n\sec(\pi/N)} \times \left\{\frac{1}{2}\left(\tan\alpha + \frac{\tan^3\alpha}{3}\right) + \frac{K}{2}f_1(\alpha)\right\}\right],$$

$$F(\alpha) = \frac{1}{4}[\log(\tan\alpha + \sec\alpha) + \tan\alpha\sec\alpha];$$

$$f_1(\alpha) = \int_0^{\alpha} \left[\cos(N\theta) + \frac{2}{N}\sin(N\theta)\right] \sec^4\theta d\theta$$

and

$$f(\alpha) = \int_0^{\alpha} [\cos(N\theta) + \frac{2}{N}\sin(N\theta)] \sec^3\theta d\theta \ ; \quad \alpha = \frac{\pi}{4}.$$

Closed Die

$$\frac{p_{av}}{\sigma_0} = \Bigg[1 - \frac{(1-2\eta)\mu b}{(1+\eta)}\Bigg\{\frac{\{1 + x(1 - \frac{r_m}{\sec(\pi/N)})\}}{\tan(2\pi/N)h}$$
$$\times \left\{\frac{1}{3}|\tan\theta\sec\theta + \ln(\sec\theta + \tan\theta)| - \left|\frac{0.1\tan\theta}{b}\right| + |\theta|\frac{0.001}{2b}\right\}$$
$$+ \frac{x}{nA\sec(\pi/N)\tan(2\pi/N)}\frac{b}{4}$$
$$\times \left\{\tan\theta + \frac{\tan^3_\theta}{3} - \frac{0.02\tan\theta}{b^2} + \frac{0.0001_\theta}{b^4}\right\}\Bigg\}_0^{2\frac{\pi}{N}}$$
$$- \frac{\mu}{\tan(2\pi/N)b}(1+x)h\Bigg]^{-1} \times \sqrt{\frac{2}{3}}\frac{\rho^K F}{2(1+\eta)\tan(2\pi/N)},$$

$$p = \Bigg[1 - \frac{(1-2\eta)\mu b}{(1+\eta)}\Bigg\{\frac{\{1 + x(1 - \frac{r_m}{\sec(\pi/N)})\}}{\tan(2\pi/N)h}$$
$$\times \left\{\frac{1}{3}|\tan\theta\sec\theta + \ln(\sec\theta + \tan\theta)| - \left|\frac{0.1\tan\theta}{b}\right| + |\theta|\frac{0.001}{2b}\right\}$$
$$+ \frac{x}{nA\sec(\pi/N)\tan(2\pi/N)}\frac{b}{4}$$
$$\times \left\{\tan\theta + \frac{\tan^3_\theta}{3} - \frac{0.02\tan\theta}{b^2} + \frac{0.0001_\theta}{b^4}\right\}\Bigg\}^{2\frac{\pi}{N}}$$
$$- \frac{\mu}{\tan(2\pi/N)b}(1+x)h\Bigg]^{-1} \times \sqrt{\frac{2}{3}}\frac{N\rho^K F\sigma_0 A^2}{2(1+\eta)\tan(2\pi/N)}$$

External power j^* supplied by the platens is

$$J^* = \int F_i U_i ds = NPU = Np_{av}Ub^2\tan(2\pi/N) = W_i + W_{f1} + W_{f2}$$

where N = number of sides = 4.

W_{f1} = Rate of energy dissipation at the bottom and top of the preform

W_{f2} = Rate of energy dissipation at four faces of the preform.

$$W_i = \frac{2}{\sqrt{3}}\sigma_0^* \int_v \left[\frac{1}{2}(\overset{\bullet}{\varepsilon}{}_r^2 + \overset{\bullet}{\varepsilon}{}_\theta^2 + \overset{\bullet}{\varepsilon}{}_z^2 + \overset{\bullet}{\varepsilon}{}_{r\theta}^2)\right]^{\frac{1}{2}} dV;$$

$$\sigma_0^* = \frac{\rho^k \sigma_0}{(1-2\eta)}; \quad dv = hrd\theta dr,$$

$$W_i = N\frac{4\rho^k\sigma_0 UC}{\sqrt{3}\sqrt{2}(1-2\eta)}\left| \int_0^{\frac{2\pi}{N}} \left[\sec^3\theta \ln r \left(1-\frac{\cos^2\theta}{2}\right)\right]^{\frac{1}{2}}\right|_{0.1}^{\frac{b}{\cos\theta}} d\theta;$$

$$C = \frac{(1-2\eta)}{2(1+\eta)}$$

Lower limit for r is taken as $r_0 = 0.1$. There is little difference in the value of W_i

For $r_0 = 0.1$ to $r_0 = 0.75$; $W_i = N\sqrt{\frac{2}{3}}\frac{\rho^k\sigma_0 UCb^2}{(1-2\eta)}F$, where

$$F = \left[\left(\ln\frac{b}{\cos\theta} - \ln 0.1\right)\sqrt{1-\frac{\cos^2\theta}{2}}\{\tan\theta\sec\theta + \ln(\sec\theta+\tan\theta)\}\right]_0^{\frac{2\pi}{N}}$$

For Square shape: $W_i = 4\sqrt{\frac{2}{3}}\frac{\rho^k\sigma_0 UCb^2}{(1-2\eta)}F$, where

$$F = \left[\left(\ln\frac{b}{\cos\theta} - \ln 0.1\right)\sqrt{1-\frac{\cos^2\theta}{2}}\{\tan\theta\sec\theta + \ln(\sec\theta+\tan\theta)\}\right]_0^{\frac{2\pi}{N}}$$

$$W_{f1} = \int_S \tau|\Delta v| ds;$$

$$\tau = \mu\left[p + \rho_0\phi_0\left(1-\frac{r_m - r}{nA\sec(\pi/N)}\right)\right]|\Delta v| = \frac{(1-2\eta)U}{2(1+\eta)rh}\left(\frac{b^2}{\cos^2\theta} - r^2\right)$$

$$W_{f1} = 2N\frac{U(1-2\eta)}{2(1+\eta)h}\int_0^{\frac{2\pi}{N}}\int_{0.1}^{\frac{R_0}{\cos\theta}} \mu\left[p+\rho_0\phi_0\left(1-\frac{r_m - r}{nA\sec(\pi/N)}\right)\right] \times (b^2\sec^2\theta - r^2)d\theta dr$$

$$W_{f1} = N\frac{U(1-2\eta)\mu\{p+\rho_0\phi_0(1-\frac{r_m}{\sec(\pi/N)})\}b^3}{(1+\eta)h} \times \frac{1}{3}|\tan\theta\sec\theta + \ln(\sec\theta+\tan\theta)|_0^{\frac{2\pi}{N}} - \left|\frac{0.1\tan\theta}{b}\right|_0^{\frac{2\pi}{N}} - |\theta|_0^{\frac{2\pi}{N}}\frac{0.001}{3b^3}\Bigg]$$

$$+ N\frac{U(1-2\eta)b^4}{4(1+\eta)h}\frac{\mu\rho_0\phi_0}{nA\sec(\pi/N)}$$
$$\times\left[\tan\theta+\frac{\tan^3\theta}{3}-\frac{0.01\times 2b^2\tan\theta}{b^2}+\frac{0.0001\theta}{b^4}\right]_0^{\frac{2\pi}{N}},$$
$$W_{f1}=N\frac{U(1-2\eta)\mu}{(1+\eta)h}\left[\left\{p+\rho_0\phi\left(1-\frac{r_m}{\sec(\pi/N)}\right)\right\}\right.$$
$$\times b^3\left\{\frac{1}{3}|\tan\theta\sec\theta+\ln(\sec\theta+\tan\theta)|-\left|\frac{0.1\tan\theta}{b}\right|-|\theta|\frac{0.001}{3b^3}\right\}$$
$$\left.+\frac{\rho_0\phi_0}{nA\sec(\pi/N)}\frac{b^4}{4}\left\{\tan\theta+\frac{\tan^3_\theta}{3}-\frac{0.01\times 2\tan\theta}{b^2}+\frac{0.0001\theta}{b^4}\right\}\right]_0^{\frac{2\pi}{N}},$$
$$W_{f1}=4\frac{U(1-2\eta)\mu}{(1+\eta)h}\left[\left\{p+\rho_0\phi_0\left(1-\frac{r_m}{\sec(\pi/N)}\right)\right\}\right.$$
$$\times b^3\left\{\frac{1}{3}|\tan\theta\sec\theta+\ln(\sec\theta+\tan\theta)|\right.$$
$$\left.-\left|\frac{0.1\tan\theta}{b}\right|-|\theta|\frac{0.001}{3b^3}\right\}-\frac{\rho_0\phi_0}{nA\sec(\pi/N)}\frac{b^4}{4}$$
$$\left.\times\left\{\tan\theta+\frac{\tan^3\theta}{3}-\frac{0.01\times\tan\theta}{b^2}+\frac{0.0001\theta}{b^4}\right\}\right]_0^{\frac{2\pi}{N}},$$
$$W_{f2}=N\int_0^h\tau|U_z|2bdz$$

Very little possibility of sticking phenomenon at flat surfaces.

$$W_{f2}=\frac{2NU\mu b}{h}[(p+\rho_0\phi_0)|z|_0^h];\ W_{f2}=\frac{2NU\mu b}{h}[(p+\rho_0\phi_0)\frac{h^2}{2}]$$

For Square shape: $W_{f2}=\frac{8U\mu b}{h}\left[(p+\rho_0\phi_0)\frac{h^2}{2}\right]$

The velocity fields and strain rate in the case of open and closed die are

3. Result and Discussion

Tabata and Masaki (1977) proposed yield criterion for porous metal

$$\rho^k=\sqrt{3J_2'}\pm 3\eta\sigma_m$$

$\eta=0.54(1-\rho)^{1.2}$ for $\sigma_m\le 0$; $\eta=0.55(1-\rho)^{0.83}$ for $\sigma_m<0$ and $k=2$.

Open die	Closed die
$U_r = \frac{(1-2\eta)}{2(1+\eta)}\left[\frac{U_r}{h} + rB\cos(N\theta)\right]$ $U_\theta = -\frac{(1-2\eta)}{2(1+\eta)}\left[\frac{2rB\sin(N\theta)}{N}\right]$ $U_z = -\frac{zU}{h}$ $\varepsilon_r^\bullet = \frac{(1-2\eta)}{2(1+\eta)}\left[\frac{U}{h} + B\cos(N\theta)\right]$ $\varepsilon_\theta^\bullet = \frac{(1-2\eta)}{2(1+\eta)}\left[\frac{U}{h} + B\cos(N\theta)\right]$ $\varepsilon_Z^\bullet = -\frac{U}{h}$ $\varepsilon_{r\theta}^\bullet = -\frac{(1-2\eta)}{2(1+\eta)}\frac{hB\sin(N\theta)}{2}$ The strain components satisfies the compressibility equation $\varepsilon_r^\bullet + \varepsilon_\theta^\bullet + \varepsilon_z^\bullet = \pm 2\eta(\varepsilon_z^\bullet - \varepsilon_r^\bullet)$ U denotes the velocity of the top die, the bottom die is assumed stationary 'B' is unknown constant	$U_\theta = 0$; $U_z = -\frac{U}{t}Z$ The normal-strain components satisfy the compressibility equation for porous materials $\varepsilon_r^\bullet + \varepsilon_\theta^\bullet + \frac{(1-2\eta)}{2(1+\eta)}\varepsilon_z^\bullet = 0$ $U_r = \frac{(1-2\eta)U}{2(1+\eta)rt}\left(\frac{R_0^2}{\cos^2\theta} - r^2\right)$ The velocity field satisfy the boundary conditions $U_r = 0$ at $r = R_0/\cos\theta$ At the top surface $U_Z = -U =$ velocity of ram The strain rate field areas: $\varepsilon_r^\bullet = \frac{(1-2\eta)U}{2(1+\eta)t}\left(\frac{R_0^2}{r^2\cos^2\theta} + 1\right)$ $\varepsilon_\theta^\bullet = -\frac{(1-2\eta)U}{2(1+\eta)t}\left(\frac{R_0^2}{r^2\cos^2\theta}\right) - 1)$ $\varepsilon_z^\bullet = -\frac{U}{t}$ $\varepsilon_{r\theta}^\bullet = -\frac{(1-2\eta)U}{2(1+\eta)t}\left(\frac{R_0^2}{r^2}\sec^2\theta\tan\theta\right)$

The yield criterion and compatibility equation are as follows

$$\sigma_1 = \frac{\rho^k\sigma_0}{(1-2\eta)} + \frac{(1+\eta)}{(1-2\eta)}\sigma_2 \; ; \quad \varepsilon_r = \frac{(2\eta-1)}{2(1+\eta)}\ln\frac{t_2}{t_1}$$

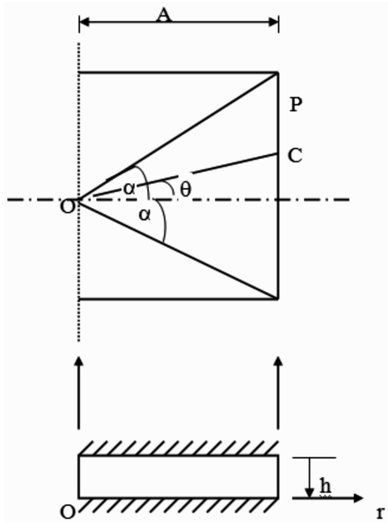
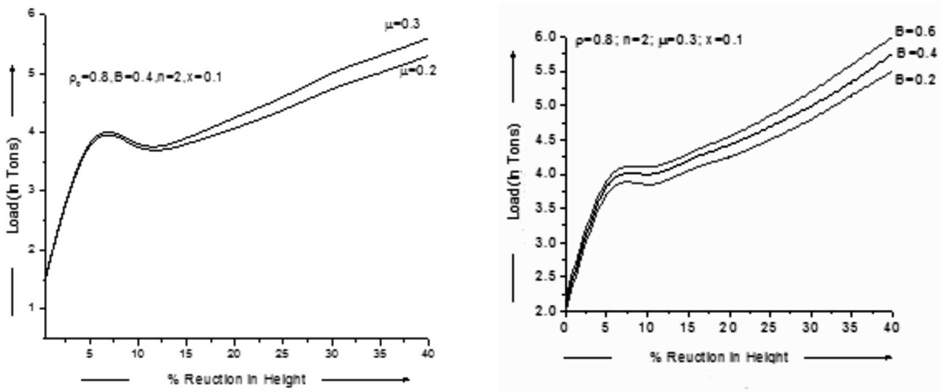


Figure 1

The material flow developed during the forging of square disc is not axisymmetric, but the complete disc can be divided into triangular regions with identical flow. Since the material flow in such region is identical, there is no shear strain or velocity disc continuity along the radial planes. We are considering non-uniformity of flow along thickness of the disc. Figure 1 shows the compaction arrangement of copper preform. Figure 2(a) and 2(b) shows the variation of Pressure load and % Reduction in the height of the preform in the case of Open die In this case, as shown in the figure the graph increases and after a particular reduction takes slight downwards direction and afterwards again increases. It is just due to the fact that filling of the pores takes place therefore slightly less pressure is required. Figure 3(a) and 3(b) suggests that in the case of closed die, more load is required. Since the flow pattern goes on changing at different reductions.The load is calculated when the die is nearly full as the load is maximum at complete filling of die.

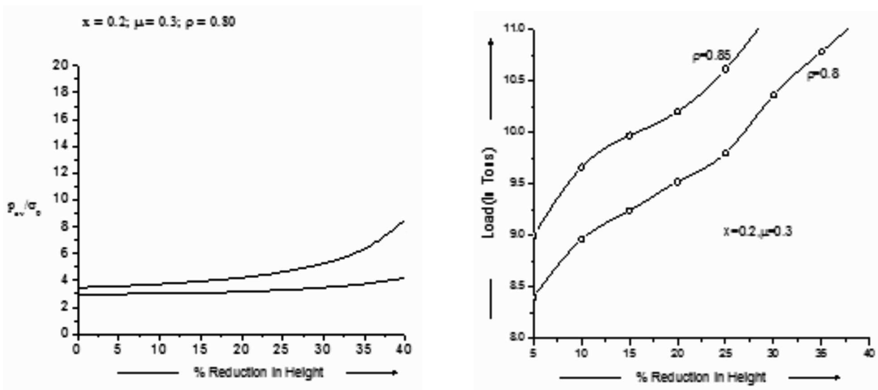
4. Conclusion

The deformation characteristics of the sintered metal powder preforms are much complex than wrought materials. It is sensitive to hydrostatic stress and instead of volume constancy; mass constancy is assumed during the deformation of the metal powder preform. The experimental and theoretical results agrees



(a) (b)

Figure 2



(a) (b)

Figure 3

satisfactorily, The P_{av}/σ_0 and the pressure load obtained theoretically agrees satisfactorily with the experimental results. Selection criteria of the velocity field for the particular shape and size of the preform is of great importance,as very careful investigation is required in the case of closed die. So it is expected that the result of this paper and better mathematical model will help the academician and researcher who are working in this field. It is expected that the present work will be of great importance for the assessment of die load. Results obtained considering the upper bound approach.

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